10/31/16 9AM

1. Quiz Thurs, webwork C4, C5, WA #6 this week

2. W/ neighbors: How would you determine if a fn $f(x)$ has an abs min/max on $(-\infty, \infty)$?

3. Sign Attendance

<u>Ex</u>: Find two numbers that differ by 110 where the product of those numbers is as small as possible.

<u>First comment</u>: Almost all problems in reality are a bit vague. First task is to clarify your assumptions.

w/ calc, we need to work w/ real numbers, i.e. $(-\infty, \infty)$, i.e. $\longleftrightarrow$

<u>Goal</u>: Find real numbers x and y with $y - x = 110$ and $x \cdot y$ minimized.

Second comment: We can do calc in MA113 using only one variable. Second task is to rewrite problem using only one variable.

I'll choose x. write $y = x+110$ and sub it into product formula.

$$\text{Product} = x \cdot y = x \cdot (x+110).$$

Goal: minimize $P(x) = x(x+110) = x^2 + 110x$.

Compute critical #'s of $P(x)$: $P'(x) = 2x + 110$

$\Rightarrow P'(x) = 0$ when $x = -55$.

This is only critical #.

To check max or min, compute $P''(x) = 2 > 0$.

So, by 2^{nd} der. test, $x = -55$ gives a local min for $P(x)$.

Need to confirm this is an abs. min.

We will use:

<u>1st der test for abs min/max:</u> If f is cts on (a,b) and c is a critical # of f in (a,b), then:

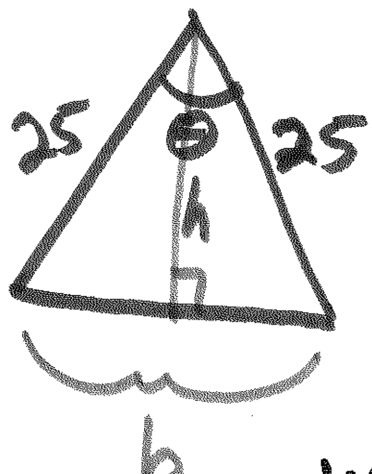
(A) If $f'(x) > 0$ for $x < c$ and $f'(x) < 0$ when $x > c$, then $f(c)$ is abs max of f on (a,b).

(B) If $f'(x) < 0$ for $x < c$ and $f'(x) > 0$ when $x > c$, then $f(c)$ is abs min of f on (a,b).

Since $P'(x) < 0$ for $x < -55$, and $P'(x) > 0$ for $x > -55$, $x = -55$ yields an abs. min for P.

Thus, our minimum product occurs when $x = -55$, $y = 55$. Since $y - x = 110$.

__Ex__: Find the angle θ that maximizes area of isoceles triangle w/ legs of length 25.



Find θ maximizing area:

$$A = \frac{1}{2} b \cdot h.$$

__Strategy:__

Find b w/ max area, then compute θ for that b, w/ $b>0$.

Following strategy, write A as a function of b. This requires h as a fn of b: $25^2 = h^2 + \left(\frac{b}{2}\right)^2 \Rightarrow$

$$h = \sqrt{25^2 - \left(\frac{b}{2}\right)^2} = \sqrt{25^2 - \frac{b^2}{4}}.$$

So, $A(b) = \frac{1}{2} \cdot b \cdot \sqrt{25^2 - \frac{b^2}{4}}$.

Product rule, or bring b inside $\sqrt{\ }$ using $b = \sqrt{b^2}$ since $b > 0$.

__Claim:__ $A'(b) = \frac{1}{2}\left[\dfrac{25^2 - \frac{b^2}{2}}{\sqrt{25^2 - \frac{b^2}{4}}}\right]$

Find critical #'s on $(0,\infty)$:

$A'(b) = 0 \Rightarrow 25^2 - \frac{b^2}{2} = 0 \Rightarrow b = \sqrt{1250}$.

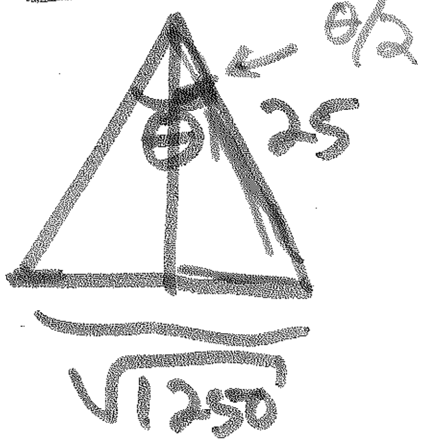
__NOTE:__ Don't use $-\sqrt{1250}$ since $b > 0$.

__Check:__ $A'(b) > 0$ for $b < \sqrt{1250}$,

$A'(b) < 0$ for $b > \sqrt{1250}$.

So by 1st der test, $\sqrt{1250}$ is abs max of A on $(0,\infty)$

Final step:



$$\frac{\theta}{2} = \cancel{\arctan}\ \arcsin\left(\frac{\sqrt{1250}/2}{25}\right)$$

$$\Rightarrow \theta = 2\,\cancel{\arctan}\ \arcsin\left(\frac{\sqrt{1250}/2}{25}\right).$$

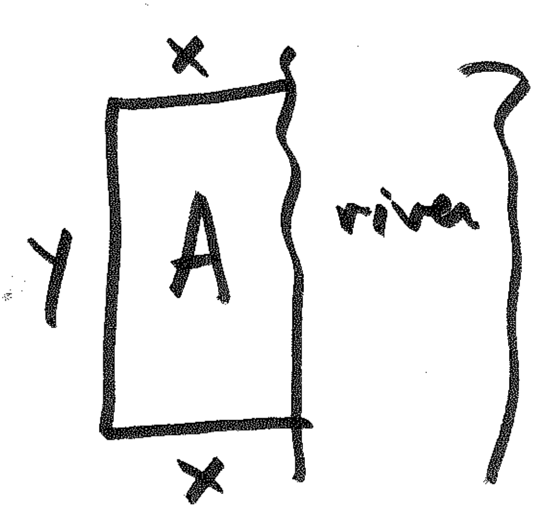
since $\tan\left(\frac{\theta}{2}\right) = \frac{\sqrt{1250}/2}{25}$

11/2/16 9AM

0. Sign Attendance
1. Webwork, Quiz tomorrow, WA#6 Fri
2. Consider MathExcel for MA114, Calc II
3. Why does $x^4+17x^3+6x^2-22x+1$ have an abs **min**? No computations allowed!

Explain to your neighbor verbally ... Think about this function...

<u>Ex</u>: Suppose farmer has 100m of fence & wants to enclose a max area along a straight river w/ rectangular field. What should dim^n's be?



x by y field w/ fence perimeter $2x+y$, and y of perimeter is river.

<u>Given</u>: $2x+y=100$. (*)

<u>Goal</u>: Maximize $A = x \cdot y$.

What are possible x and y values?

$x, y > 0$. Know $y = 100-2x$ by (*)

So $y = 100 - 2x > 0$. Thus, $100 > 2x$, hence $50 > x$.

Conclusion: $50 > x > 0$ are possible x-values.

Strategy: Write A as a fn of x, maximize on $(0,50)$.

Write $A = x \cdot y = x \cdot (100 - 2x) = -2x^2 + 100x$.

Critical values are: $A'(x) = -4x + 100$ defined on $(0,50)$

So, $A'(x) = 0$ when

$-4x + 100 = 0 \Rightarrow x = 25$.

Check signs: $\left.\begin{array}{l} A'(x) < 0 \text{ for } x > 25, \\ A'(x) > 0 \text{ for } x < 25 \end{array}\right\}$ by linearity of $A'(x)$.

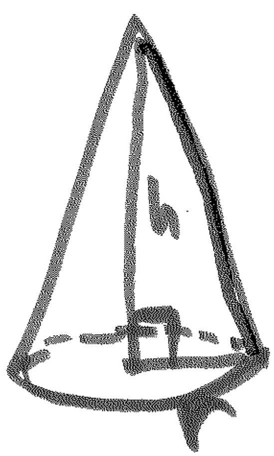
So, 1st der. test for abs. max $\Rightarrow A(x)$ is max at $x = 25$.

when $x = 25$, $y = 100 - 2 \cdot 25 = 50$.

So, dim$^{\underline{n}}$s of area are $25_m \times 50$ m.

Ex: Find the dimensions of a cylindrical cone of minimal cost having volume $125\,m^3$, if the base material costs 3 times more than side material.

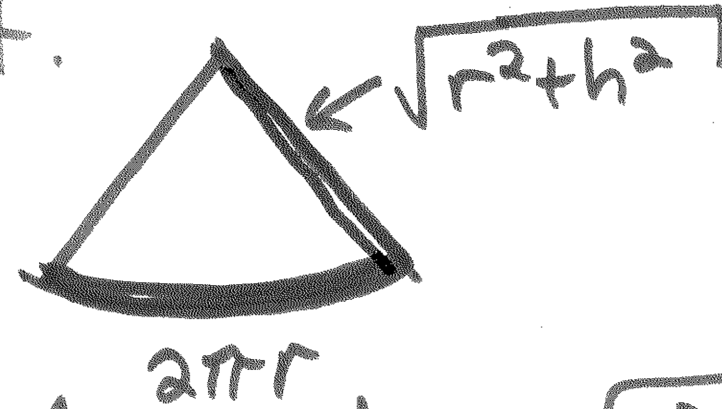
Recall: $V = \frac{1}{3}\pi r^2 h = 125$ ← Given.



Cost = 3 · Area of base + Area of conical surface.

Area of base is πr^2.

For surface, think about slicing cone & laying it out flat.



squared

Area of full circle of radius $\sqrt{r^2+h^2}$ is $\pi(r^2+h^2)$.

$$\frac{\text{Area of sector}}{\text{Area of full circle}} = \frac{\text{length of arc}}{\text{full circumference}}.$$

$$\Rightarrow \quad \frac{\text{surface Area}}{\pi(r^2+h^2)} = \frac{\cancel{2\pi} r}{\cancel{2\pi}\sqrt{r^2+h^2}}.$$

Surface area $= \pi r \cdot \dfrac{(r^2+h^2)}{\sqrt{r^2+h^2}} = \pi r\sqrt{r^2+h^2}$.

$$\text{Cost} = 3\cdot\pi r^2 + \pi r\sqrt{r^2+h^2}.$$

<u>Recall</u>: $125 = \frac{1}{3}\pi r^2 h \Rightarrow h = \dfrac{125}{\frac{1}{3}\pi r^2} = \dfrac{375}{\pi r^2}$.

So, $C(r) = 3\pi r^2 + \pi r\sqrt{r^2 + \left(\dfrac{375}{\pi r^2}\right)^2}$.

cost

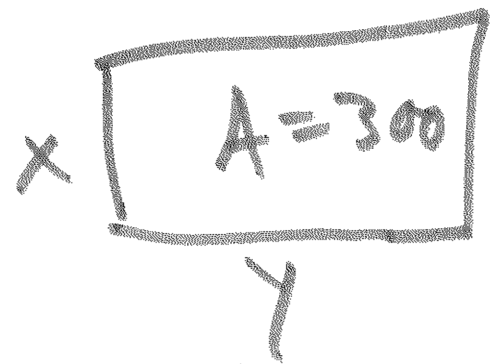
This is for $r>0$.

Minimize on $(0,\infty)$, then find h for that min r.

This is hard! The main task for this problem in MA 113 is to set up the math. model.

Ex: Find the dim^{n}'s of a rectangle w/ area 300 cm^2 having min. perimeter.

$$300 = x \cdot y \Rightarrow y = \frac{300}{x}.$$



Goal: Minimize $2x + 2y$ = Perimeter.

Rewrite: Minimize $P(x) = 2x + 2 \cdot \frac{300}{x}$ for $x > 0$.

$P'(x) = 2 - \frac{600}{x^2}$, so crit. value in $(0, \infty)$ is $\sqrt{300}$.

Check signs of $P'(x)$ + see this is an abs min. So dim^{ns} are

$x = \sqrt{300}$, $y = \sqrt{300}$.

11/4/16 9AM

Projector broken!

Board day....

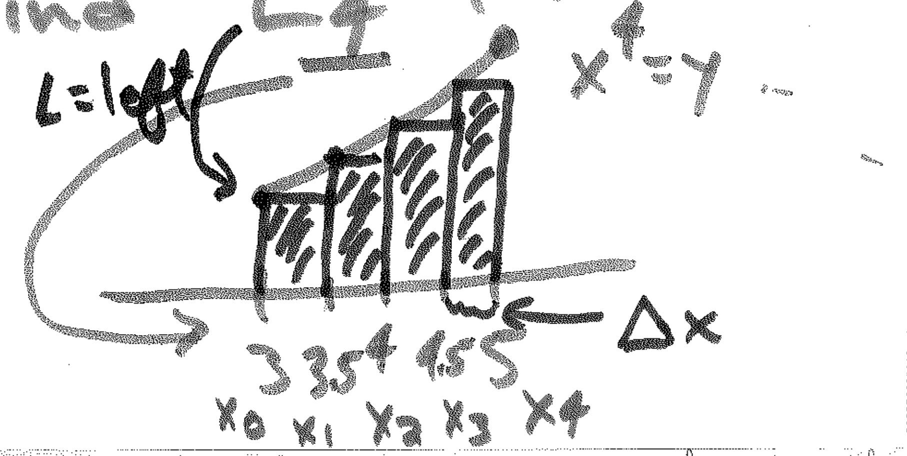
11/14/16 9AM

[1] Exam 3 Tuesday night – See Canvas announcement

[2] Summation formulas will be provided for Σi^2, Σi, etc.

[3] No Webwork or Quiz this week. Webwork D1 due Mon, Nov 21.

(4) w/neighbors: Find L_4 for x^4 on $[3,5]$.



$\underline{\Delta x}$: $\quad \dfrac{5-3}{4} = \dfrac{2}{4} = \dfrac{1}{2}$.

<u>Points to evaluate x^4</u>: $\quad 3, 3.5, 4, 4.5$. $\quad$ set $x_0 = 3$.

$\qquad x_0, x_1, x_2, x_3$. $\qquad x_i = 3 + i \cdot \frac{1}{2}$.

left-most endpt. $\qquad \Delta x$

<u>Sum</u>:

$$\frac{1}{2}\cdot 3^4 + \frac{1}{2}\cdot(3.5)^4 + \frac{1}{2}(4)^4 + \frac{1}{2}(4.5)^4$$

$$= 7177/16 = 448.5625.$$

$$\Delta x \cdot x_0^4 + \Delta x \cdot x_1^4 + \Delta x \cdot x_2^4 + \Delta x \cdot x_3^4$$

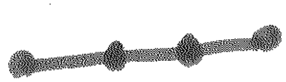
$$\sum_{i=0}^{3} \Delta x \cdot x_i^4 .$$

<u>Q</u>: why i from 0 to 3? <u>A</u>: If you use n subintervals, you produce $n+1$ endpts.

x_0 = left-most endpt.

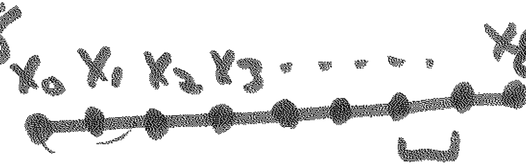
$x_i = x_0 + i \cdot \Delta x$

<u>Ex</u>: $n=3$



4 dots

<u>Ex</u>: $n=8$



$\underbrace{\ }_{\Delta x}$

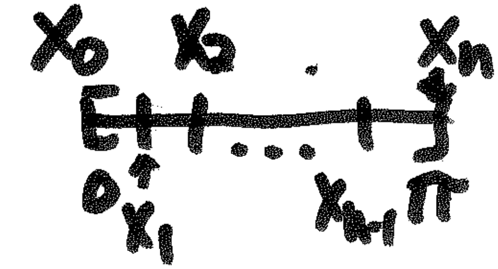
9 endpts.

<u>Ex</u>: Which of these expresses R_n for $f(x) = \sqrt{x} \cdot \sin(x)$ on $[0, \pi]$?

A) $\displaystyle\sum_{i=0}^{n-1} \left(\sqrt{\frac{i}{\pi}} \sin\left(\frac{i}{\pi}\right) \cdot \frac{\pi}{n} \right)$

* B) $\displaystyle\sum_{i=1}^{n} \left(\sqrt{\frac{i\pi}{n}} \sin\left(\frac{i\pi}{n}\right) \cdot \frac{\pi}{n} \right)$ ← Correct.

C) $\displaystyle\sum_{i=1}^{n} \left(\sqrt{\frac{i}{n}} \sin\left(\frac{i}{n}\right) \cdot \frac{i}{n} \right)$



<u>Observations</u>:

- $\Delta x = \dfrac{\pi - 0}{n} = \dfrac{\pi}{n}$.
- R_n means $\displaystyle\sum_{i=1}^{n} \Delta x \cdot f(x_i)$

$x_0 = 0, \quad x_i = 0 + i\Delta x$

$$= i \cdot \frac{\pi}{n}.$$

I see $R_n = \sum_{i=1}^{n} \frac{\pi}{n} \cdot \sqrt{i\frac{\pi}{n}} \sin\left(\frac{i\pi}{n}\right).$

Ex: Let A be area btwn x-axis and graph of $f(x) = 2x^2 - x + 3$ over $[2,4]$. Compute A.

I'll choose to compute $\lim_{n\to\infty} R_n = A$.

- $\Delta x = \frac{4-2}{n} = \frac{2}{n}$.

- $x_0 = 2$. $x_i = \underbrace{2}_{x_0} + i \cdot \underbrace{\frac{2}{n}}_{\Delta x}$.

<u>1st goal:</u> Simplify R_n.

$$R_n = \sum_{i=1}^{n} f(x_i) \cdot \Delta x = \Delta x \cdot \sum_{i=1}^{n} f(x_i) =$$

$$\frac{2}{n} \sum_{i=1}^{n} f\left(2 + \frac{2i}{n}\right) = ???$$

simplify $f\left(2+\frac{2i}{n}\right)$ before plugging in.

$$\begin{aligned} f\left(2+\frac{2i}{n}\right) &= 2\left(2+\frac{2i}{n}\right)^2 - \left(2+\frac{2i}{n}\right) + 3 \\ &= 2\left(4 + \frac{8i}{n} + \frac{4i^2}{n^2}\right) - 2 - \frac{2i}{n} + 3 \\ &= \frac{8}{n^2} \cdot i^2 + \frac{14}{n} i + 9. \end{aligned}$$

$$??? = \frac{2}{n} \cdot \sum_{i=1}^{n} \left(\frac{8}{n^2} i^2 + \frac{14}{n} i + 9 \right) = \circledast$$

<u>Claim</u>: This is a good simplified form.

Next, use our formulas for $\sum_{i=1}^{n} i$ and $\sum_{i=1}^{n} i^2$ to write R_n as a function of n.

<u>Picture for sum</u>:

	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$ ...
+	$\frac{8}{n^2} \cdot 1^2$	$\frac{8}{n^2} \cdot 2^2$	$\frac{8}{n^2} \cdot 3^2$	$\frac{8}{n^2} \cdot 4^2$	...
+	$\frac{14}{n} \cdot 1$	$\frac{14}{n} \cdot 2$	$\frac{14}{n} \cdot 3$	$\frac{14}{n} \cdot 4$	...
+	9	9	9	9	...

$$\circledast = \frac{2}{n}\left[\sum_{i=1}^{n} \frac{8}{n^2} i^2 + \sum_{i=1}^{n} \frac{14}{n} \cdot i + \sum_{i=1}^{n} 9\right]$$

$$= \frac{2}{n}\frac{8}{n^2}\sum_{i=1}^{n} i^2 + \frac{2}{n}\cdot\frac{14}{n}\sum_{i=1}^{n} i + \frac{2}{n}\cdot 9\sum_{i=1}^{n} 1$$

$$= \frac{16}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{28}{n^2}\cdot\frac{n(n+1)}{2} + \frac{18}{n}\cdot n$$

formulas

$$= \frac{112}{3} + \frac{22}{n} + \frac{8}{3n^2}.$$

simplify

So, $A = \lim_{n\to\infty} R_n = \lim_{n\to\infty}\left(\frac{112}{3} + \frac{22}{n} + \frac{8}{3n^2}\right) = \frac{112}{3}.$

answer!