10/26/16   9AM

$\boxed{1}$ Webwork, Quiz, + WA#5 this week!

$\boxed{2}$ With your neighbors:

a) Find critical points of $f(x) = 3x^5 - 5x^3 + 3$

b) Find the critical points of $f'(x)$.

Today: §4.3, derivatives + shapes of graphs.

Builds on max/min discussion in §4.1, + on our MVT discussion in §4.2. Goal for today is to complete toolbox for local max/min pts.

Increasing / Decreasing test:

- If $f'(x) > 0$ on (a,b), then f is increasing on (a,b).
- If $f'(x) < 0$ on (a,b), then f is decreasing on (a,b).

First derivative Test: If c is a critical number of f, f is cts, then

(a) If f' changes from pos to neg at c, then $f(c)$ is a local max.

(b) If f' changes from neg. to pos. at c, then $f(c)$ is a local min.

(c) If f' does not change sign at c, $f(c)$ is neither a local max nor a local min.

→ Desmos Ex

Second Derivative info:

Def$^{\underline{n}}$: If the graph of f lies above all of its tangent lines on (a,b) (locally for each point in (a,b)), then f is called <u>concave up</u> on (a,b).

If graph similarly is below its tangent lines, call it concave down.

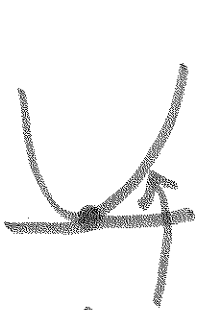
<u>Concavity Test</u>: (a) If $f''(x) > 0$ for all x in (a,b), then f is concave up on (a,b).

(b) If $f''(x) < 0$ for all x in (a,b), then f is concave down on (a,b).

Def$^{\underline{n}}$: A point on graph of $f(x)$ is an **inflection point** if f is cts at that point and f changes concavity across that point.

2nd Derivative Test: Suppose f'' is cts near c.

(a) If $f'(c)=0$ + $f''(c)>0$, then f has a local min at c.



$f''(c)>0$ means graph is above horiz. tang.

(b) If $f'(c)=0$ + $f''(c)<0$, then f has a local max at c.

$f''(c)<0$ means graph is below tang. line.

__Ex:__ For $h(x) = 3x^5 - 5x^3 + 3$, find all local max/min pts, intervals of inc/dec, and intervals of concavity.

__Step 1:__ Compute $h'(x) = 15x^4 - 15x^2$

$$= 15x^2(x^2 - 1)$$

$$= 15x^2(x+1)(x-1).$$

So, $h'(x)$ is defined at all x, and $h'(x) = 0$ for $x = 0, 1, -1$.

These are critical number x-values.

To use 2^{nd} der. test to check local max/mins, compute $h''(x) = 60x^3 - 30x = 30x(2x^2 - 1)$.

Apply 2nd der. test at $x=0,1,-1$.

$$h''(1) = 30\cdot 1\underbrace{(2\cdot 1^2-1)}_{=1} = 30 \quad > 0$$

$$h''(-1) = 30\cdot(-1)\cdot(2\cdot(-1)^2-1) = -30. < 0$$

$$h''(0) = 30\cdot 0\cdot(2\cdot 0^2-1) = 0.$$ ← issue.... can't use 2nd der. test here.... need 1st der. test here.

<u>Conclude:</u> h has a local min at 1, h has a local max at -1 } by 2nd der. test.

Let's check sign of $h'(x)$ on intervals.

sign of $h'(x)$	$+$		$-$		$-$		$+$
	$h'(-2)=180$	-1	$h'(-\frac{1}{2})=-2.8$	0	$h'(\frac{1}{2})=-2.8$	1	$h'(2)=180$

Since h' does not change sign at 0,
so h has no local max/min there.

Interval of increase are $(-\infty, -1) \cup (1, \infty)$

" " decrease are $(-1, 0) \cup (0, 1)$.

<u>NOTE</u>: Similar analysis gives

concave up on $(\frac{-1}{\sqrt{2}}, 0) \cup (\frac{1}{\sqrt{2}}, \infty)$

and conc. down on $(-\infty, \frac{-1}{\sqrt{2}}) \cup (0, \frac{1}{\sqrt{2}})$.

10/28/16 9AM

$\boxed{1}$ Turn in WA #5, webwork tonight!

$\boxed{2}$ w/ neighbors, compute

$\lim_{x \to \infty} \frac{x^2+2}{x^3+x+1}$ using our technique from before.

---

§4.4: L'Hospital's Rule

↳ pf in textbook, read it.

Ex: $\lim_{x \to \infty} \frac{x^2+2}{x^3+x+1} = \lim_{x \to \infty} \frac{\frac{1}{x^3}(x^2+2)}{\frac{1}{x^3}(x^3+x+1)}$

$= \lim_{x \to \infty} \frac{\frac{1}{x} + \frac{2}{x^3}}{1 + \frac{1}{x^2} + \frac{1}{x^3}} = \frac{0}{1} = 0.$

Previous method!

But .... how to handle:

Ex: $\lim_{x \to \infty} \frac{x^2}{e^x}$?

Can't use division technique here!

L'Hospital's Rule: Suppose f, g are diff and $g'(x) \neq 0$ near a (a might be ∞ or $-\infty$).

Then if either (A) $\lim_{x \to a} f(x) = 0 = \lim_{x \to a} g(x)$

or (B) $\lim_{x \to a} f(x) = \pm\infty$ and $\lim_{x \to a} g(x) = \pm\infty$,

Then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$, if ~~R.H.~~ right-hand limit exists.

Ex: $\lim_{x \to \infty} \frac{x^2+2}{x^3+x+1} \underset{L'H}{=} \lim_{x \to \infty} \frac{2x}{3x^2+1} \underset{L'H}{=} \lim_{x \to \infty} \frac{2}{6x} = 0$

Ex: $\displaystyle\lim_{x\to\infty} \frac{x^2}{e^x} \underset{\text{L'H}}{=} \lim_{x\to\infty} \frac{2x}{e^x} \underset{\text{L'H}}{=} \lim_{x\to\infty} \frac{2}{e^x} = 0.$

Ex: $\displaystyle\lim_{x\to 2} \frac{x^3-8}{x^4+2x-20} \underset{\substack{\text{L'H} \\ \text{since form } \frac{0}{0}}}{=} \lim_{x\to 2} \frac{3x^2}{4x^3+2} \underset{\substack{\text{direct} \\ \text{sub. aka} \\ \text{continuity}}}{=} \frac{12}{34}$

Ex: $\displaystyle\lim_{x\to 1} \frac{x^2-9}{x+3} \underset{\substack{\text{direct} \\ \text{sub}}}{=} \frac{1^2-9}{1+3} = \frac{-8}{4} = -2.$

NO NEED for L'H Rule!

Ex: $\displaystyle\lim_{x\to 2} \frac{4-x^2}{\sin(\pi x)} \underset{\substack{\text{L'H} \\ \text{type } \frac{0}{0}}}{=} \lim_{x\to 2} \frac{-2x}{\pi\cdot\cos(\pi x)} \underset{\substack{\text{direct} \\ \text{sub}}}{=} \frac{-4}{\pi\cdot\cos(2\pi)} = \frac{-4}{\pi}.$

Ex: $\displaystyle\lim_{x\to 1} \frac{x-1}{\ln(x)} \underset{\substack{\text{L'H of} \\ \text{type } \frac{0}{0}}}{=} \lim_{x\to 1} \frac{1}{1/x} = 1.$

Another form: L'Hospital for products.

Say $\displaystyle\lim_{x\to a} f(x) = 0$ and $\displaystyle\lim_{x\to a} g(x) = \pm\infty$.

what is $\displaystyle\lim_{x\to a} f(x)\cdot g(x)$? If we write

$\displaystyle\lim_{x\to a} \frac{f}{1/g}$ or $\displaystyle\lim_{x\to a} \frac{g}{1/f}$ for this problem,

equal

we can use L'H rule.

Ex: $\lim_{x \to 0^+} \underbrace{x}_{0} \cdot \underbrace{\ln(x)}_{-\infty} \underset{\text{rewrite}}{=} \lim_{x \to 0^+} \frac{\ln(x)}{1/x} \underset{\text{rewrite}}{=}$

$\lim_{x \to 0^+} \frac{\ln(x)}{x^{-1}} \underset{\substack{\text{L'H} \\ \text{type } \frac{-\infty}{\infty}}}{=} \lim_{x \to 0^+} \frac{\frac{1}{x}}{-x^{-2}} \underset{\text{algebra}}{=} \lim_{x \to 0^+} -x = 0.$

Ex: $\lim_{x \to \infty} \frac{x^7}{e^x} \underset{\text{L'H}}{=} \lim_{x \to \infty} \frac{7x^6}{e^x} \underset{\text{L'H}}{=} \lim_{x \to \infty} \frac{7 \cdot 6 \cdot x^5}{e^x} =$

$\underset{\text{use L'H again + again}}{\cdots\cdots} = \lim_{x \to \infty} \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{e^x} \underset{\substack{\text{not L'H} \\ \text{says } e^x \to \infty}}{=} 0.$

Ex: $$\lim_{x\to\infty} \frac{x^n}{e^x} \underset{L'H}{=} \cdots\cdots \underset{L'H}{=} \lim_{x\to\infty} \frac{n(n-1)(n-2)\cdots 3\cdot 2\cdot 1}{e^x}$$

$$= 0.$$

Consequences: ① Need a notation for this.

$$n(n-1)(n-2)\cdots 3\cdot 2\cdot 1 = n!$$

n "factorial"

② e^x "grows faster" than any polynomial.