9/28/16 ~~9AM~~ 12 PM

$\boxed{1}$ Sign Attendance

$\boxed{2}$ Quiz Thurs, written Assig. Fri, Webworks too!

$\boxed{3}$ Compute $\frac{d}{dx}\left(3e^x + x^7 - 3x^e\right)$ "e"

$= 3e^x + 7x^6 - 3 \cdot e x^{e-1}$

§3.2: Product + Quotient Rules.

Q: What is $(f \cdot g)'$?

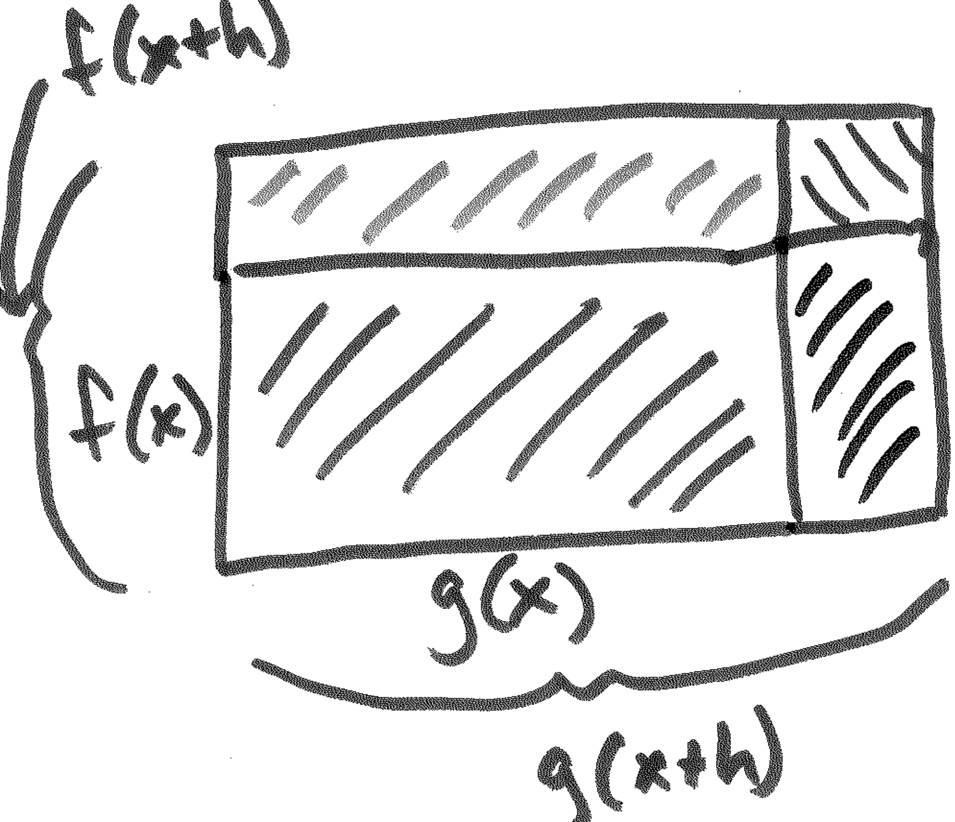
$$\left(f(x)\cdot g(x)\right)' = \lim_{h\to 0} \frac{f(x+h)\cdot g(x+h) - f(x)\cdot g(x)}{h}$$

$$= (*)$$

NOTE: $$F'(x) = \lim_{h\to 0} \frac{F(x+h) - F(x)}{h}.$$

$(*)$ is tough.... not possible to "get rid" of h in denominator.

Picture for numerator: Assume f + g are increasing.



Blue lower-left rect is area $f(x) \cdot g(x)$.

$f(x+h)g(x+h) - f(x)g(x)$ is area outside this but within larger rect.

So, $f(x+h)g(x+h) - f(x) \cdot g(x)$ equals

$$\big(f(x+h)-f(x)\big)\,g(x) + \big(g(x+h)-g(x)\big)\cdot f(x) + \big(f(x+h)-f(x)\big)\cdot\big(g(x+h)-g(x)\big)$$

NOTE: If you expand this + cancel, you get
$f(x+h)\,g(x+h) - f(x)\,g(x)$.

$$(f\cdot g)' = \circledast = \lim_{h\to 0}\left(\frac{(f(x+h)-f(x))g(x) + (g(x+h)-g(x))f(x) + (f(x+h)-f(x))(g(x+h)-g(x))}{h}\right)$$

$$= \lim_{h\to 0}\frac{(f(x+h)-f(x))g(x)}{h} + \lim_{h\to 0}\frac{(g(x+h)-g(x))f(x)}{h} + \lim_{h\to 0}\frac{(f(x+h)-f(x))(g(x+h)-g(x))}{h}$$

$$= g(x)\cdot\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} + f(x)\cdot\lim_{h\to 0}\frac{g(x+h)-g(x)}{h} + \left(\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\right)\cdot\left(\lim_{h\to 0} g(x+h)-g(x)\right) = \bigstar$$

If f and g are both diff, then 1) f' + g' exist and 2) g is cts.

$= g(x) \cdot f'(x) + f(x) \cdot g'(x) + f'(x) \cdot 0$.

by continuity.

Thm: If f, g diff, then $(f \cdot g)' = g \cdot f' + f \cdot g'$
(Product Rule)

Ex: Find $\frac{d}{dx}(x^2)$ using only product rule, not $\frac{d}{dx}(x^2) = 2x$ by power rule.

Note: $x^2 = x \cdot x$.

$$\frac{d}{dx}(x \cdot x) = \frac{d}{dx}(x) \cdot x + x \cdot \frac{d}{dx}(x)$$
$$= 1 \cdot x + x \cdot 1$$
$$= 2x.$$

<u>Ex</u>: Find $\frac{d}{dx}(x^7)$ knowing only
- prod. rule
- $\frac{d}{dx}(x^2) = 2x$,
- $\frac{d}{dx}(x^5) = 5x^4$.

<u>NOTE</u>: $x^7 = x^{5+2} = x^5 \cdot x^2$.

So, $\frac{d}{dx}(x^7) = \frac{d}{dx}(x^5 \cdot x^2) = 5x^4 \cdot x^2 + x^5 \cdot 2x$

$= 5x^6 + 2x^6$

$= 7x^6$.

<u>Ex</u>: $\frac{d}{dx}\Big(\underbrace{(x^3+x)}_{f}\underbrace{e^x}_{g}\Big) = \underbrace{(3x^2+1)}_{f'} \cdot \underbrace{e^x}_{g} + \underbrace{(x^3+x)}_{f} \cdot \underbrace{e^x}_{g'}$

__Ex:__ $\frac{d}{dx}\left(\frac{e^x}{\sqrt{x}}\right) = \frac{d}{dx}\left(e^x \cdot x^{-1/2}\right)$

$= \frac{d}{dx}(e^x) \cdot x^{-1/2} + e^x \cdot \frac{d}{dx}\left(x^{-1/2}\right) = e^x \cdot x^{-1/2} + e^x \cdot \frac{-1}{2} \cdot x^{-3/2}.$

__Q:__ What if you needed: $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{d}{dx}\left(f \cdot (g)^{-1}\right)$

$\underset{\text{Prod Rule}}{=} \frac{df}{dx} \cdot (g)^{-1} + f \cdot \frac{d}{dx}\left((g)^{-1}\right).$ $(g)^{-1}$ is a composition of fns.

Next wk, we will learn __chain rule__ for $(f \circ g)'$, and this will work.

This method results in:

<u>Quotient Rule:</u> $\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2}$.

<u>Ex:</u> $\frac{d}{dx}\left(\frac{e^x}{1+x^2}\right) = \frac{(1+x^2) \cdot e^x - e^x \cdot 2x}{(1+x^2)^2}$.

<u>Ex:</u> Find $f''''(x)$ for $f(x) = \frac{e^x}{x^2}$.

Rewrite $f(x) = e^x \cdot x^{-2}$, use prod. rule.

$$f'(x) = e^x \cdot x^{-2} + e^x \cdot -2 \cdot x^{-3}$$
$$= e^x \cdot x^{-2} - 2e^x x^{-3}.$$

9/30/16 12PM

$\boxed{1}$ Written assignments due today!

Goal for today: $\frac{d}{dx}(\text{trig fns})$.

Thm: $\frac{d}{dx}(\sin x) = \cos x$ ← Focus today!

$\frac{d}{dx}(\cos x) = -\sin x$. ← similar...

NOTE: All other trig fns can be handled using these + Quot. Rule (i.e. prod. rule + chain rule)

SEE §3.3 !!!

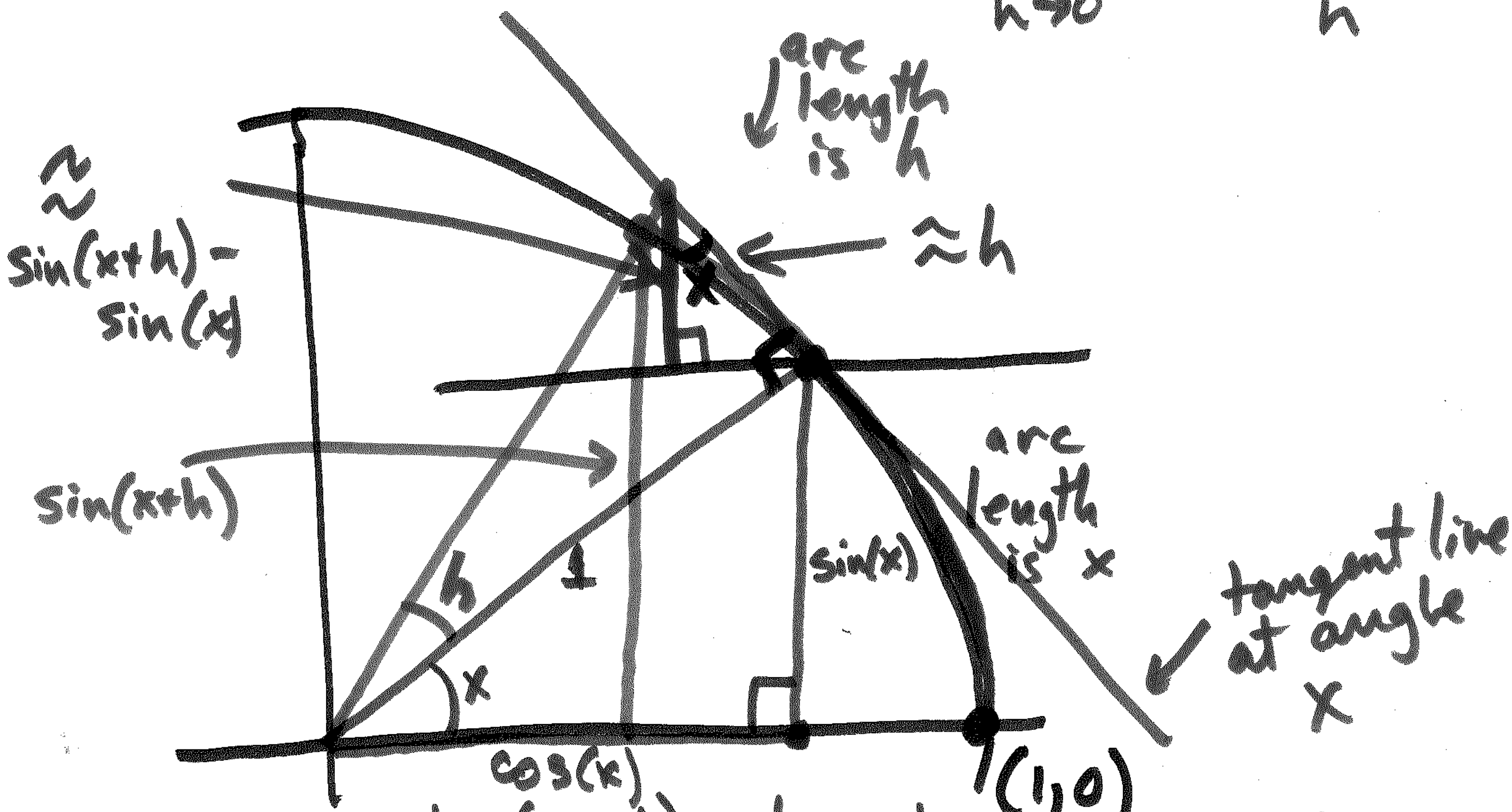
What is $\frac{d}{dx}(\sin x)$? 3 ~~answers...~~ approaches....

I Look @ graph of $\sin x$.

→ see desmos, guess $\frac{d}{dx}\sin x = \cos x$.

II Look @ unit circle ——

unit circle quadrant. $(\sin x)' = \lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h}$



In 2 triangles (right) w/ angle x, compare $\frac{\text{adj}}{\text{hyp}}$ for $\angle x$. $\frac{\cos(x)}{1} \approx \frac{\sin(x+h) - \sin(x)}{h}$.

Guess (again) $\frac{d}{dx}\sin x = \cos x$.

III Use $\frac{d}{dx}\sin x = \lim_{h\to 0} \frac{\sin(x+h)-\sin(x)}{h} =$

↑ angle sum form. for $\sin(x+h)$

$$\lim_{h\to 0} \frac{\sin x\cos h + \cos x \sin h - \sin(x)}{h} =$$

$$\lim_{h\to 0} \frac{(\sin x)(\cos h - 1)}{h} + \lim_{h\to 0} \frac{(\cos x)(\sin h)}{h} =$$

$$(\sin x)\left(\lim_{h\to 0} \frac{\cos h - 1}{h}\right) + (\cos x)\left(\lim_{h\to 0} \frac{\sin h}{h}\right) = *$$

Recall: $\sin(x+h) = \sin x \cosh + \cos x \sin h$.

If we want $\circledast = \cos x$, we need:

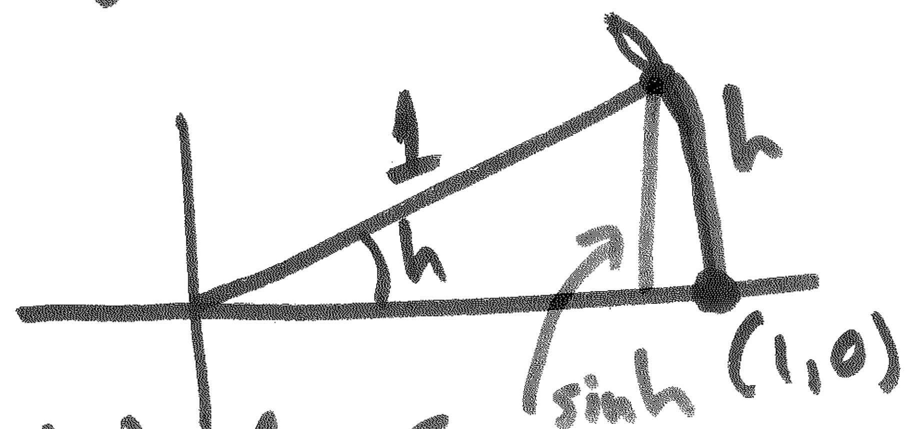
Thm: #1. $\displaystyle\lim_{h\to 0} \frac{\sin h}{h} = 1$.

#2. $\displaystyle\lim_{h\to 0} \frac{\cos h - 1}{h} = 0$

$\boxed{\text{So, } \circledast = \sin x \cdot 0 + \cos x \cdot 1 = \cos x. \text{ done!}}$

Note: #2 is a consequence of #1.
See §3.3.

Idea for why #1 true:



Full Proof in §3.3.

Ex: Find $\lim_{x \to 0} \frac{\sin^2(3x)}{x^2} = \star$.

$$\star = \lim_{x \to 0} \left(\frac{\sin(3x)}{x} \cdot \frac{\sin(3x)}{x} \right)$$

$$= \lim_{x \to 0} \frac{\sin(3x)}{x} \cdot \lim_{x \to 0} \frac{\sin(3x)}{x}$$

$$= \left(\lim_{x \to 0} \frac{3 \cdot \sin(3x)}{3 \cdot x} \right)^2 = 3^2 \left(\lim_{x \to 0} \frac{\sin 3x}{3x} \right)^2$$

$$= 9 \cdot 1 = 9.$$

Ex:

$$f(x) = \sin x$$

$$f'(x) = \cos x$$

$$f''(x) = -\sin x$$

match!

$$f'''(x) = -\cos x$$

$$f^{(4)}(x) = \sin x$$

Related to Euler's formula:

$$e^{ix} = \cos x + i\sin x.$$