8/29/2016

1. Mathskeller and Study now open for tutoring — see Canvas announcement.
2. Webwork A1 and A2 due this week. WA#1 due Friday. See Canvas.
3. Quiz #1 Thursday. See Canvas.

Q: With neighbors: Why do we define $2^0 = 1$? Does this make sense? Why or why not?

Q: (a) If a population of bacteria doubles every hour and has a starting population of 162, how many bact. are there after 5 hours? 5.25 hours?

A: (a) Let $P(t)$ = pop. at time $t$.

Then $P(t) = 162 \cdot 2^t$.

So, $P(5) = 162 \cdot 2^5 = 5{,}184$ bact.

$P(5.25) = 162 \cdot 2^{5.25} =$

$162 \cdot 2^{5+1/4} = 162 \cdot 2^5 \cdot \sqrt[4]{2}$

$\approx 6164.84\ldots$ bact.

(b) Same setup: 162 bact, x2 every hour. When will the population reach 10,000 bact.?

$$P(t) = 162 \cdot 2^t.$$

Solve $10{,}000 = 162 \cdot 2^t$.

$$\Rightarrow \frac{10{,}000}{162} = 2^t$$

$$= \log_2\left(\frac{10{,}000}{162}\right) = t.$$

in hours.

$\simeq 5.94\ldots$ by WolframAlpha

**Review**: For any $b > 0$, set $f(x) = b^x$. For integer x-values, repeat multiplication.

e.g. $b^6 = b \cdot b \cdot b \cdot b \cdot b \cdot b$

If x is a fraction, repeat mult. & take a root.

e.g. $b^{67/10} = \sqrt[10]{b^{67}}$.

$$\underbrace{b^{67/10} \cdot b^{67/10} \cdots b^{67/10}}_{\text{10 times}} = b^{67}.$$

If $x$ is irrational, e.g. $x = \pi$,
we can only approximate $b^x$.

$\pi \approx 3.14$, so

$$b^{\pi} \approx b^{3.14} = b^{314/100} = \sqrt[100]{b^{314}}.$$

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Logs: If $f(x) = b^x$, then write

$f^{-1}(x) = \log_b(x)$. ← Mysterious function!

i.e. $y = b^x$ when $x = \log_b(y)$.

Read in book: Domains & Ranges for these, Laws of Exp's, Laws of logs.

# "Natural" Logs and e:

There is a "best" base b to use for exponentials & logs. We call it

$$e \approx 2.718\,28\ldots$$

← NOT a definition!

The reason e is awesome is

$$e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{3\cdot 2} + \frac{x^4}{4\cdot 3\cdot 2} + \frac{x^5}{5\cdot 4\cdot 3\cdot 2} + \ldots$$

Follow this pattern to get a polynomial w/ desired accuracy. (Calc II, + last day of Calc I).

We write $\log_e(x)$ as $\ln(x)$.

natural log.

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Here is an unsolved math problem worth $1 million. (Equivalent to Riemann Hypothesis).

Q: For every positive integer $n$, is

$$\left(\begin{array}{c}\text{Sum of all}\\ \text{divisors of } n\end{array}\right) \leq \left(1+\tfrac{1}{2}+\tfrac{1}{3}+\cdots+\tfrac{1}{n}\right) + \ln\left(1+\tfrac{1}{2}+\tfrac{1}{3}+\cdots+\tfrac{1}{n}\right) \cdot e^{\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}\right)}$$

?

$n = 6$

$$1+2+3+6 \leq \left(1+\frac{1}{2}+\cdots+\frac{1}{6}\right) + \ln\left(1+\cdots+\frac{1}{6}\right) e^{\left(1+\cdots+\frac{1}{6}\right)}$$

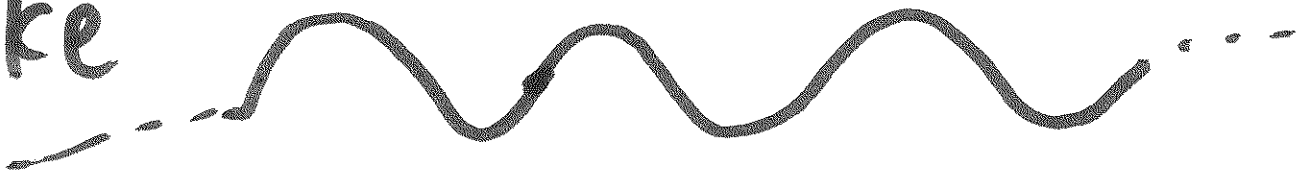
✓

8/31/2016

[1] Please review today's Canvas announcement regarding Exams.

[2] Discuss with your neighbors:

(a) What is the definition of $\sin\theta$?

(b) Why does the graph of $\sin\theta$ look like 

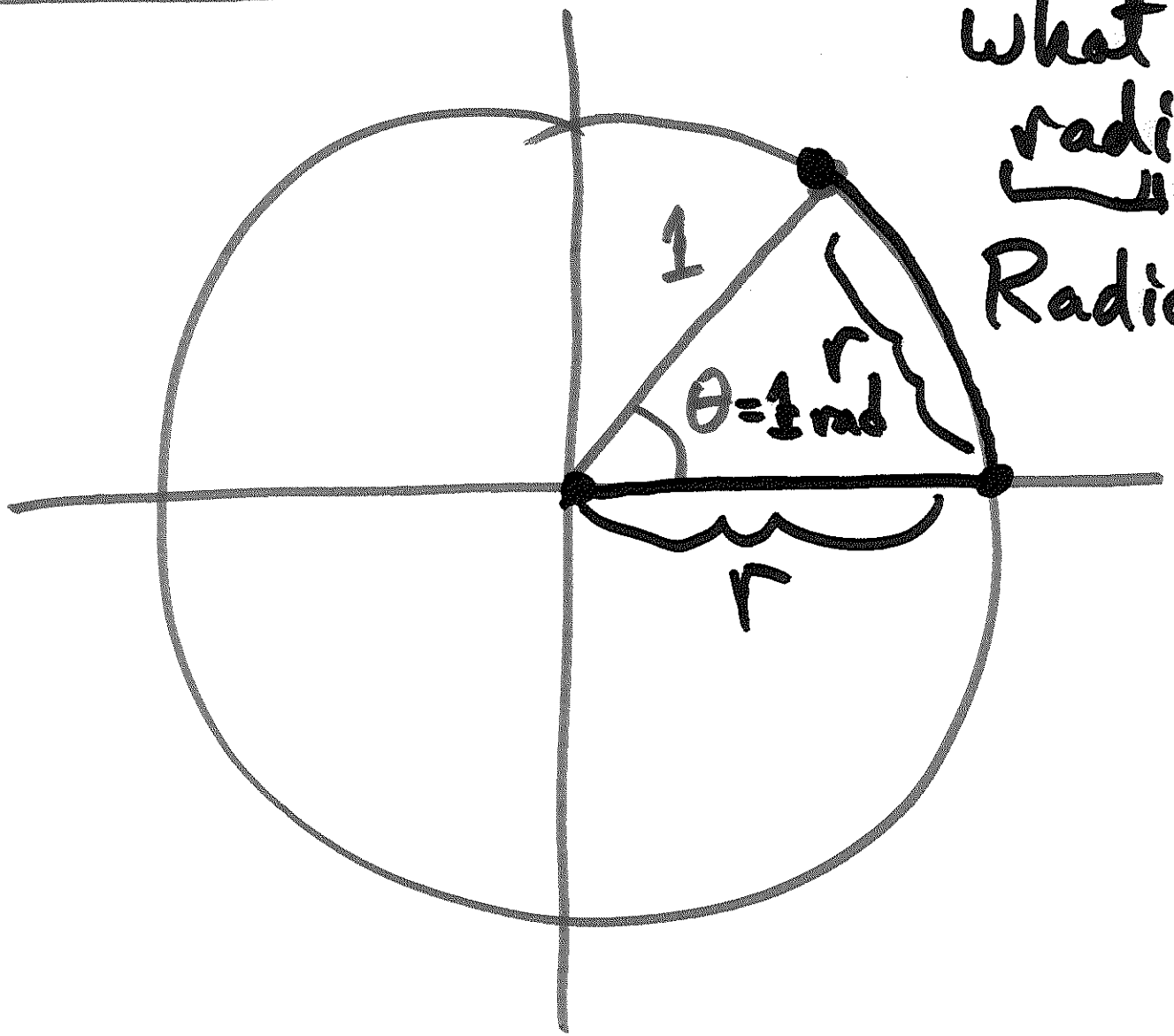
Def$^{n}$ of $\sin\theta$: Students say:

- Opp over Hyp
- On unit circle, the "y"
- Ratio of opp over Hyp
- Reciprocal of $\csc\theta$.
- $y/r$

Pictures

- triangle
- unit circle
- graph of $\sin\theta$

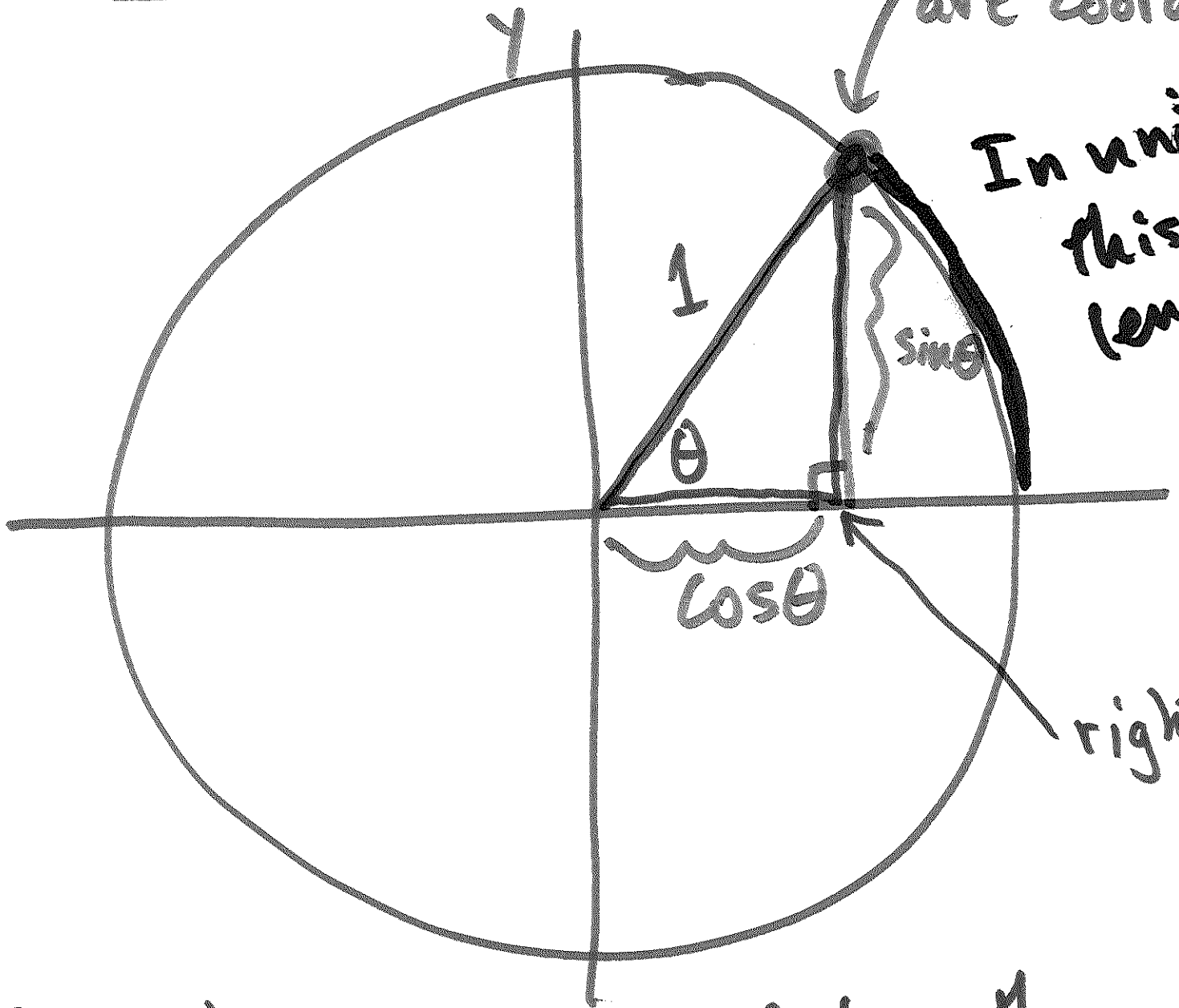
# Unit Circle:




What is a radian?

Radial Angle

# Unit Circle:




we say $(\cos\theta, \sin\theta)$ are coords of this pt.

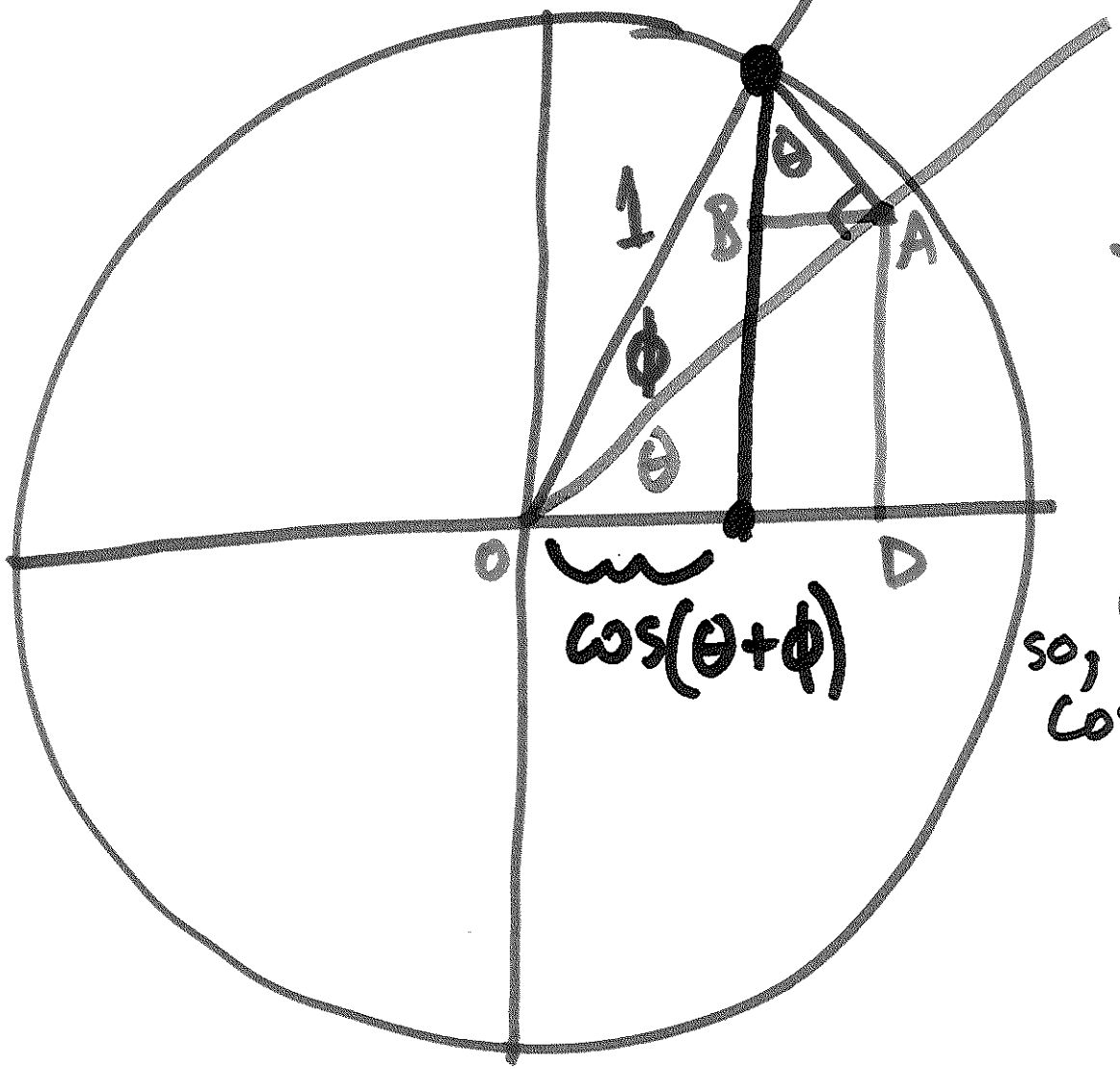
In unit circle, this arc length is $\theta$.

right triangle

**Q:** Why do $\sin\theta$ + $\cos\theta$ have the graphs they do?

# What does this tell us?

$$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$



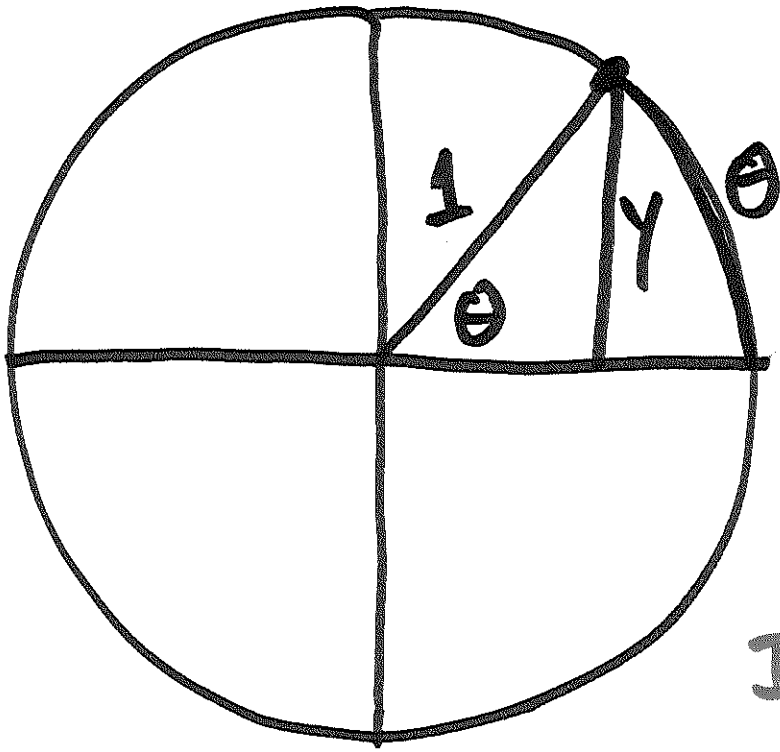

Using similar triangles, show

$BA = \sin\theta\sin\phi$

$OD = \cos\theta\cos\phi$

so, $\cos(\theta+\phi) = OD - BA$.

# Inverse Trig: Read §1.5; in class focus on $\sin\theta$.




$\theta$ = length of arc in a unit circle.

By def$^n$, $y = \sin\theta$.

$\theta$ = arc length $\longmapsto$ line length $y = \sin\theta$.

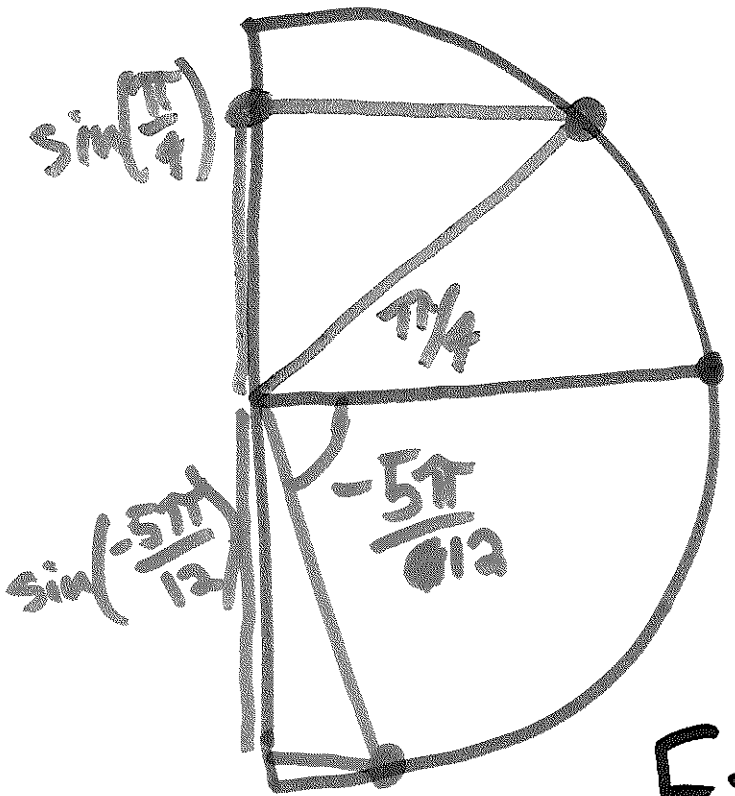
Invert to get

$y$ = line length $\longmapsto$ arc length is $\theta = \sin^{-1} y$ = $\arcsin y$.

**One Problem:**

$\sin\theta$ is not 1-1 on $(-\infty, \infty)$.

To ensure inverse of sin exists, restrict $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.




Thus, $y = \sin\theta$ must satisfy

$$-1 \leq y \leq 1.$$

- This is our standard choice of domain + range.

**Ex:** $\arcsin(\sin(\pi))$

$= \arcsin(0)$

$= 0.$