10/31/16 12PM

$\boxed{1}$ Sign attendance

$\boxed{2}$ Quiz Thurs, 2 webworks, and WA #6 this week

$\boxed{3}$ w/ neighbors: How would you determine if a fn $f(x)$ has an abs max/min on $(-\infty, \infty)$?

$\boxed{\text{§4.7 Optimization}}$

Ex: Find two numbers that differ by 110 where the product of these numbers is as small as possible.

First remark: Most "real" problems are vague + require clarification.

Goal: Find x + y real numbers where

$$y - x = 110 \longleftarrow \boxed{y = x + 110} \; * $$

and $x \cdot y$ is minimum possible value.

Second remark: After clarification, have a clear strategy.

Strategy: Write product $x \cdot y$ as a function of x + apply min/max techniques.

$P(x) = x \cdot y \overset{*}{=} x \cdot (x+110) = x^2 + 110x.$

Minimize $P(x)$.

<u>Critical numbers:</u> $P'(x) = 0 \Rightarrow$

$2x + 110 = 0 \Rightarrow$

$x = -55.$

Since $P'(x)$ is defined for all x in $(-\infty, \infty)$,

$x = -55$ yields only critical #.

2nd der. test $\Rightarrow$ $P''(x) = 2 > 0$, so

P has a local min at $x = -55$.

Q: How can we conclude that this is an abs. min on $(-\infty, \infty)$?

<u>Need:</u> First Derivative Test for Abs. Extreme Values

If f is cts on (a,b) and if c is a critical number of f in (a,b), then:

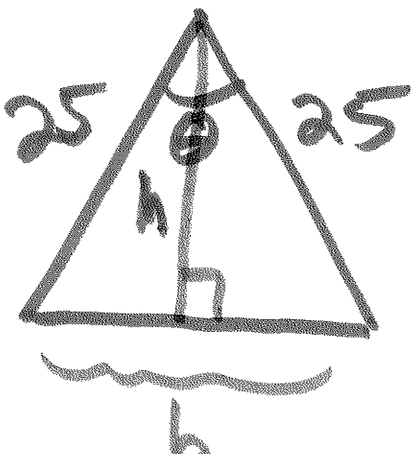
(A) If $f'(x) > 0$ for $x < c$ and $f'(x) < 0$ for $x > c$, then $f(c)$ is an abs. max of $f(x)$ on (a,b).

(B) If $f'(x) < 0$ for $x < c$ and $f'(x) > 0$ for $x > c$, then $f(c)$ is an abs min of $f(x)$ on (a,b).

To finish problem, since $P'(x) < 0$ for $x < -55$ + $P'(x) > 0$ for $x > -55$, $-55 = x$ gives an abs. min.

So, our numbers are $x = -55$, $y = -55 + 110 = 55$.

Ex: Find the angle θ that maximizes the area of the isoceles triangle w/ legs of length 25.



I know $A = \frac{1}{2} b \cdot h$.

This gives me relationship

$h^2 + \left(\frac{b}{2}\right)^2 = 25^2$ by Pyth. Thm.

Goal: Write A as a fn of b, maximize it, then use that b-value to find θ.

Q: What are possible values of b?

$0 < b < 50$

Maximize $A(b) = \frac{1}{2} b \cdot h = \frac{1}{2} \cdot b \cdot \sqrt{25^2 - \frac{b^2}{4}}$

on b in $(0, 50)$.

<u>Exercise:</u> Show $A'(b) = \frac{1}{2}\left[\frac{25^2 - \frac{b^2}{2}}{\sqrt{25^2 - \frac{b^2}{4}}}\right]$.

using (a) Product rule

or (b) bring b under $\sqrt{\ }$

using $b = \sqrt{b^2}$ since $b > 0$.

<u>Note:</u> Domain of den. of $A'(b)$ is $(-50, 50)$.

So, $A'(b)$ is defined on $(0, 50)$, for critical numbers we only need to solve $A'(b) = 0$.

Observe $A'(b)=0$ when $25^2-\frac{b^2}{2}=0$,

so, $b=\pm\sqrt{1250}$. Since $b>0$,

we have only $b=\sqrt{1250}$ as a critical number.

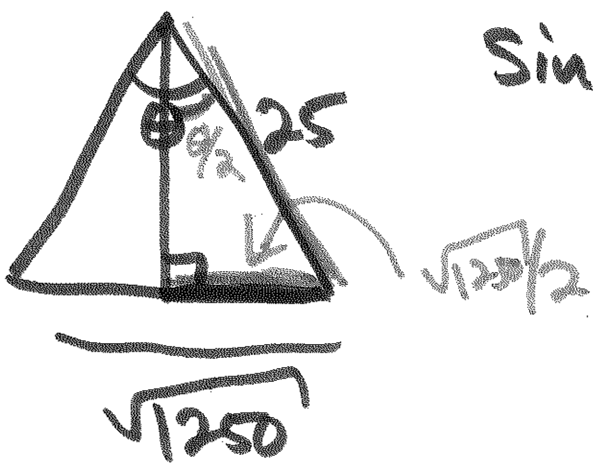
<u>Exercise</u>: Explain why $A'(b)>0$ for b in $(0,\sqrt{1250})$

and $A'(b)<0$ for b in $(\sqrt{1250},50)$.

By First der. test for abs max/min,

$b=\sqrt{1250}$ is abs. max of A on $(0,50)$.

Notes like this are helpful.



$\sin\left(\frac{\theta}{2}\right)=\frac{\sqrt{1250}/2}{25}$, so

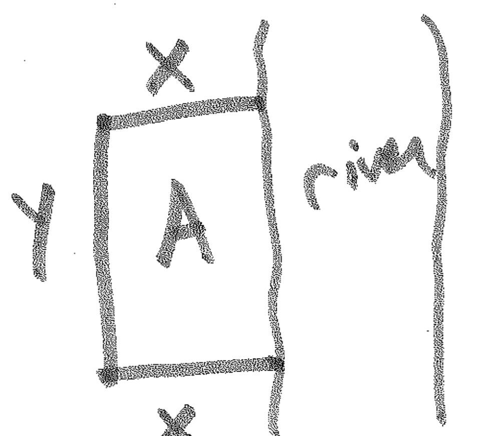
$\theta=2\cdot\arcsin\left(\frac{\sqrt{1250}}{50}\right)$.

11/2/16 **12PM**

0. Sign attendance
1. Webwork, Quiz tomorrow, WA#6 Fri
2. Consider Math Excel for MA114, Calc II.
3. Why does $x^4+17x^3+6x^2-22x+1$ have an abs. min? No computations allowed!

Explain to your neighbor verbally.... Think about this function....

Ex: Suppose farmer has 100m of fence and wants to enclose a max rectangular area along a river w/ a straight bank. What dim^n should fencing have?



Given: $2x + y = 100$

Goal: maximize $A = x \cdot y$.

Strategy: substitute $y = 100 - 2x$ and minimize $A(x) = x \cdot (100 - 2x)$
$= -2x^2 + 100x$

What x-values matter? $x > 0$.

$y = 100 - 2x > 0 \Rightarrow 50 > x$.

So, x is in $(0, 50)$.

Let's find critical values: $A'(x) = -4x + 100$.

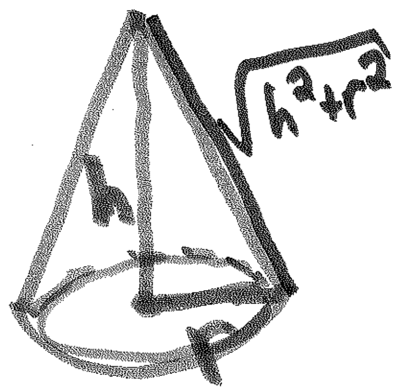
Since $A'(x)$ is defined for all x in $(0,50)$, critical values only when $A'(x) = 0$, i.e.

$$-4x + 100 = 0 \Rightarrow x = 25\text{ m}.$$

Since $A'(x)$ is a line, clear that $A'(x) < 0$ for $x > 25$, and $A'(x) > 0$ for $x < 25$, so $x = 25$ is an abs max on $(0,50)$.

$\Rightarrow$ Dim^{n}s are $x = 25^{m}$, $y = 50^{m}$.

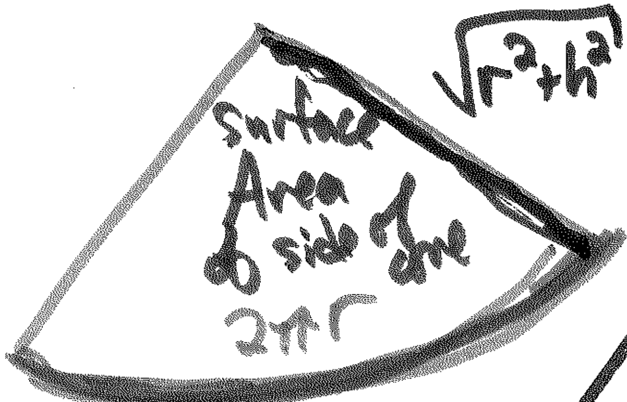
Ex: Find the dim^{n}s of a cylindrical cone of minimal cost having volume $125\,m^3$ if the base material costs 3 times more than the side material. Reminder: $V = \frac{1}{3}\pi r^2 h$.



$$V = 125 = \frac{1}{3}\pi r^2 \cdot h$$

$$\text{Cost} = 3\cdot(\text{area of base}) + (\text{area of side surface}).$$

- Area of base: πr^2
- Area of side surface: Cut the cone and unroll it to a flat shape.



$$\frac{\boxed{\text{Area of side of cone}}}{\underbrace{\text{total area of disk of radius } \sqrt{r^2+h^2}}_{= \pi\cdot(r^2+h^2)}} = \frac{2\pi \cdot r}{\underbrace{\sqrt{r^2+h^2}\,\pi \cdot 2}_{\text{total circ.}}}$$

Solve to get

Area of side of cone $= \pi r \cdot \sqrt{r^2+h^2}$.

$$\boxed{\frac{a}{b} = \frac{c}{d} \quad \Rightarrow a = \frac{b\cdot c}{d}}$$

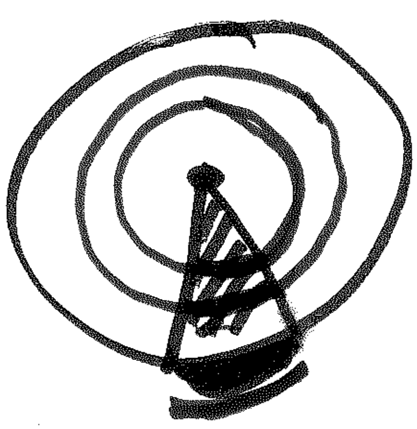
where:

$b = \pi(r^2+h^2)$

$c = 2\pi r$

$d = \sqrt{r^2+h^2}\cdot\pi\cdot 2$

Ratio:



Cost $= C = 3 \cdot \pi r^2 + \pi r \sqrt{r^2 + h^2}$

Use $h = \dfrac{125}{\frac{1}{3}\pi r^2}$ from volume, so $h = \dfrac{375}{\pi r^2}$.

<u>New Goal</u>: minimize

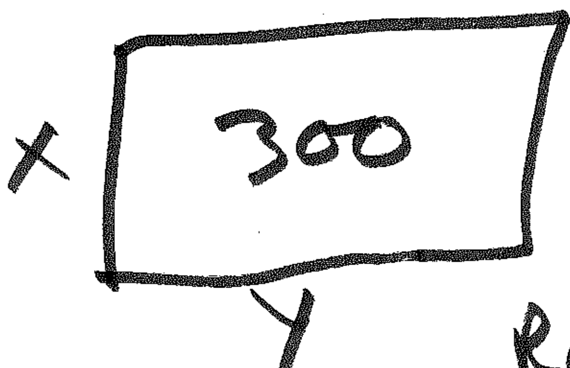
$$C(r) = 3 \cdot \pi r^2 + \pi r \sqrt{r^2 + \left(\frac{375}{\pi r^2}\right)^2}$$

for $r > 0$.

<u>This is hard!</u> In principle, compute $C'(r)$, solve $C'(r) = 0$, find an abs min.

Ex: Find dim$^{\underline{n}}$s of a rectangle w/ area $300m^2$ having minimum perimeter.

(You should be looking for squares...)



$300 = x \cdot y \Rightarrow y = \frac{300}{x}$.

minimize $2x + 2y$ = Perimeter.

Rewrite $P(x) = 2x + 2 \cdot \frac{300}{x} = 2x + \frac{600}{x}$.

I can use $x > 0$ for $P(x)$.

$P'(x) = 2 - \frac{600}{x^2} \Rightarrow x = \sqrt{300}$ is the critical #.

1st der. test $\Rightarrow$ this gives an abs. min.

So, $x = \sqrt{300}$ m, $y = \sqrt{300}$ m is soln.

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$\boxed{1}$ Turn in WA #6

$\boxed{2}$ Today: §4.9., Antiderivatives

What's next? Integration. The Fund. Thm of Calculus will say we can compute an integral using either

- Antiderivatives

or

- Riemann Sums.

The next few weeks will explain how this works.

Def$^{\underline{n}}$: $F(x)$ is an <u>antiderivative</u> of $f(x)$ on (a,b) if $F'(x)=f(x)$ on (a,b).

Thm: If $F'(x)=f(x)$ on (a,b), then every other antiderivative of $f(x)$ is of the form $F(x)+C$ for some constant C.

Notation: Write $\int f(x)\,dx = F(x) + C$ "indefinite integral"

~~#~~ for the most general form of an antiderivative of $f(x)$.

Ex: Find $\int x^3\,dx = \frac{x^4}{4} + C$

antider of x^3

Ex: $\int \sin x\,dx = -\cos x + C$

der of const is zero.

note: der of $-\cos x$ is $-\,-\sin x = \sin x$

Ex: $\int e^x\,dx = e^x + C$.

Ex: Find $\int \frac{1}{x}\,dx$ on $(-\infty, 0)$. Sol$^{\underline{n}}$ is $\ln|x| + C$

by chain rule

Ex: If $F'(x) = f(x)$, $G'(x) = g(x)$, find

$$\int [f(x) + g(x)]\,dx = F(x) + G(x) + C$$

Ex: Find an antiderivative of

$$3\cos x + \frac{3x^5 - 7\sqrt[3]{x}}{x^2}.$$

Rewrite: $3\cos x + \dfrac{3x^5}{x^2} - \dfrac{7(x)^{1/3}}{x^2}$

$$= 3\cos x + 3x^3 - 7 \cdot x^{-5/3}$$

Antiderivative: $3\sin x + 3 \cdot \dfrac{x^4}{4} - 7 \cdot \dfrac{x^{-5/3+1}}{-\frac{5}{3}+1}$

$$= 3\sin x + \frac{3}{4}x^4 + \frac{21}{2}x^{-2/3}$$

No "+C" is okay, since Q asked only an antiderivative.

Ex: $\displaystyle\int x^{m/n}\,dx = \frac{x^{\frac{m}{n}+1}}{\frac{m}{n}+1} = \frac{n}{m+n}x^{\frac{m+n}{n}}$

<u>Ex</u>: If $f''(x) = 12x^2 + 6x - 4$, with $f(0) = 4$, $f(1) = 1$, find $f(x)$.

<u>Strategy</u>:
- Compute $f'(x)$ by antider's.
- Compute $f(x)$ by antider's.

} Pick up 2 constants.

- Use sample points to find value of constants.

$$f'(x) = 12\frac{x^3}{3} + 6\frac{x^2}{2} - 4x + C$$

$$f(x) = \frac{12}{3}\cdot\frac{x^4}{4} + \frac{6}{2}\cdot\frac{x^3}{3} - 4\frac{x^2}{2} + Cx + D.$$

constants.

$$= x^4 + x^3 - 2x^2 + Cx + D.$$

$4 = f(0) = 0^4 + 0^3 - 2\cdot 0^2 + C\cdot 0 + D = D$, So $D = 4$.

$1 = f(1) = \cancel{1^4} + \cancel{1^3} - \cancel{2\cdot 1^2} + C\cdot 1 + 4 = C + 4$. So, $C + 4 = 1$, hence $C = -3$.

<u>Soln</u>: $f(x) = x^4 + x^3 - 2x^2 - 3x + 4$.

<u>Ex</u>: If $f''(x) = 3x + 2$, $f'(0) = -5$, $f(0) = 6$.

Find $f(x)$.

$\rightarrow f'(x) = 3\frac{x^2}{2} + 2x + C$, $f'(0) = -5 \Rightarrow C = -5$

$\rightarrow f(x) = \frac{3}{2}\cdot\frac{x^3}{3} + 2\frac{x^2}{2} - 5x + D$, $f(0) = 6 \Rightarrow D = 6$.

<u>Soln</u>: $\frac{x^3}{2} + x^2 - 5x + 6$.

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11-7-16

Section 5.1: Areas & Distances

Distance Problem: Given a fixed velocity function, find the distance travelled in a certain time.

Ex. If $v(t) = 6$ m/s, then from $t = 5$s to $t = 7$s, object move

$$6(7-5) = 12\text{m} \quad \left(\begin{array}{c}\text{distance} = \\ \text{rate} \times \text{time}\end{array}\right)$$

When $v(t)$ is not constant, we can try to solve this by first approximating the distance travelled.

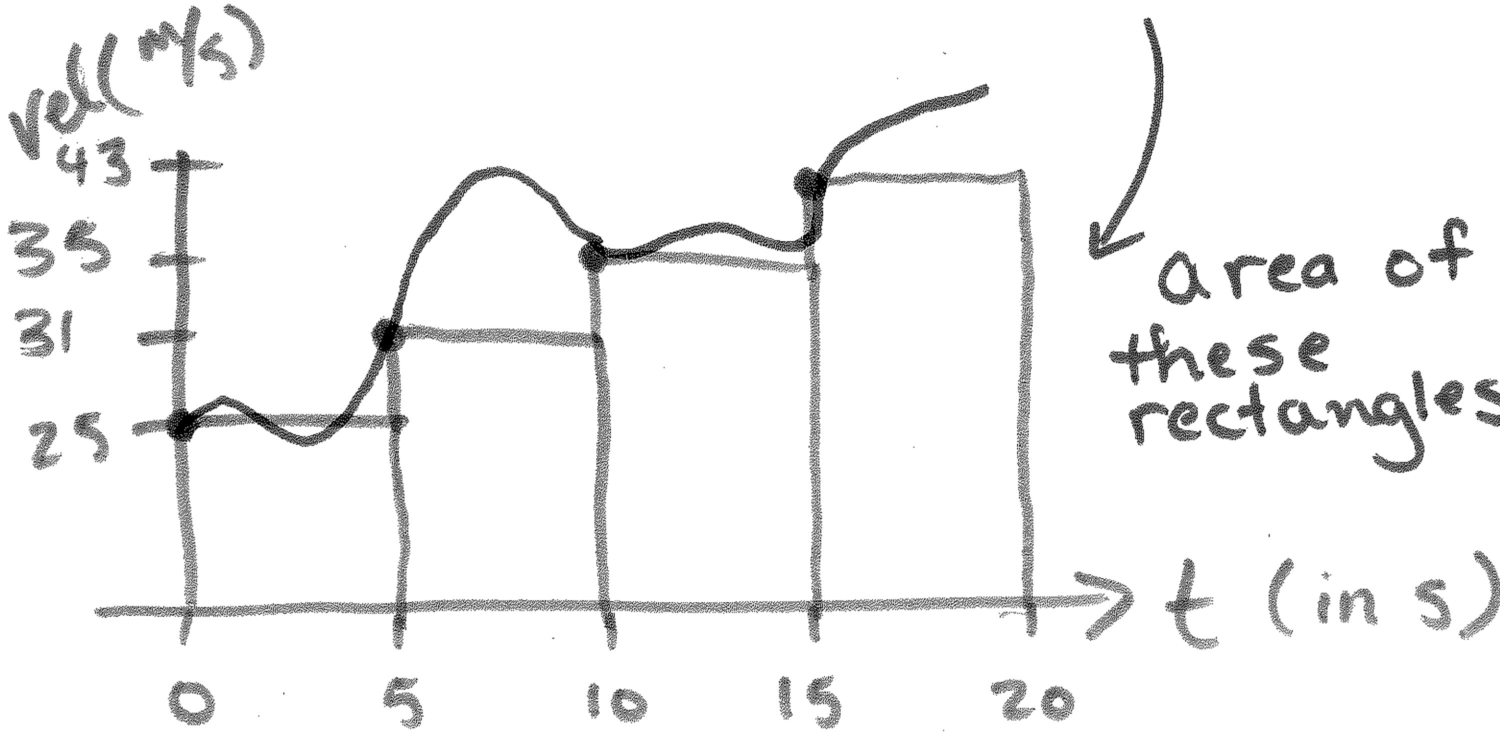
Ex.

time (s)	0	5	10	15	20	25	30
vel (m/s)	25	31	35	43	47	45	41

Estimate distance traveled by assuming $v(t)$ is constant over the subintervals.

? First 20 seconds.

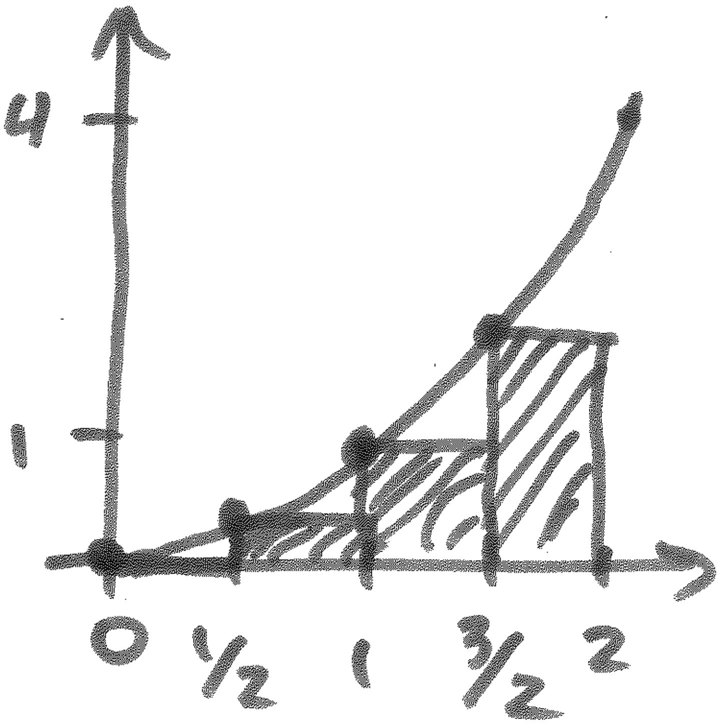
$$25(5-0)+31(10-5)+35(15-10)$$
$$+\,43(20-15) \quad = 670\,\text{m}$$



area of these rectangles

<u>Ex.</u> Suppose $v(t) = t^2$ m/s.

Estimate distance traveled between $t=0$ and $t=2$.



Let's use sample points in a table.

t	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
vel	0	$\frac{1}{4}$	1	$\frac{9}{4}$	4

$$0\left(\tfrac{1}{2}-0\right) + \tfrac{1}{4}\left(1-\tfrac{1}{2}\right) + 1\left(\tfrac{3}{2}-1\right) + \tfrac{9}{4}\left(2-\tfrac{3}{2}\right) = \tfrac{14}{8} \text{ m}$$

$$= v(0)\left(\tfrac{1}{2}-0\right) + v\left(\tfrac{1}{2}\right)\left(1-\tfrac{1}{2}\right) + v(1)\left(\tfrac{3}{2}-1\right) + v\left(\tfrac{3}{2}\right)\left(2-\tfrac{3}{2}\right)$$

This gives the areas of the rectangles.

1st Key Idea

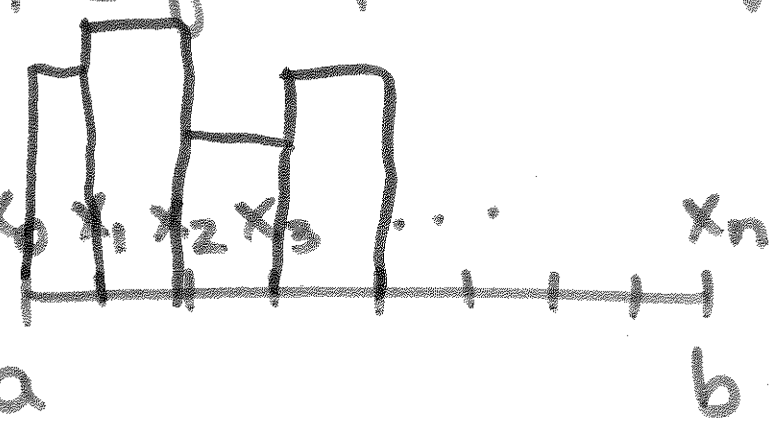
Approximate the total distance by adding areas of rectangles.

2nd Key Idea

This also approximates the area between the x-axis and $y = t^2$ on $[0, 2]$.

<u>Let's make this precise.</u>

Given a cts function $f(x)$ on $[a,b]$, break the interval into n equal pieces of length $\Delta x = \frac{b-a}{n}$



Set $x_0 = a$

$x_1 = a + \Delta x$

$x_2 = a + 2(\Delta x)$

$x_3 = a + 3(\Delta x)$

$\vdots$

$x_n = b$

$$x_n = a + n(\Delta x) = a + (b-a) = b$$

<u>Def'n/Theorem</u>: The area A of the region S lying between the x-axis and the graph of $y = f(x)$ on $[a,b]$ is

$$A = \lim_{n\to\infty}\left[f(x_0)\Delta x + f(x_1)(\Delta x) + \dots + f(x_{n-1})\Delta x\right]$$

$= L_n$ <u>left</u> endpoint approx.

$$\lim_{n\to\infty} \underbrace{\left(f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x\right)}_{R_n \,=\, \text{right endpoint approx.}}$$

<u>Note:</u> M_n is the sum using midpoints, see Ex. 3 5.1

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Calculate area under a curve by approximating it using rectangles.

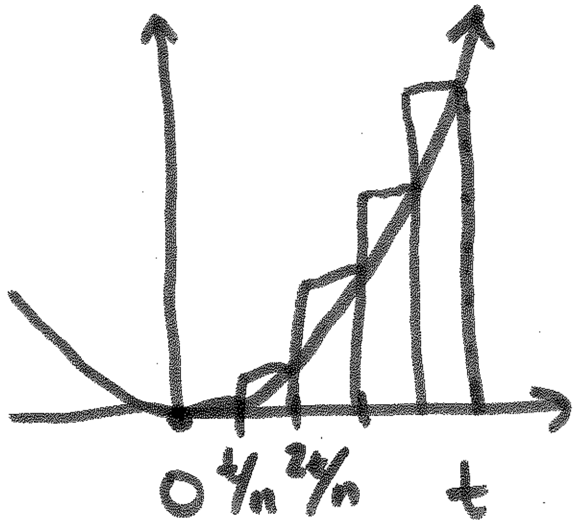
Note: We frequently use sigma notation.

$$\sum_{i=0}^{n} a_i = a_0 + a_1 + a_2 + \ldots + a_n$$

Ex. $$\sum_{i=0}^{n-1} f(x_i)\,\Delta x = \overbrace{\underbrace{f(x_0)}_{\text{height}}\underbrace{\Delta x}_{\text{width}}}^{\text{Area of } \square} + f(x_1)\,\Delta x + \ldots + f(x_{n-1})\,\Delta x$$

Ex. Find area "under" $y=x^2$ for $[0,t]$ using R_n.

right endpoints.



Goal:

$$A = \lim_{n\to\infty} R_n \ ?$$

$$\Delta x = \frac{t-0}{n} = \frac{t}{n}$$

$$R_n = \underbrace{\left(\frac{t}{n}\right)^2}_{f(x_0)}\underbrace{\frac{t}{n}}_{\Delta x} + \underbrace{\left(\frac{2t}{n}\right)^2}_{f(x_1)}\underbrace{\frac{t}{n}}_{\Delta x} + \left(\frac{3t}{n}\right)^2\frac{t}{n} + \cdots + \left(\frac{nt}{n}\right)^2\frac{t}{n}$$

$$= \frac{t^3}{n^3}\left(1^2+2^2+3^2+\cdots+n^2\right)$$

$$= \frac{t^3}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$ from H.S.

Perhaps recall from high school,

$$1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^2+2^2+3^2+\dots+7^2 = \frac{7\cdot 8\cdot 15}{6} =$$

$$2n^3+\dots$$

$$R_n = \frac{t^3}{6}\cdot\frac{n(n+1)(2n+1)}{1n^3}$$

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{t^3}{6}\,\frac{2}{1} = \boxed{\frac{t^3}{3}}$$

area under $y=x^2$ from $[0,t]$.

Q: What is the area for $t=4$?

$$A = \frac{64}{3}$$

Section 5.2: The Definite Integral

Notes: • Read def'n 2 in 5.2 of Stewart for the full definition of the definite integral.

• Our ultimate goal is to connect areas, integrals, and antiderivatives.

Def'n: Given a function $f(x)$ on $[a,b]$, write

integrand, differential

$$\int_a^b \underbrace{f(x)}_{\text{integrand}}\, dx = \lim_{n\to\infty} R_n \quad \leftarrow \text{right endpoints}$$

$$= \lim_{n\to\infty} L_n \quad \leftarrow \text{left endpoints}$$

$$\lim_{n\to\infty} M_n \quad \leftarrow \text{midpoints}$$

integral symbol

limits of integration

<u>Remark</u>: You can pick sample points more randomly than "right", "left", or "mid."
In practice, these are the easiest.

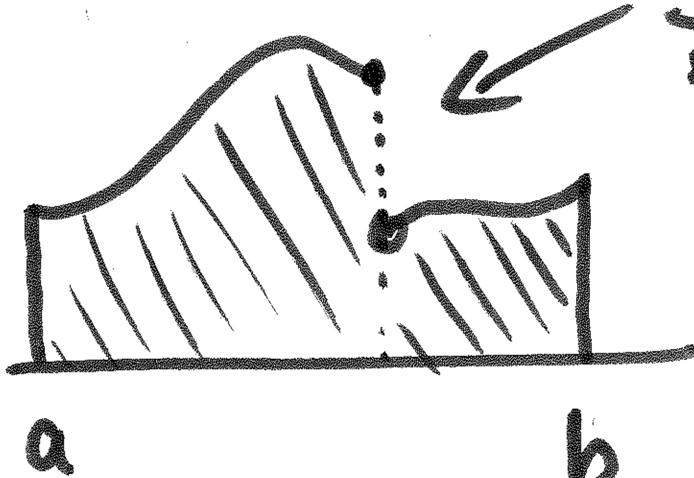
$$R_n = \sum_{i=1}^{n} f(x_i)\,\Delta x$$

When this approximating area, we call it a "Riemann Sum".

<u>Thm:</u> $\int_a^b f(x)\,dx$ exists if

- f is continuous on $[a,b]$.
- f has a finite number of jump discontinuities

<u>Ex.</u>



jump disc. but Area makes sense.

<u>Usefeel Tools:</u>

$$1+2+3+4+\dots+n=\sum_{i=1}^{n} i=\frac{n(n+1)}{2}$$

$$1^2+2^2+3^2+\dots+n^2=\sum_{i=1}^{n} i^2=\cancel{\frac{n(n+1)(2n+1)}{6}}$$

$$=\frac{n(n+1)(2n+1)}{6}$$

$$1^3+2^3+3^3+\cdots+n^3=\sum_{i=1}^{n} i^3=\left[\frac{n(n+1)}{2}\right]^2$$

Ex. Evaluate $\int_0^3 (x^3-6x)\,dx$.

$$\Delta x = \frac{3-0}{n} = \frac{3}{n} \qquad x_0 = 0 \quad x_2 = \frac{6}{n}$$
$$x_1 = \frac{3}{n} \quad x_3 = \frac{9}{n}$$
$$\ldots \; x_i = \frac{i3}{n}$$

$$\int_0^3 (x^3-6x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x. \; \text{✱}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{3i}{n}\right)\frac{3}{n}$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left[\left(\frac{3i}{n}\right)^3 - 6\left(\frac{3i}{n}\right)\right]$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left[\frac{27i^3}{n^3} - \frac{18i}{n}\right]$$

$$= \lim_{n\to\infty} \frac{3}{n} \left[\sum_{i=1}^{n} \frac{27i^3}{n^3} - \sum_{i=1}^{n} \frac{18i}{n}\right]$$

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$$= \lim_{n\to\infty}\left(\frac{81}{n^4}\sum_{i=1}^{n} i^3\right) - \left(\frac{54}{n^2}\sum_{i=1}^{n} i\right)$$

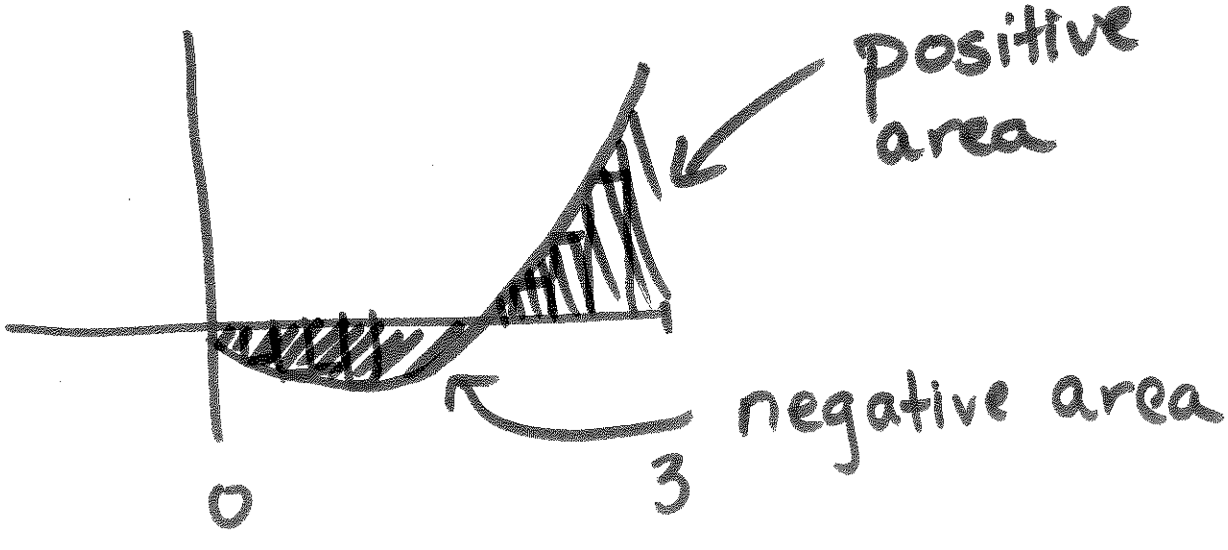
$$= \lim_{n\to\infty}\left(\frac{81}{n^4}\left[\frac{n(n+1)}{2}\right]^2\right) - \left(\frac{54}{n^2}\,\frac{n(n+1)}{2}\right)$$

$$= \lim_{n\to\infty}\left[\frac{81}{4}\,\frac{\overbrace{n^2(n+1)^2}^{\boxed{n^4+\ldots}}}{n^4} - \frac{54}{2}\,\frac{\overbrace{n(n+1)}^{\boxed{n^2+\ldots}}}{n^2}\right]$$

$$= \frac{81}{4}(1) - \frac{54}{2}(1)$$

$$= \boxed{-\frac{27}{4}}$$ Whew!

$f(x) = x^3 - 6x$



$\int_a^b f(x)\,dx$ = net signed area between $y = f(x)$, the x-axis, and $x = a$, $x = b$.

Remark: read §5.2 about the Midpoint Rule.

Properties of the Def. Integral

(A) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

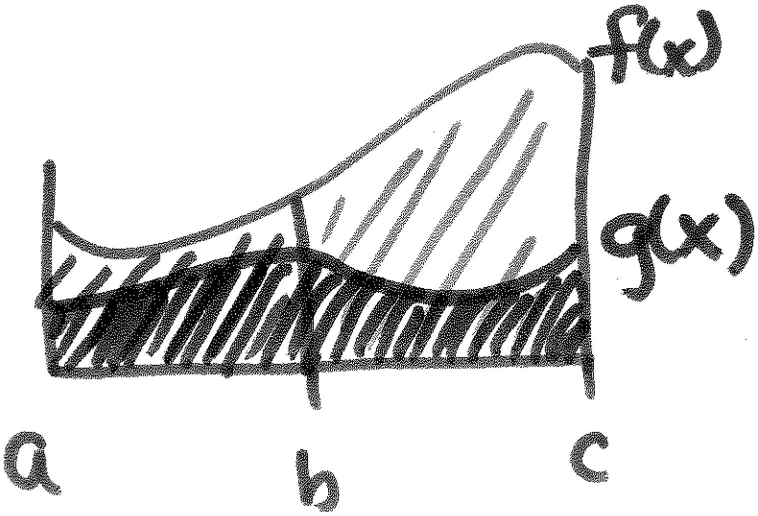
(B) $\int_a^a f(x)\,dx = 0$

(C) $\int_a^b cf(x) \pm g(x)\,dx$

$$= c\int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

Ex. $\int_0^3 x^3 - 6x\,dx = \int_0^3 x^3\,dx - 6\int_0^3 x\,dx$

(D) $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx$



$= \int_a^c f(x)\,dx$

(E) If $f(x) \geq 0$ on $[a,b]$,

$\int_a^b f(x)\,dx \geq 0.$ (usual area)

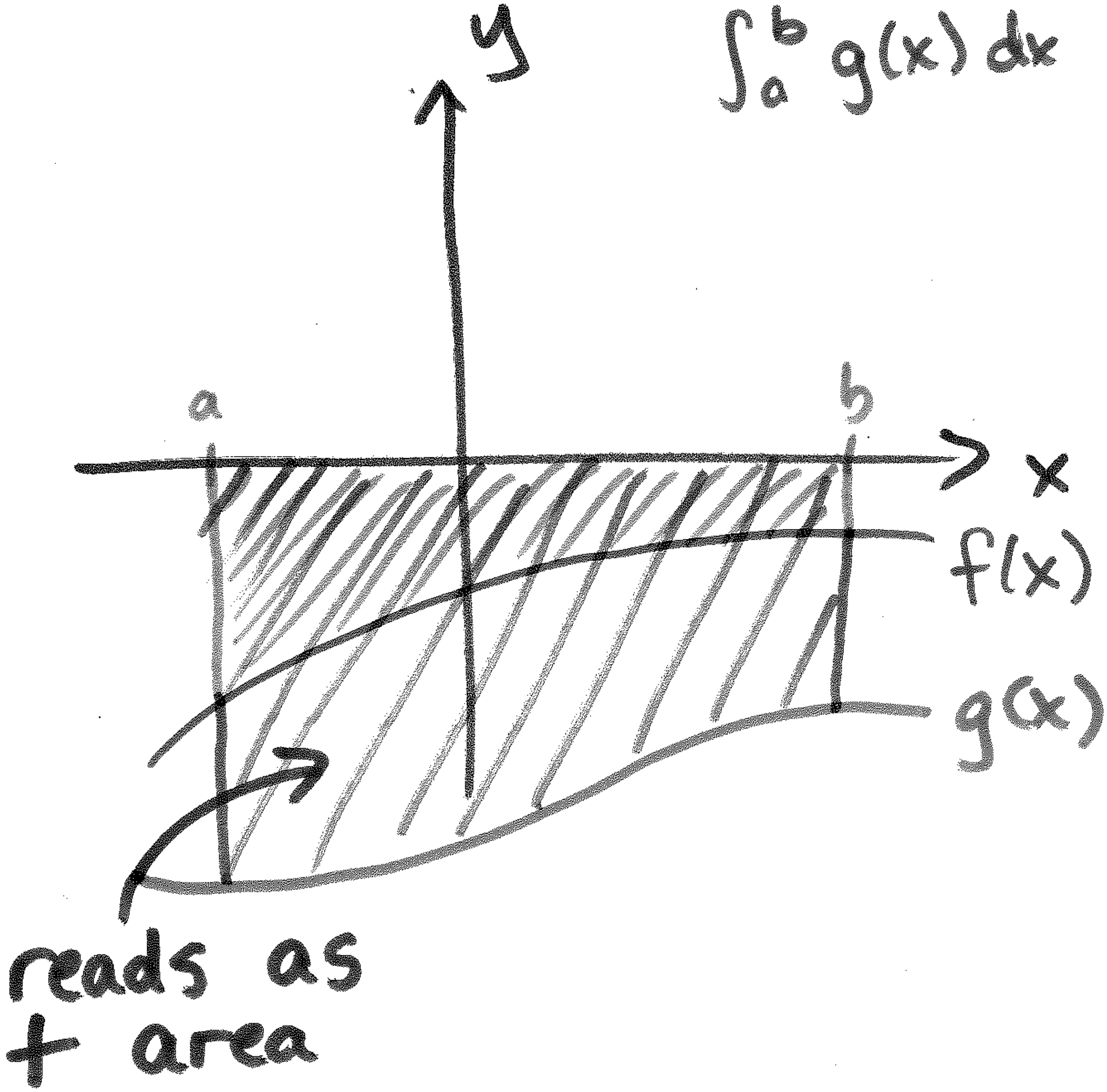
(F) If $f(x) \geq g(x)$ on $[a,b]$,

$f(x) - g(x) \geq 0$, and so

$\int_a^b f(x) - g(x)\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx \geq 0$

thus, $\underbrace{\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx}_{*}$.

$$\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$$



Review for Exam 3

<u>Ex.</u> The half-life of cesium-137 is 30 years. If we begin with 100 mg of it, how long will it take before we have 1mg left?

$$M(t) = \underset{\uparrow\,100}{M_0}\, e^{rt} \qquad \frac{1}{2} = e^{r(30)}$$

$$50 = 100e^{rt}$$

$$\frac{1}{2} = e^{rt}$$

$$\ln\left(\frac{1}{2}\right) = r(30)$$

$$\boxed{-\frac{\ln(2)}{30} = r}$$

$$1 = 100e^{-\frac{\ln 2}{30}t}$$

$$\frac{1}{100} = e^{-\frac{\ln 2}{30}t} \Rightarrow -\ln(100) = -\frac{\ln 2}{30}t$$

$$t = \frac{30\ln(100)}{\ln(2)}$$

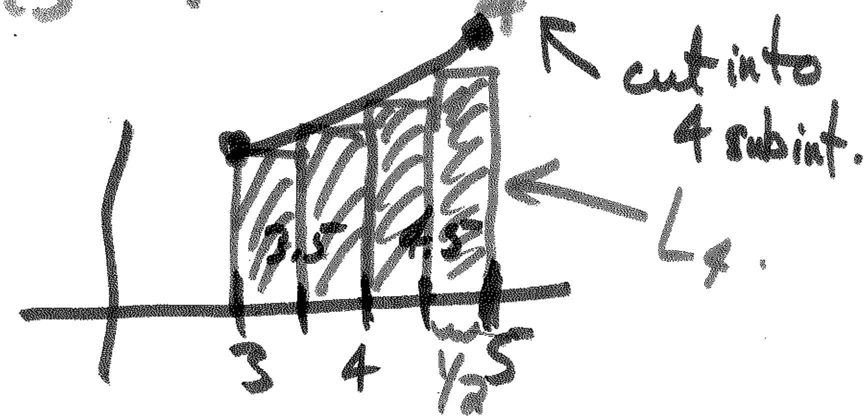
11/14/16 [redacted] 12PM

1. Exam 3 Tuesday night - See Canvas announcement

2. Summation formulas will be provided for $\sum i^2$, $\sum i$, etc.

3. **No webwork on Quiz this week. Webwork D1 due Mon Nov 21.**

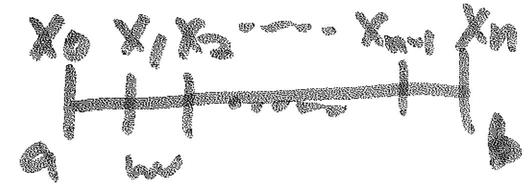
4. [redacted] w/neighbors: Find L_4 for x^4 on $[3,5]$.



$$L_4 = \underline{3}^4 \cdot \frac{1}{2} + \underline{(3.5)}^4 \cdot \frac{1}{2} + \underline{4}^4 \left(\frac{1}{2}\right) + \underline{(4.5)}^4 \frac{1}{2}.$$

$$= \frac{7177}{16} = 448.5625.$$

n pieces



$$\Delta x = \frac{b-a}{n}.$$

e.g. $n = 8$.

n pieces $\Rightarrow$ $n+1$ possible endpts.

2 other ways to write the sum.

- $\displaystyle\sum_{i=0}^{3} \left(3 + \frac{i}{2}\right)^4 \cdot \frac{1}{2}$

<u>NOTE:</u> $x_i = x_0 + i\Delta x$.

- $\displaystyle\sum_{i=0}^{3} (x_0 + i\Delta x)^4 \cdot \Delta x$.

$$R_n = \sum_{i=1}^{n} f(x_0 + i\Delta x)\Delta x \qquad L_n = \sum_{i=0}^{n-1} f(x_0 + i\Delta x)\Delta x$$

Ex: which of these expresses R_n for $\sqrt{x}\cdot\sin(x)$ on $[0,\pi]$?

A) $\displaystyle\sum_{i=0}^{n-1}\left(\sqrt{\frac{i}{\pi}}\sin\left(\frac{i}{\pi}\right)\frac{\pi}{n}\right)$

★ B) $\displaystyle\sum_{i=1}^{n}\left(\underbrace{\sqrt{\frac{i\pi}{n}}\sin\left(\frac{i\pi}{n}\right)}_{f(x_i)}\cdot\underbrace{\frac{\pi}{n}}_{\Delta x}\right)$

correct for R_n

C) $\displaystyle\sum_{i=1}^{n}\left(\sqrt{\frac{i}{n}}\sin\left(\frac{i}{n}\right)\frac{i}{n}\right)$

$\Delta x = \frac{\pi - 0}{n} = \frac{\pi}{n}$.

$x_0 = 0$, so

$x_i = 0 + i\Delta x$

$= \frac{i\pi}{n}$.

Ex: Does there exist a diff. function $f(x)$ s.t. $f(0) = -1$, $f(2) = 4$, and $f'(x) \le 2$ for all x?

Interval $[0,2]$. Apply MVT on this interval to a hypothetical fn $f(x)$, There is a c s.t. c in $(0,2)$ and

$$f'(c) = \frac{f(2)-f(0)}{2-0} = \frac{4-(-1)}{2} = \frac{5}{2}.$$

But, given this, $f'(x)$ can't always be ≤ 2 for all x, since $\frac{5}{2} > 2$. So answer is no.

Ex: Let A be area btwn x-axis and $f(x)=x^2-9$ over $[2,4]$. Compute A.

Recommend: Given a choice, use R_n, since sum is from 1 to n.

$\Delta x = \frac{4-2}{n} = \frac{2}{n}$. $\quad x_0 = 2,$

$x_i = 2 + i\cdot\frac{2}{n}$.

$$R_n = \sum_{i=1}^{n} f(x_i)\,\Delta x = \sum_{i=1}^{n}\left(\left(2+\frac{i\cdot 2}{n}\right)^2-9\right)\cdot\frac{2}{n}.$$

$$= \frac{2}{n}\cdot\sum_{i=1}^{n}\left(\left(2+\frac{i\cdot 2}{n}\right)^2-9\right) = \frac{2}{n}\cdot\sum_{i=1}^{n}\left(4+\frac{8i}{n}+\frac{4i^2}{n^2}-9\right)$$

$$= \frac{2}{n} \cdot \sum_{i=1}^{n} \left(\frac{4i^2}{n^2} + \frac{8i}{n} - 5 \right) = (*)$$

$$\begin{array}{ccccc}
i=1 & i=2 & i=3 & \cdots & i=n \\
\frac{4 \cdot 1^2}{n^2} & \frac{4 \cdot 2^2}{n^2} & \frac{4 \cdot 3^2}{n^2} & \cdots & \frac{4 \cdot n^2}{n^2} \\
\frac{8 \cdot 1}{n} & \frac{8 \cdot 2}{n} & \frac{8 \cdot 3}{n} & \cdots & \frac{8 \cdot n}{n} \\
-5 & -5 & -5 & \cdots & -5
\end{array}$$

$$(*) = \frac{2}{n} \cdot \left(\sum_{i=1}^{n} \frac{4i^2}{n^2} + \sum_{i=1}^{n} \frac{8i}{n} + \sum_{i=1}^{n} -5 \right)$$

$$= \frac{2}{n}\cdot\frac{4}{n^2}\sum_{i=1}^{n} i^2 + \frac{2}{n}\cdot\frac{8}{n}\sum_{i=1}^{n} i - \frac{5\cdot 2}{n}\sum_{i=1}^{n} 1$$

$$\underbrace{= \frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{16}{n^2}\,\frac{n(n+1)}{2} - \frac{10}{n}\cdot n}_{}$$

$= R_n.$

<u>NOTE</u>: $\lim_{n\to\infty} R_n = \lim_{n\to\infty}\left(\quad \right).$

Use $\frac{1}{n^3}$ $\frac{1}{n^2}$ limit technique.