10/21/16 12PM

$\boxed{1}$ No curve on exam (Avg exam2 grade in my classes was 81)

$\boxed{2}$ Check Canvas Announcements.

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§4.1: Max + Min Values

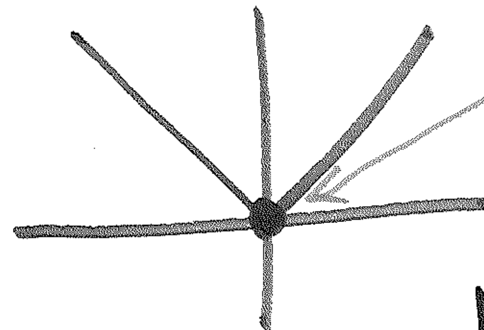
Ex: If velocity $v(x)$ of an object is given by $v(x) = 3x^4 - 16x^3 + 18x^2$.

Q: When is object moving in the negative direction most quickly?

See desmos, conclude

max + min values occur where derivative is zero!

<u>Ex</u>: $f(x) = |x|$.



min is $f(0) = 0$.

Max/Min can also occur where $f'(x)$ is not defined.

<u>Required def$\underline{{}^{n}}$s</u>: Say c is in domain of f.

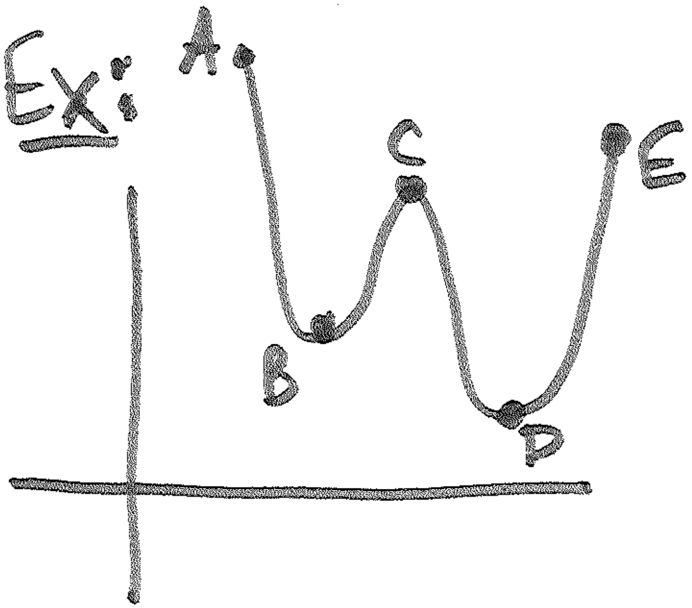
we say $f(c)$ is the

- <u>absolute max</u> if $f(c) \geq f(x)$ for all x in domain of f.
- <u>absolute min</u> if $f(c) \leq f(x)$ for all x in domain of f.

We say $f(c)$ is a

- **local max** if $f(c) \geq f(x)$ for all x near c.
- **local min** if $f(c) \leq f(x)$ for all x near c.

Ex:



A = max, abs & local
B = min, local
C = max, local
D = min, abs & local
E = max, local

Def$^{\underline{n}}$: A critical number of f is a number c in domain(f) such that either

$f'(c) = 0$ or $f'(c)$ not defined.

__Fermat's Thm:__ If f has a local max or a local min at c, then c is a critical number of f. __Pf:__ See §4.1.

__Extreme Value Thm:__ If f is cts on $[a,b]$, then f attains an abs. max and an abs. min on $[a,b]$.

Super-tough to justify fully! To see why this is true, take MA 471G.

__Rmk:__ First Thm says "Study f'"

Second Thm says "abs max + mins can be found".

Method for finding abs max + min values.

Assume f defined on $[a,b]$.

1. Solve $f'(x)=0$ and find c in domain(f) where f' not defined. This gives a set of points to check; evaluate f on each.

2. Find values of f at endpoints of $[a,b]$.

3. Pick the largest + least of the values from **1.** + **2.**. These are abs max + min of f.

Original Ex: Consider $v(x) = 3x^4 - 16x^3 + 18x^2$ on $[-1, 5]$.

1. Compute $v'(x) = 12x(x-1)(x-3)$.

power rule + factor.

So, $v'(x) = 0$ when $x = 0, 1, 3$.

Also, $v'(x)$ is defined for all x.

Evaluate $\underline{v(0) = 0}$, $\underline{v(1) = 5}$, $\underline{v(3) = -27}$.

2. Check $v(x)$ for $x = -1$ + 5, i.e. endpts.

$\underline{v(-1) = 37}$, $\underline{v(5) = 325}$.

3. Of these 5 values, min is -27, max is 325. So, abs min of $v(x)$ is -27.

<u>10/24/16</u> 12PM

$\boxed{1}$ See Canvas for announcement about Assgns this week.

<u>§4.2:</u> Mean Value Theorem (M.V.T.)

<u>Ex</u>: Say $f(x) = x^3 + 2x^2 - x$ on $[-1, 2]$.
To get from $(-1, f(-1)) = (-1, 2)$ to $(2, f(2)) = (2, 14)$ along secant line, we move w/ slope $\frac{f(2) - f(-1)}{2 - (-1)} = 4$.

NOTE: by inspection, at $x = \frac{-4+\sqrt{76}}{6}$)

we have slope of tangent line to f equal to 4. So,

Slope of secant line = slope of tangent line (in this case)...

M.V. Thm: Suppose f is

- cts on $[a,b]$

and

- diff on (a,b).

Then there is some value c in (a,b) where

$$\underbrace{\frac{f(b)-f(a)}{b-a}}_{\text{slope of secant line}} = \underbrace{f'(c)}_{\text{slope of tang. line}}.$$

Application #1:

Thm: If $f'(x)=0$ on (a,b), then f is constant on (a,b).

Suppose x and z in (a,b).

Q: How can I express using x and z that I want f to be constant?

We want to show $f(x)=f(z)$.

If this is true for every x + z, then f is constant.

Let's use M.V.T. to show this. Use $[x,z]$.

Note: f is diff on (a,b), so it is cts on (a,b).

Thus f is diff on (x,z) and cts $[x,z]$.

So, M.V.T. implies there is a "c" in (x,z) with

$$\frac{f(z)-f(x)}{z-x}=f'(c)=0.$$

So, $\dfrac{f(z)-f(x)}{z-x}=0$ implies $f(z)-f(x)=0$, thus $f(x)=f(z)$. ∎

<u>Corollary</u>: If $f'(x)=g'(x)$ on (a,b). Then $f=g+\text{constant}$ on (a,b).

<u>why?</u> Since $\frac{d}{dx}(f-g)=f'(x)-g'(x)=0$, then $f-g=\text{constant}$.

2 more applications:

Ex: Suppose $f(x)$ is cts on $[6, 15]$ + diff on $(6, 15)$. If $f(6) = -2$ and $f'(x) \leq 10$, what is the largest value that $f(15)$ might take?

By M.V.T.,

$$\frac{f(15) - f(6)}{15 - 6} = f'(c) \leq 10 \quad \text{for some } c \text{ in } (6, 15).$$

assumption in problem.

So, $\frac{f(15) - f(6)}{15 - 6} \leq 10 \Rightarrow f(15) - (-2) \leq 10(15 - 6)$

$\Rightarrow f(15) \leq 88$

So, 88 is largest possible value.

__Ex__: Show that $f(x) = 4x^5 + x^3 + 7x - 2$ has __exactly__ one real root.

__Step 1__: Use I.V.T. to show $f(x)$ has a real root.

Observe $f(0) = -2$ & $f(1) = 10$.

Since $-2 < 0 < 10$, and f is cts, by I.V.T. there is a soln to $f(x) = 0$.

__Step 2__: Use M.V.T. to show there are no other real roots. Suppose f has 2 real roots, call them a and b. Then $f(a) = 0 = f(b)$, so M.V.T. implies there is a c in (a,b) with $f'(c) = \frac{f(b)-f(a)}{b-a} = 0$.

__But__, $f'(x) = 20x^4 + 3x^2 + 7 \geq 7$ for all x.

This is impossible! f must have only one root.