10/3/16 12PM

[1] Check Canvas announcements for assignments this week.

[2] Today: Chain Rule!

This week: inst. vel $\Longrightarrow$ derivatives of fns.

So far: we know $\frac{d}{dx}$ for
- polys, exps, trig
- $+, -, \times, \div$ of fns

This week we will study:
- $f \circ g$
- $\frac{d}{dx}$ (inverse fns)
  $\rightarrow$ logs + inverse trig.

Chain Rule: The derivative of $(f \circ g)(x)$ is the product of $f'(g(x))$ and $g'(x)$ if f, g are differentiable.

NOTE: This is NOT !!! the same as $\underline{f'(x) \cdot g'(x)}$.
↑ nooooo!....

Form #1: $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$.

Form #2: **Leibniz notation:** $y = f(u)$, $u = g(x)$.

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{df}{du} \cdot \frac{dg}{dx}.$$

Ex: $f(x) = x^3$, $g(x) = x^2$. $F(x) = (f \circ g)(x)$

$$= (x^2)^3 = x^6.$$

Check w/ power rule: $F'(x) = 6x^5$.

Form #1: $f'(x) = 3x^2$
$g'(x) = 2x$

$$\Rightarrow f'(g(x)) \cdot g'(x) =$$
$$3 \cdot (x^2)^2 \cdot 2x = 3 \cdot 2 \cdot x^4 \cdot x$$
$$= 6 \cdot x^5. \checkmark$$
$$= F'(x).$$

Form #2: $y = f(u) = u^3$.
$u = g(x) = x^2$.

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 3u^2 \cdot 2x = 3(x^2)^2 \cdot 2x$$
$$= 6 \cdot x^5 = F'(x).$$

Ex: If $F(x) = \sqrt[3]{x^4+x}$, find $F'(x)$.

Q: Write $F = f \circ g$. $f = \sqrt[3]{x} = x^{1/3}$ "outside"

$g = x^4 + x$ "inside"

$$F'(x) = f'(g(x)) \cdot g'(x)$$

$$= \frac{1}{3}(x^4+x)^{-2/3} \cdot (4x^3+1).$$

Ex: $F(x) = (x^4-1)^{900}$, Find $F'(x)$.

$y = f(u) = u^{900}$, $u = g(x) = x^4 - 1.$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 900u^{899} \cdot (4x^3) = 900(x^4-1)^{899} \cdot 4x^3.$$

Ex: Find $\frac{d}{dx}\left(e^{\cos(x^2+1)}\right)$.

$$\boxed{\begin{aligned} y &= f(v) \\ v &= g(u) \\ u &= h(x) \end{aligned}}$$

Want: $e^{\cos(x^2+1)} = f \circ g \circ h$.

with $y = f(v) = e^v$

$v = g(u) = \cos(u)$

$u = h(x) = x^2+1$.

Then: $\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx} = e^v \cdot (-\sin(u)) \cdot (2x)$

$= e^{\cos(x^2+1)} \cdot \left(-\sin\left(x^2+1\right)\right) \cdot 2x$.

NOTE: $(f \circ g \circ h)'(x) = f'((g \circ h)(x)) \cdot g'(h(x)) \cdot h'(x)$.

Ex: (Quotient Rule)

Prod. rule

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{d}{dx}\left(f\cdot(g)^{-1}\right) = \frac{df}{dx}\cdot(g)^{-1} + f\cdot\frac{d}{dx}\left((g)^{-1}\right)$$

$$= \frac{\frac{df}{dx}}{g} + f\cdot\frac{-1}{g^2}\cdot\frac{dg}{dx} = \frac{\frac{df}{dx}}{g} - \frac{f\cdot\frac{dg}{dx}}{g^2}$$

$$= \frac{\frac{df}{dx}\cdot g - f\cdot\frac{dg}{dx}}{g^2}.$$

Ex: We can now handle

$$\frac{d}{dx}(b^x) \text{ for } b>0.$$

Ex: $b=1$, $1^x=1$, so $\frac{d}{dx}(1^x)=\frac{d}{dx}(1)=0.$

Ex: $b=e$, $\frac{d}{dx}e^x=e^x.$

Thm: $\frac{d}{dx}b^x=\ln(b)\cdot b^x.$

why? $b^x=e^{\ln(b^x)}=e^{\ln(b)\cdot x}$ $f=e^u$, $g=\ln(b)\cdot x$

$$\Rightarrow \frac{d}{dx}b^x=e^{\ln(b)\cdot x}\cdot\ln(b)=\ln(b)\cdot b^x.$$

__Ex__: Find $F'(x)$ for $F(x) = \sin(x^3)\cdot\cos(x^3)$.

$= \sin(x^3)\cdot\cos(x^3)$

① Prod rule then chain:

$$\frac{d}{dx}\left(\sin(x^3)\cdot\cos(x^3)\right) = \frac{d}{dx}\left(\sin(x^3)\right)\cdot\cos(x^3) + \sin(x^3)\cdot\frac{d}{dx}\left(\cos(x^3)\right) =$$

$$\cos(x^3)\cdot 3x^2\cdot\cos(x^3) + \sin(x^3)\cdot\left(-\sin(x^3)\cdot 3x^2\right)$$

$$= 3x^2\cos^2(x^3) - 3x^2\sin^2(x^3).$$

② $y = f(u) = \sin(u)\cdot\cos(u)$, $u = g(x) = x^3$.

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \left[\cos(u)\cdot\cos(u) + \sin(u)\cdot\left(-\sin(u)\right)\right]\cdot 3x^2$$

$$= \left[\cos(x^3)\cdot\cos(x^3) - \sin(x^3)\sin(x^3)\right]\cdot 3x^2.$$

10/5/16 12PM

Today: Implicit diff + derivatives of inverse fns.

Ex: $x^2+y^2=2^2=4$.

Idea: near a point (x,y) on the graph of this curve, y is a function of x. **Only** near a specific point. I treat y as a function of x, and apply $\frac{d}{dx}$ to both sides of eqn.

$$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(x^2) + \underbrace{\frac{d}{dx}(y^2)}$$

chain rule:
<u>outside</u>: u^2
<u>inside</u>: $u = y(x)$.

$$\frac{d}{dx}(4) = 0 = 2x + 2y \cdot \frac{dy}{dx}$$

Calc III:

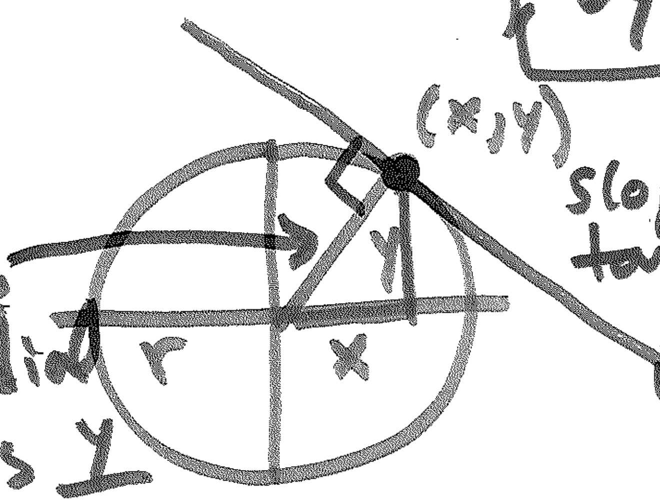
$$\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z}$$

<u>Step 2</u>: Set equal + solve for $\frac{dy}{dx}$.

$$0 = 2x + 2y\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x}{y}.$$

slope of radial line is $\frac{y}{x}$



slope of tangent line is $\frac{-x}{y}$.

Ex: $x^4+y^4=16$, Find $\frac{dy}{dx}$ = slope of tangent line at (x,y)

$\frac{d}{dx}(16)=0.$

$\frac{d}{dx}(x^4+\underbrace{y^4}_{\text{chain rule}\circledast}) = 4x^3+4y^3\cdot\frac{dy}{dx}$

Set equal: $0 = 4x^3+4y^3\cdot\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{-x^3}{y^3}.$

$\circledast$ $y^4 = (y(x))^4 \Rightarrow \frac{d}{dx}y^4 = \frac{d}{dx}(y(x))^4 = 4y^3\cdot\frac{dy}{dx}.$

<u>Ex</u>: Find $\frac{dy}{dx}$ for $\cos(x+y) = x\sin(y)$.

$$\frac{d}{dx}\Big(\underbrace{\cos}_{\text{out.}}(\underbrace{x+y}_{\text{ins.}})\Big) = \big[-\sin(x+y)\big]\cdot\frac{d}{dx}(x+y)$$

$$= \big[-\sin(x+y)\big]\left(1+\frac{dy}{dx}\right).$$

$$\frac{d}{dx}\big(x\cdot\sin(y)\big) = \frac{d}{dx}(x)\cdot\sin(y) + x\cdot\frac{d}{dx}\Big(\overbrace{\sin}^{\text{out}}(\overbrace{y}^{\text{ins.}})\Big)$$

$$= 1\cdot\sin(y) + x\cdot\cos(y)\cdot\frac{dy}{dx}$$

Set equal, solve for $\frac{dy}{dx}$: $\big[-\sin(x+y)\big]\left(1+\frac{dy}{dx}\right) = \sin(y) + x\cdot\cos y\cdot dy/dx$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sin y + \sin(x+y)}{\sin(x+y) + x\cos y}.$$

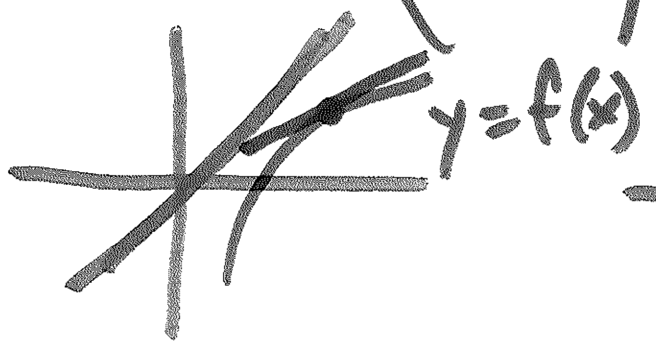
<u>Algebra</u>: $-\sin(x+y) = \frac{dy}{dx}\sin(x+y) = \sin y + x\cdot\cos y\cdot\frac{dy}{dx}$

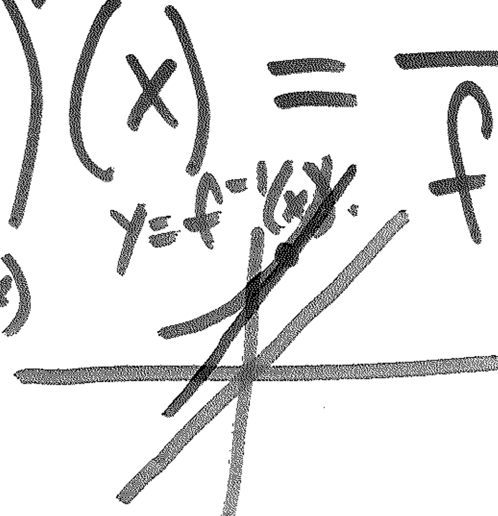
$$\Rightarrow \quad \frac{dy}{dx}\left(\underline{\sin(x+y) - x\cos y}\right) = \sin y + \sin(x+y)$$

$\Rightarrow$ divide both sides by

<u>Thm</u> <u>(Problem 77(a) in §3.5)</u>: Suppose f is diff, and f is 1-1, and f^{-1} is diff. Then

$$\left(f^{-1}\right)'(x) = \frac{1}{f'\left(f^{-1}(x)\right)}.$$



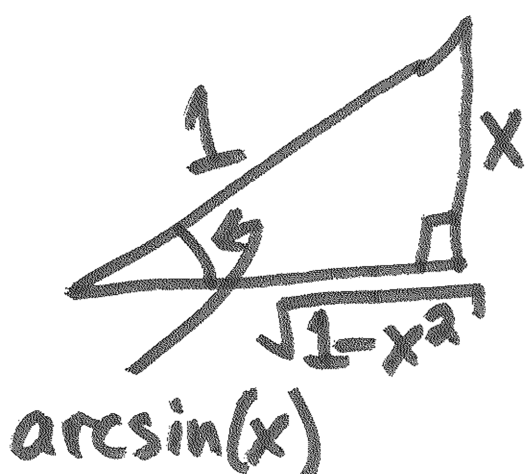


Ex: $f = \sin(x)$, $f^{-1} = \arcsin(x)$.

$f' = \cos(x)$.

$$\frac{d}{dx}\left(f^{-1}(x)\right) = \frac{d}{dx}\arcsin(x) = \frac{1}{\cos(\arcsin(x))}$$

See §3.5 for derivatives of all inv. trig fns!



$\cos(\arcsin(x)) = \sqrt{1-x^2}$.

So, $\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}$.

Ex: $f = \tan(x)$, $f^{-1} = \arctan(x)$.

$f' = \sec^2(x)$.

$$\frac{d}{dx}\arctan(x) = \frac{1}{\sec^2(\arctan(x))} \overset{\text{trig id.}}{=} \frac{1}{1+\tan^2(\arctan(x))}$$

$= 1/(1+x^2)$.

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$\boxed{1}$ Turn in W.A.

$\boxed{2}$ Today: derivatives of logs!

Recall: $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$.

Why? Chain rule!

NOTE: $(f \circ f^{-1})(x) = x$. Apply $\frac{d}{dx}$ to both sides

right side

$$\frac{d}{dx}(f \circ f^{-1})(x) = \underbrace{f'(f^{-1}(x)) \cdot (f^{-1})'(x) = 1}_{\Downarrow \text{ divide by } f'(f^{-1}(x))} = \frac{d}{dx}(x)$$

↑ left side

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}.$$

<u>Recall</u>: $\underbrace{\log_b(x)}_{=f^{-1}}$ is inverse fn for $\underbrace{b^x}_{=f}$.

So, we need $f'(x) = \ln(b)\cdot b^x$, as we've seen.

So, using formula above,

$$\frac{d}{dx}\log_b(x) = \frac{1}{\ln(b)\cdot b^{\log_b(x)}} = \frac{1}{\ln(b)\cdot x}$$

by inverse functions.

Know this!

Looking at $b = e$, we get

$$\frac{d}{dx}\ln(x) = \frac{1}{\ln(e)\cdot x} = \frac{1}{1\cdot x} = \frac{1}{x}. \leftarrow (*)$$

Know this!

NOTE: This provides a way to estimate e.

Recall: e is the value so that

$$\lim_{h\to 0} \frac{e^h - 1}{h} = 1. \leftarrow$$ This doesn't help compute e.

Set $f(x) = \ln(x)$, $f'(x) = \frac{1}{x}$. Note $f'(1) = 1$.

So, $\underbrace{1}_{\text{Left}} = f'(1) = \lim_{h\to 0} \frac{\ln(1+h) - \ln(1)}{h} = \lim_{h\to 0} \frac{\ln(1+h)}{h}$.

$$= \lim_{h\to 0} \frac{1}{h}\cdot \ln(1+h) = \underbrace{\lim_{h\to 0} \ln\left((1+h)^{1/h}\right)}_{\text{Right}}$$

exponentiate both sides.

$$\Rightarrow \quad e^1 = e^{\lim_{h\to 0} \ln\left((1+h)^{1/h}\right)} \underset{\substack{\uparrow \\ \text{by continuity} \\ \text{of } e^x}}{=} \lim_{h\to 0} e^{\ln\left((1+h)^{1/h}\right)} = \lim_{h\to 0} (1+h)^{1/h}.$$

Aside: If $f(x)$ is cts, then $f\left(\lim_{x\to a} g(x)\right) = \lim_{x\to a} f(g(x))$

Let's estimate e.

Let's try $h = \frac{1}{1000}$.

$$e \approx \left(1+\frac{1}{1000}\right)^{1/\frac{1}{1000}} = \left(\frac{1001}{1000}\right)^{1000} \approx 2.716\ldots$$

$$e \approx \left(1+\frac{1}{19276}\right)^{19276} = \left(\frac{19277}{19276}\right)^{19276} \approx \underline{2.7182\ldots}$$

at least 4 digits of accuracy.

So, this pattern shows

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n. \quad \left(\text{when } h = \frac{1}{n}\right)$$

Ex: Find $\frac{dy}{dx}$ for $y = \ln(2x)$.

Chain Rule: $\frac{d}{dx}\left(\underbrace{\ln}_{\text{outside}}(\underbrace{2x}_{\text{ins.}})\right) = \frac{1}{2x} \cdot 2 = \frac{1}{x}$.

Huh? weird.

Let's try $\ln(5x)$. $\Rightarrow \frac{d}{dx}\ln(5x) = \frac{1}{5x} \cdot 5 = \frac{1}{x}$.

NOT the best first step for logs ...

Use log laws $\ln(2 \cdot x) = \ln(2) + \ln(x)$.

$$\Rightarrow \frac{d}{dx}\ln(2x) = \frac{d}{dx}\left(\underbrace{\ln(2)}_{\text{constant}} + \ln(x)\right) = 0 + \frac{1}{x} = \frac{1}{x}.$$

Ex: $\frac{d}{dx}\ln(x^5) = \frac{d}{dx}(5\cdot\ln(x))$

$= 5\cdot\frac{d}{dx}\ln(x) = \frac{5}{x}$.

Chain rule gives $\frac{1}{x^5}\cdot 5x^4 = \frac{5}{x}$.

Ex: $\frac{d}{dx}\ln(x^3+x+1) = \frac{1}{x^3+x+1}\cdot(3x^2+1)$.

use chain rule

Ex: See Example 6 in §3.6 for

$\frac{d}{dx}\ln|x| = \frac{1}{x}$.

good to know!

<u>Ex</u>: $\frac{d}{dx}\left[\ln\left(\frac{x^2+1}{\sqrt{x-1}}\right)\right] =$

$$\frac{d}{dx}\left[\ln(x^2+1) - \ln\left(\underset{(x-1)^{1/2}}{\cancel{\sqrt{x-1}}}\right)\right] =$$

$$\frac{d}{dx}\left[\ln(x^2+1) - \frac{1}{2}\ln(x-1)\right] =$$

$$\frac{1}{x^2+1}\cdot 2x - \frac{1}{2}\cdot\frac{1}{x-1}\cdot 1 =$$

$$\frac{2x}{x^2+1} - \frac{1}{2(x-1)}.$$

__Ex:__ $\frac{d}{dx}\left((\ln(x))^{2/3}\right) = \frac{2}{3}(\ln(x))^{-1/3} \cdot \frac{1}{x}$

chain rule w/ power rule

$= \frac{2}{3x\sqrt[3]{\ln(x)}}.$

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$\boxed{1}$ See Canvas Announcements

$\boxed{2}$ Exam 2: You need to Know Def$^{\underline{n}}$ $\boxed{4}$ in §2.7, State + Apply it!

$\boxed{3}$ Sign Attendance

§3.7: Rates of Change

<u>Notation</u>: If $y = f(x)$, then for $f(x_1) = y_1$, $f(x_2) = y_2$, we write

$$\frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}.$$

<u>NOTE</u>: As $x_2 \to x_1$, we get $\frac{\Delta f}{\Delta x} \to \frac{df}{dx} = \frac{dy}{dx}$.

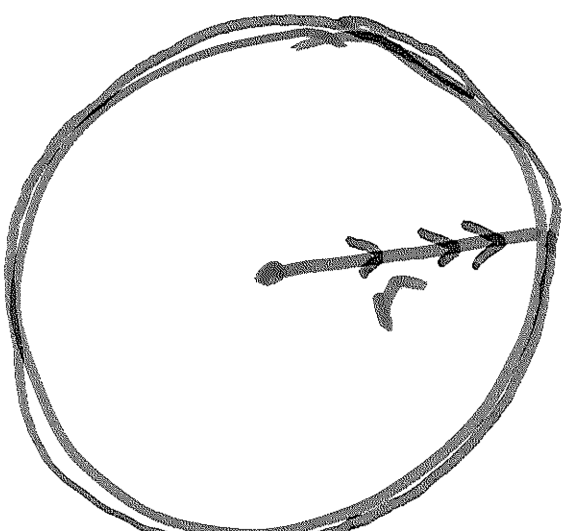
<u>we say</u>:
- $\frac{\Delta y}{\Delta x}$ is average rate of change from x_1 to x_2.
- $\frac{dy}{dx}$ is inst. rate of change at x.

Pay attention to units!

<u>NOTE</u>: "rate of change" = R.O.C.

__Ex:__ what is __R.O.C.__ of area of a circle with respect to radius?

$A(r) = \pi \cdot r^2 \Rightarrow$ Apply $\frac{d}{dr}$ to both sides.



$$\frac{dA}{dr} = \underbrace{2 \cdot \pi \cdot r}_{2\pi r \,=\, \text{circumference of circle}}$$

__Q:__ If r is in meters, & A is in m^2, what are units for $\frac{dA}{dr}$?

$\frac{dA}{dr}$ units $m^2/m = m$ ← unit measures circumference, so it agrees!

<u>NOTE</u>: Read §3.7 about Ohm's Law.

$$V = I \cdot R$$

voltage = current · resistance

we will ask you to find $\frac{dI}{dR}$, or $\frac{dR}{dV}$, etc.

You need to do some algebra to rewrite $V = IR$ as $\frac{V}{I} = R$, or $\frac{V}{R} = I$, etc.

<u>Ex (Biology)</u>: If $f(t)$ = population of bact. at t seconds, then

- $\frac{\Delta f}{\Delta t}$ = average growth rate of pop.
- $\frac{df}{dt}$ = inst. growth rate of pop.

eg. $f(t) = 203 \cdot 2^t$

Q: How quickly is the pop. growing at $t = 5$ sec?

A: We can approximate this w/ $f'(5)$.

$$f'(t) = \frac{d}{dt}(203 \cdot 2^t) = 203 \cdot \frac{d}{dt}(2^t)$$

$$= 203 \cdot \ln(2) \cdot 2^t.$$

formula

Aside: $\frac{d}{dt}(2^t) = \frac{d}{dt}\left(e^{\ln(2^t)}\right)$

$$= \frac{d}{dt}\left(e^{t \cdot \ln(2)}\right) = \ln(2) \cdot e^{t\ln(2)}$$

$$= \ln(2) \cdot 2^t.$$

So, $f'(5) = 203 \cdot \ln(2) \cdot 2^5$

$\approx 4502.68\ldots$ bact/sec.

<u>Ex (Physics)</u>: $f(t) =$ position in m at t sec

$f'(t) =$ velocity in m/s

$f''(t) =$ acceleration in m/s^2

$f'''(t) =$ jerk

<u>NOTE</u>: <u>Q</u>: when is an object moving in positive direction?

<u>A</u>: Compute $f'(t)$, find t w/ $f'(t) > 0$.

$\mapsto$ Read Example #1 in §3.7 for an extended example w/ lots of discussion.

Ex (Economics):

Let $C(x)$ = cost of producing x items.

Difference in cost between x and $x+1$

is $C(x+1) - C(x) \approx \underbrace{C'(x)}_{\text{"Marginal Cost" at } x}$.

If you are given a cost fn C + asked to find marginal cost, compute $C'(x)$.

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$\boxed{1}$ Sign attendance

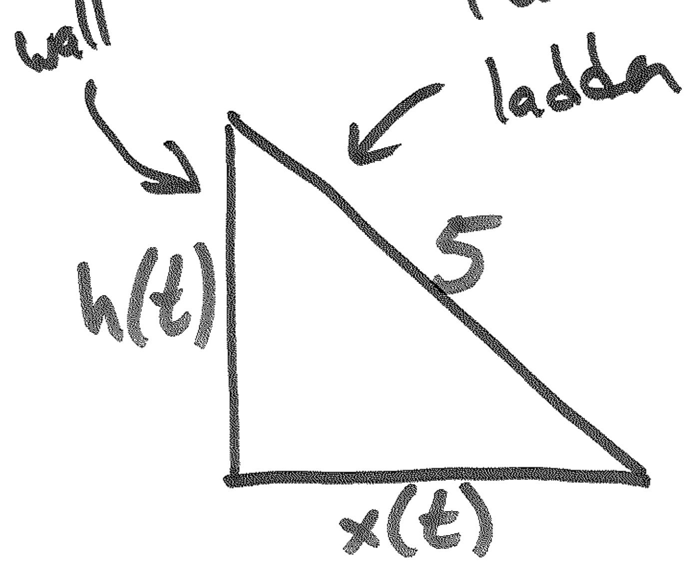
$\boxed{2}$ Exam 2 next week!

Today: § 3.9 – Related Rates

Idea: You have two (or more) functions $f(x)$ + $g(x)$ of the same variable x.
They are related through some algebraic/functional expression. Apply $\frac{d}{dx}$ to both sides of equality, get an equation involving $f(x)$, $g(x)$, $\frac{df}{dx}$, $\frac{dg}{dx}$. use this!

Ex: **Ladder Problem:** A 5-meter ladder rests against a wall, w/ bottom of ladder 1.5 m from base of wall. Suppose ladder slides away from wall on floor at 0.8 m/s, top keeping contact w/ wall. What is velocity of top of ladder after 1 sec?

Step 1: Draw a picture, introduce variables + functions, restate the problem w/ these.



t = time in sec

$x(t)$ = bottom length in m

$h(t)$ = height of top of ladder in m.

Problem: Find $\frac{dh}{dt}$ at $t = 1$ sec.

<u>Step 2</u>: Consider given info and create a relationship btwn your fns.

Given: $x(0) = 1.5$ m

$\frac{dx}{dt} = 0.8$ m/s for all t.

From triangle, use Pyth. Thm to write

$$(h(t))^2 + (x(t))^2 = 5^2.$$

Apply $\frac{d}{dt}$ $\Rightarrow$ $2 \cdot h(t) \cdot \frac{dh}{dt} + 2x(t) \cdot \frac{dx}{dt} = 0$

$$\Rightarrow \quad \frac{dh}{dt} = \frac{-x(t)}{h(t)} \cdot \underbrace{\frac{dx}{dt}}_{\text{not a fraction}} = \frac{-x(t) \cdot \frac{dx}{dt}}{h(t)}$$

<u>Step 3</u>: Evaluate. Want $\frac{dh}{dt}$ at $t=1$.

<u>we need</u>: $x(1)$, $h(1)$, $\frac{dx}{dt}$ at $t=1$.

Since $\frac{dx}{dt}$ is constant, we have

$$x(1) = x(0) + 0.8$$ ← change in x over 1 sec.

$$= 1.5 + 0.8 = 2.3\,m.$$

use pyth. Thm to solve

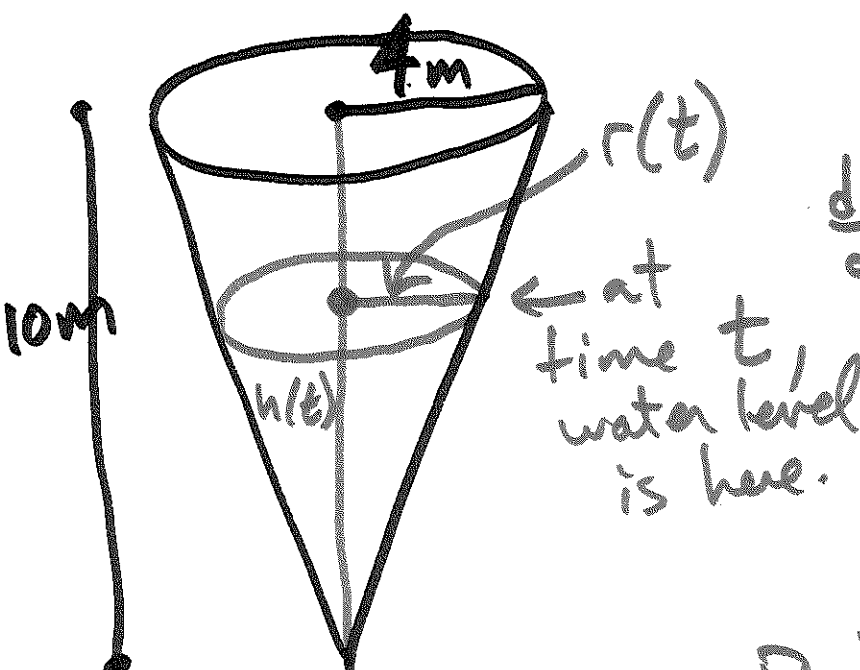
$$h(1) = \sqrt{5^2 - x(1)^2} = \sqrt{5^2 - 2.3^2} \approx 4.44\,m$$

Using our formula for $\frac{dh}{dt}$, our answer is

$$\left.\frac{dh}{dt}\right|_{t=1} = \frac{-x(1)}{h(1)} \cdot \left.\frac{dx}{dt}\right|_{t=1} \approx \frac{-2.3}{4.44} \cdot 0.8\ m/s.$$

Ex: A conical tank is 10 m high w/ a radius of 4 m at top. Water is poured in at a constant rate of $6\ m^3/min$. Find rate of H_2O-level increase when height of water is 5 m.

Soln: Picture:



we know:

radius at top 4 m

height of tank 10 m

$\frac{dV}{dt} \rightarrow$ change in volume is $6\ m^3/min$

$h(t)$ = height of water at time t.

$r(t)$ = radius of top of water at time t

$$V(t) = \frac{1}{3}\pi \cdot h(t) \cdot (r(t))^2.$$

Problem: Find $\frac{dh}{dt}$ at $h = 5$ m.

By similar triangles, $\frac{r(t)}{h(t)} = \frac{4}{10} \Rightarrow r(t) = 0.4 \cdot h(t)$.

$$\Rightarrow V(t) = \frac{1}{3} \cdot \pi \cdot (0.4)^2 \cdot (h(t))^3 .$$

$\Rightarrow$ Apply $\frac{d}{dt}$ + get $\frac{dV}{dt} = \frac{1}{\cancel{3}} \cdot \pi (0.4)^2 \cdot \cancel{3} (h(t))^2 \cdot \frac{dh}{dt}$.

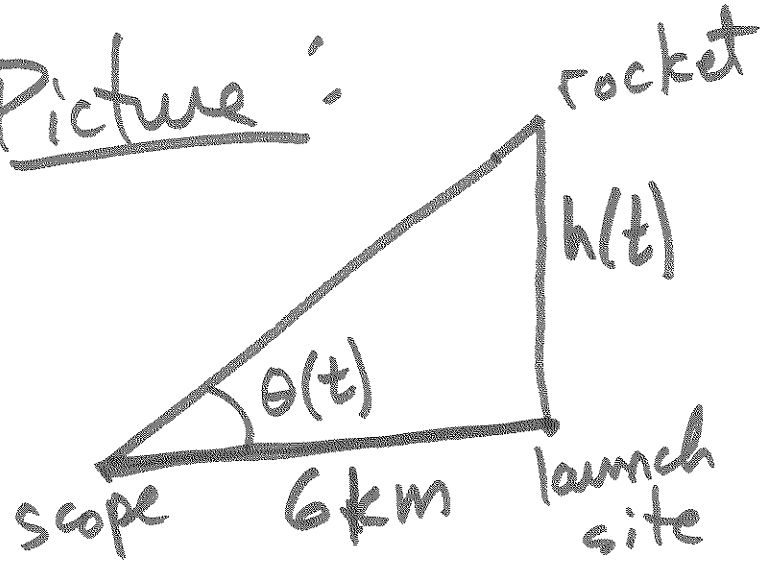
Solve for $\frac{dh}{dt}$ when $h = 5m$. Note: $\frac{dV}{dt} = 6 \, m^3/min$

So, $6 = \pi (0.4)^2 \cdot 5^2 \cdot \frac{dh}{dt}$

$$\Rightarrow \frac{6}{\pi (0.4)^2 \cdot 5^2} \, m/min = \frac{dh}{dt} .$$

<u>Ex</u>: Telescope tracking rocket launch. Scope is 6 km from launch site, rocket travels vertically. When angle btwn scope + ground is $\frac{\pi}{3}$, then the angle is changing at 0.9 rad/min.
What is the velocity of rocket when angle is $\pi/3$?

<u>Picture</u>:



t = time in min.

at $\theta = \frac{\pi}{3}$, $\frac{d\theta}{dt} = 0.9$ rad/min.

<u>Goal</u>: Find $\frac{dh}{dt}$.

NOTE: $\tan\theta = \frac{h}{6}$

Apply $\frac{d}{dt} \Rightarrow \sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{6} \cdot \frac{dh}{dt}$

Rewrite as $\frac{dh}{dt} = \frac{6}{\cos^2\theta} \cdot \frac{d\theta}{dt}$.

when $\theta = \frac{\pi}{3}$ & $\frac{d\theta}{dt} = 0.9$, $\frac{dh}{dt} = \frac{6}{(1/2)^2} \cdot 0.9$ Km/min.