10/21/16 9AM

$\boxed{1}$ No curve on exam. (Average for my sections was 81).

$\boxed{2}$ See Canvas announcements.

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§4.1: Max + Min Values.

Q: If velocity of an object is $v(x) = 3x^4 - 16x^3 + 18x^2$, when is object moving in the negative direction most quickly?

Look @ graph of $v(x)$.

<u>Note</u>: At points where $v(x)$ is "max" or "min" (locally at least), $v(x)$ has a horizontal tangent line.

Let's look at points where hor. tan. line happens, i.e. Find points where $v'(x)=0$.

$$v'(x) = 12x^3 - 48x^2 + 36x = 12x(x-3)(x-1).$$

So, $v'(x)=0$ when $x = 0, 1,$ or $3$.

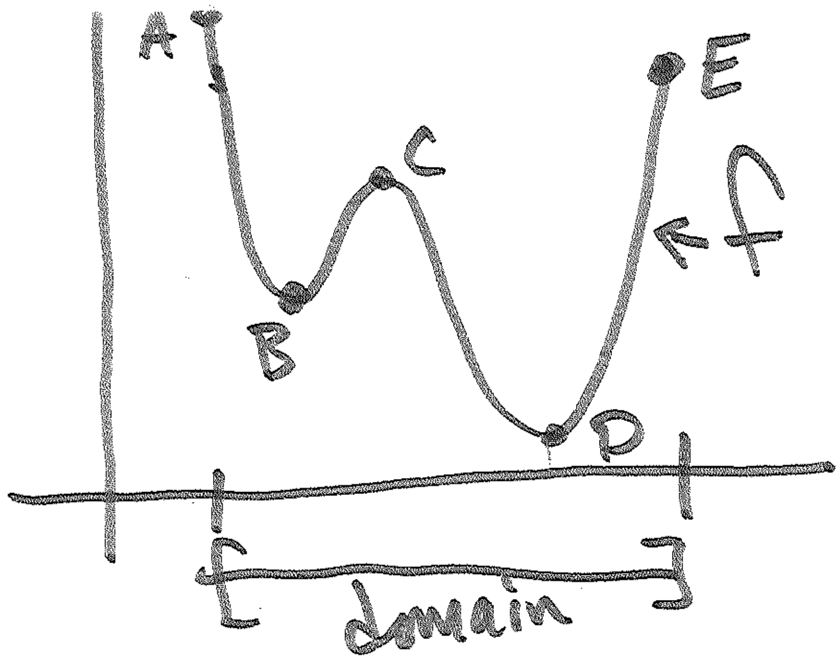
Def$^{\underline{n}}$'s to provide precise language:

Say $c$ is in domain of $f$. Then $f(c)$ is the

- absolute max of $f$ if $f(c) \geq f(x)$ for all $x$ in domain.
- absolute min of $f$ if $f(c) \leq f(x)$ for all $x$ in domain.

We have $f(c)$ is a

- local max of $f$ if $f(c) \geq f(x)$ for all $x$-values near $c$.
- local min of $f$ if $f(c) \leq f(x)$ for all $x$-values near $c$.




A = max, abs & local
B = min, local
C = max, local
D = min, abs & local
E = max, local.

<u>Def$^{\underline{n}}$</u>: A <u>critical number</u> of $f$ is a number $c$ in domain of $f$ such that either $f'(c)=0$ or $f'(c)$ does not exist.

<u>Ex</u>: $f'$ does not exist at A or E, & is zero at B, C, and D.

__Fermat's Thm:__ If $f$ has a local max or a local min at $c$, then $c$ is a critical number of $f$.

__Proof:__ See §4.1.

__Extreme Value Theorem:__ If $f$ is cts on $[a,b]$ then $f$ attains both an abs max __and__ an abs min on $[a,b]$.

__NOTE:__ Intuitively clear, very hard to fully justify as true. Take MA 471G to see why this works.

# Method for finding local + abs extreme values for $f$:

Say $f$ defined on $[a,b]$.

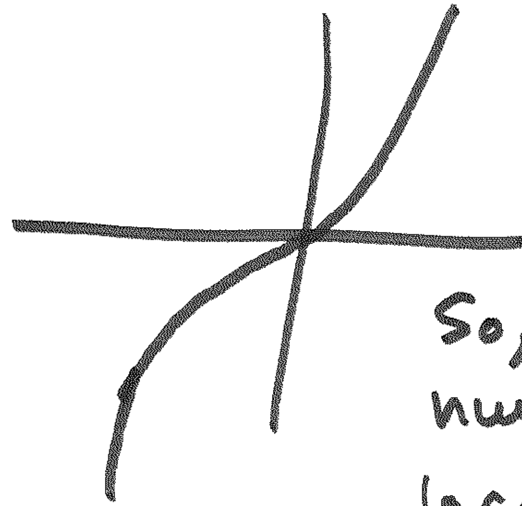
<u>1.</u> Use $f'$ to find critical values of $f$, check value of $f$ at those critical values.

<u>2.</u> Find value of $f$ at endpoints of $[a,b]$.

<u>3.</u> To find extreme values, pick largest + smallest values from <u>1.</u> + <u>2.</u>, i.e. abs max + abs min values.

<u>**NOTE**</u>: The critical values are candidates for local max/mins, but might not be! More work is needed.....

Ex: $f(x) = x^3$



$f'(x) = 3x^2$

$f'(0) = 0$

So, 0 is a critical number, but not a local min or max.

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Back to original example.

$v(x) = 3x^4 - 16x^3 + 18x^2$. Consider on $[-1, 5]$.

Find abs max + abs min values!

1. $v'(x) = 12x(x-3)(x-1)$, so critical numbers are $x = 0, 1, 3$.

Values are $v(0) = 0$, $v(1) = 5$, $v(3) = -27$

2. Check endpts.

$$v(-1)=37, \quad v(5)=325.$$

3. Pick largest + least value you computed.

$v(5)=325$ is abs. max.

$v(3)=-27$ is abs. min.

**10/24/16** **9AM**

$\boxed{1}$ See Canvas announcement for assignments this week

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**§4.2**: Mean Value Theorem

**Ex**: $f(x) = x^3 + 2x^2 - x$ on $[-1, 2]$.

**NOTE**: To get from the pt $(-1, f(-1)) = (-1, 2)$ to the pt $(2, f(2)) = (2, 14)$ along secant line, one must move w/ slope

$$\frac{f(2) - f(-1)}{2 - (-1)} = 4.$$

Observe that at $x = -\frac{4+\sqrt{76}}{6}$, we have $\underline{\text{slope of tangent line}}$ equals the $\underline{\text{slope of secant line}}$ btwn $x=-1$, + $x=2$.

(*) Mean Value Thm states this will always happen at least once on every interval.

<u>Mean Value Thm: (MVT)</u> Suppose $f$ is

- cts on $[a,b]$

and

- differentiable on $(a,b)$.

Then there is a value $c$ in $(a,b)$ where

$$\frac{f(b)-f(a)}{b-a} = f'(c).$$

slope of secant line

slope of tangent line.

**NOTE:** MVT does <u>not</u> say what $c$ <u>is</u>, only that it exists.

__Remarks:__

(A) When $f(a)=f(b)$, i.e. slope of secant line is $\frac{f(b)-f(a)}{b-a}=0$, then the MVT is called __Rolle's__ __Theorem__.

(B) How to prove this?

__Outline:__ __Step 1:__ Use Fermat's Thm + Extreme Value Thm to prove Rolle's Thm.

__Step 2:__ Use Rolle's Thm to prove full M.V.T.

For details, See §4.2, nicely written.

__Application:__

__Thm:__ If $f'(x)=0$ on $(a,b)$, then $f$ is constant on $(a,b)$.

- we want to use $f'(x)=0$ to say something about secant lines.

- For any values $x$ and $z$ in $(a,b)$, $f(x)=f(z)$.

want to show!

Suppose $x<z$ in $(a,b)$, so we can consider $[x,z]$. Since $f$ is diff on $(a,b)$, it is cts on $(a,b)$, hence cts on $[x,z]$. $f$ is also diff on $(x,z)$. By MVT, we have

$$\frac{f(z)-f(x)}{z-x} = f'(c) = 0.$$

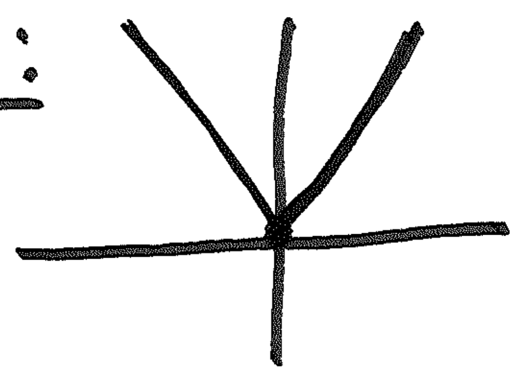
for some $c$ in $(x,z)$

So, $f(z)-f(x)=0$, thus, $f(z)=f(x)$. ■

why write full argument?

**False argument**: There is a value $c$ in $(-1,1)$ where $f'(c)=0$ for $f=|x|$.

**NOTE:**



"By MVT, $\frac{f(1)-f(-1)}{1-(-1)}=0$,

so $f'(c)=0$ somewhere"

is **wrong** since $f$ not diff on $(-1,1)$.

<u>Corollary (Pf in book):</u>

If $f'(x) = g'(x)$ on $(a,b)$, then $f = g + \text{constant}$ on $(a,b)$.

Key idea: $f-g$ has derivative $=0$ on $(a,b)$, so $f-g = \text{constant}$.

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<u>Application</u>: <u>Ex</u>: Suppose $f(x)$ is cts on $[6,15]$ and diff on $(6,15)$. If $f(6)=-2$ and $f'(x) \le 10$, what is largest possible value of $f(15)$?

Light bulb $\to$ use MVT! Based on

cts on $[6,15]$ + diff on $(6,15)$.

By MVT, There is a pt $c$ where

$$\frac{f(15)-f(6)}{15-6} = f'(c) \leq 10.$$

by problem given.

So, $f(15)-f(6) \leq 10(15-6)$,

thus $f(15)-(-2) \leq 10 \cdot 9$,

hence $f(15) \leq 88.$

So, 88 is largest poss. value of $f(15)$.

Ex: Show that $f(x) = 4x^5 + x^3 + 7x - 2$ has exactly one real root.

IVT and MVT.

Step 1: Show $f$ has at least one root.
Using $f(0) = -2$ + $f(1) = 10$, IVT implies a root exists in $(0,1)$.

Step 2: Use MVT to show this is only one.
Suppose $f$ has 2 real roots, i.e.
$f(a) = 0$ + $f(b) = 0$.

Apply MVT to get $0 = \frac{f(b) - f(a)}{b - a} = f'(c)$ ~~for some~~ c.

But, $f'(x) = 20x^4 + 3x^2 + 7 \geq 7$, so $f'(c)$ can't be zero!

This is impossible!