<u>9/2/16</u> 12 PM

1 Turn in your written assignment on front table <u>WITH</u> <u>YOUR</u> <u>SECTION</u>.

2 Discuss with neighbors:

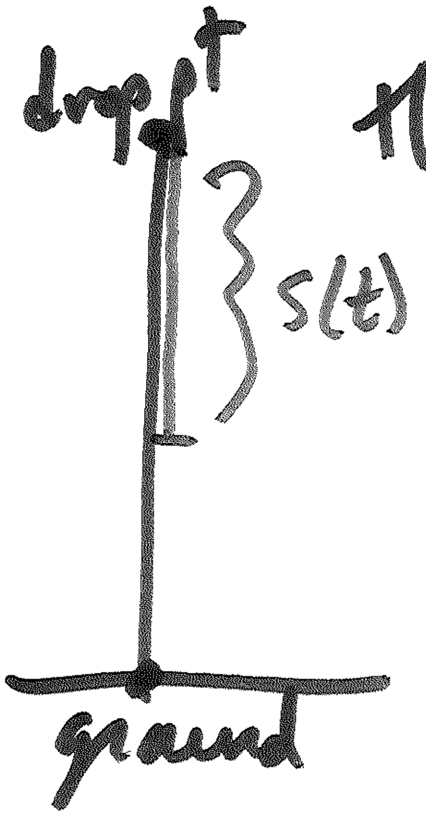
(a) What was something interesting you learned in 1st week of Calc?

(b) What is difference between <u>speed</u> and <u>velocity</u>?

Ex: Suppose we drop an object from high up. Ignore reality, apply Galileo's law and obtain:

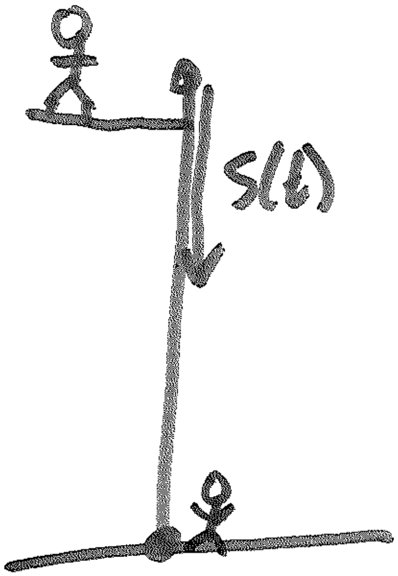
If $s(t)$ = distance (in meters) traveled in t seconds,

then $s(t) = 4.9t^2$. m/s ← units!



Q: should we consider $s(t)$ to be a positive distance or a negative distance?

A: It depends on your def$^{\underline{n}}$s and your point of reference.



From drop pt, $s(t)$ is positive dist. away from you.

From ground, object is moving toward you, i.e. "backwards", so here negative makes sense.

MORAL: Be careful w/ mathematical models; they should make sense to *you*.

Q: What is the best way to describe/define velocity?

Student responses:

on the right track

- rate of position change
- speed in terms of direction
- displacement over time
- slope of position + time graph

much further down the right track

- derivative of positive
- integral of acceleration
- vector
- slope of tangent line

These are result of reflection on Q.

There are 2 ways to think about velocity:

- velocity at a moment in time (instantaneous)
- velocity over a period of time. (average)

Start:

<u>Def</u>: Given an object w/position $s(t)$ m at time t sec, the <u>average velocity</u> of object between time t_1 + t_2 is

$$\frac{\text{change in position}}{\text{change in time}} = \frac{\Delta s}{\Delta t} = \frac{s(t_2)-s(t_1)}{t_2-t_1} \text{ m/s}.$$

<u>Q</u>: For $s(t) = 4.9t^2$ m, which is greatest?

(a) $\dfrac{\Delta s}{\Delta t}$ over $[0,1]$ $\longrightarrow$ $\dfrac{s(1)-s(0)}{1-0} = 4.9\ m/s$

(b) $\dfrac{\Delta s}{\Delta t}$ over $[1/2, 3/4]$ $\longrightarrow$ $\dfrac{s(3/4)-s(1/2)}{3/4-1/2} = 6.125\ m/s$

(c) $\dfrac{\Delta s}{\Delta t}$ over $[3, 3.1]$ $\longrightarrow$ $\dfrac{s(3.1)-s(3)}{3.1-3} =$

$29.89\ m/s$

Q: Do these average velocities make sense physically?

Q: How do we compute instantaneous velocity?

A: We can't yet. We need calculus!

But, we can estimate:

Ex: $s(t) = 4.9t^2$, estimate inst. vel at $t = 3.2$ sec.

(i) $$\frac{s(3.21) - s(3.2)}{3.21 - 3.2} = 31.409 \text{ m/s}$$

(ii) $$\frac{s(3.2) - s(3.199)}{3.2 - 3.199} = 31.3551 \text{ m/s}.$$

(ii) $$\frac{s(3.25) - s(3.15)}{3.25 - 3.15} = 31.36 \text{ m/s}.$$

Def$^{\underline{n}}$: Inst. velocity at t_0 is limiting value of $\frac{\Delta s}{\Delta t}$ for time intervals over t_0.

Ex: Estimate inst. velocity at **$t = 2$ sec.** if

t sec	1.0	1.5	1.8	1.9	2.1	2.2	2.5	3.0
s m	2.0	2.75	3.44	3.71	4.31	4.64	5.75	8.0

A: There isn't a "best" choice. Probably [1.9, 2.1]? Check several, compare.

☑ Sign attendance on table!

9/7/2016 **12PM**

$\boxed{1}$ Check Canvas Announcement for WebWork, Quiz, Written Assign. info.

$\boxed{2}$ MATH CLUB Organizing Meeting Sept 8 (tomorrow) 5-6PM, POT 745

$\boxed{3}$ With neighbors: Estimate inst. velocity of $f(x) = \sqrt{x}$ at $x=4$. Obtain accurate results to 2 significant digits.

On interval $[4, 4+t]$, t small, average vel. is

$$\frac{f(4+t)-f(4)}{4+t-4} = \frac{\sqrt{4+t}-2}{t}.$$

<u>TEST</u>: $t = 0.01 \Rightarrow \frac{\Delta f}{\Delta t} = 0.249844$

wolfram alpha

$t = 0.0001 \Rightarrow \frac{\Delta f}{\Delta t} = 0.249998$

$t = 0.00000001 \Rightarrow \frac{\Delta f}{\Delta t} = 0.25$

$= 10^{-8}$

Let's double-check; rewrite last estimate w/out negative powers of 10.

$$\frac{\Delta f}{\Delta t} = \frac{\sqrt{4+10^{-8}} - 2}{10^{-8}} = 10^{8}\left(\sqrt{4+10^{-8}} - 2\right)$$

See Desmos for crazy graph!

$$= \sqrt{4 \cdot 10^{16} + 10^{8}} - 2 \cdot 10^{8}$$

use $10^{8} = \sqrt{10^{16}}$

Instantaneous velocity is the "limiting value" of average velocity $\longrightarrow$ we need a precise mathematical way to study this phenomenon.

This leads to <u>LIMITS</u>.

Defn: Suppose $f(x)$ is defined on an open interval containing the number a. We write $\lim_{x \to a} f(x) = L$ if we can make the value of $f(x)$ as close to L as we like by restricting x to be close enough to a, (but not $x=a$).

Q: Why do we not allow $x=a$?

Hint: Think about $\dfrac{f(4+t)-f(4)}{t+4-4}$ .....

$$\text{defn}\begin{cases} f \longrightarrow \dfrac{\sqrt{4+t}-2}{t} \\ x \longrightarrow t \\ a \longrightarrow 0 \end{cases}$$

With average velocity for position function $s(t)$, on interval $[b, b+x]$, we want $x \to 0$ to estimate inst. vel.

$$\frac{\Delta s}{\Delta t} = \frac{s(b+x)-s(b)}{b+x-b} = \frac{s(b+x)-s(b)}{x}.$$

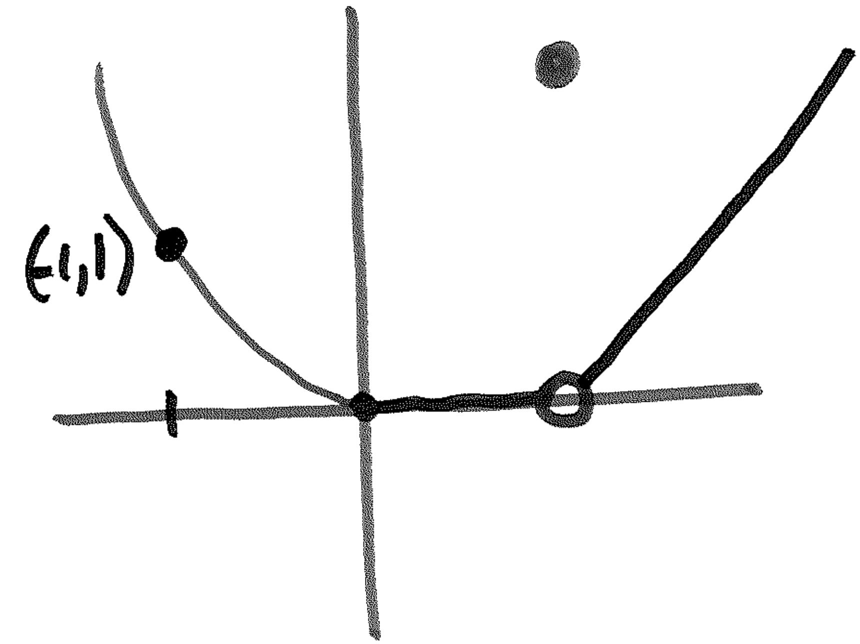
If we set $x=0$, we would divide by zero! We can't allow this!

Here, $a=0$, so we need $x \neq a = 0$.

For today, + for recitation + A4, trust your calculator (Ha!).

We need a precise set of rules for limits ⟹ See Friday's lecture (Sept 9).

<u>Ex</u>: Let $f(x) = \begin{cases} x^2 & x \le 0 \\ 0 & 0 \le x < 1 \\ 2 & x = 1 \\ x-1 & x > 1. \end{cases}$



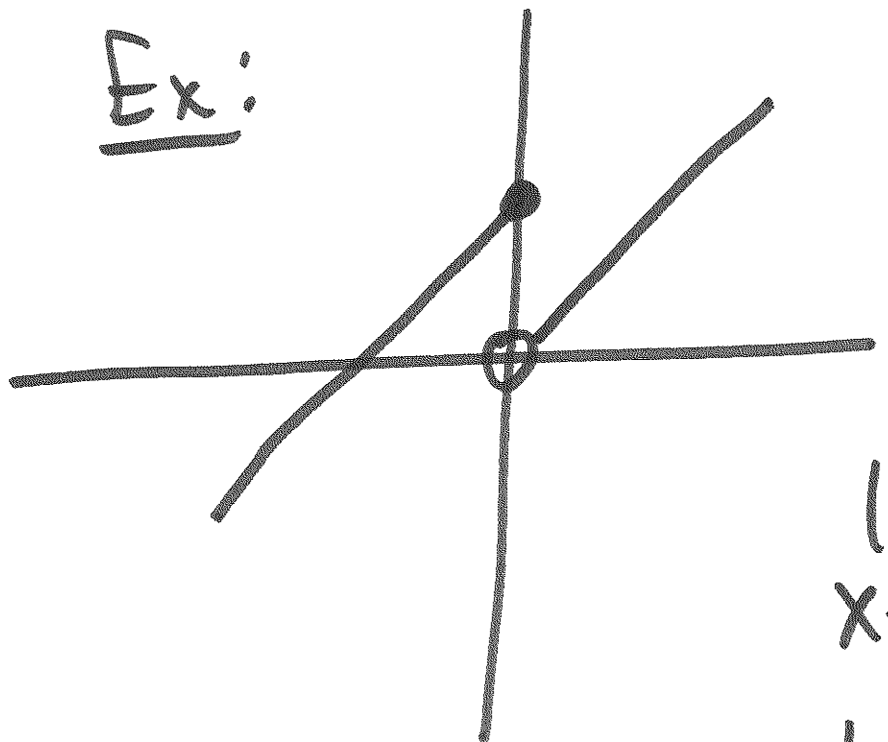
<u>Q</u>: $\lim_{x\to 1} f(x) = 0 \leftarrow$ <u>NOT</u> $f(1) = 2$.

$\lim_{x\to -1} f(x) = 1 \leftarrow$ <u>IS</u> $f(-1)$.

$\boxed{1}$ Read in §2.2 about one-sided limits

$$\lim_{x\to a^-} f(x) \quad \text{and} \quad \lim_{x\to a^+} f(x).$$

<u>Ex:</u>



$$f(x) = \begin{cases} x+1 & x \le 0 \\ x & x > 0 \end{cases}$$

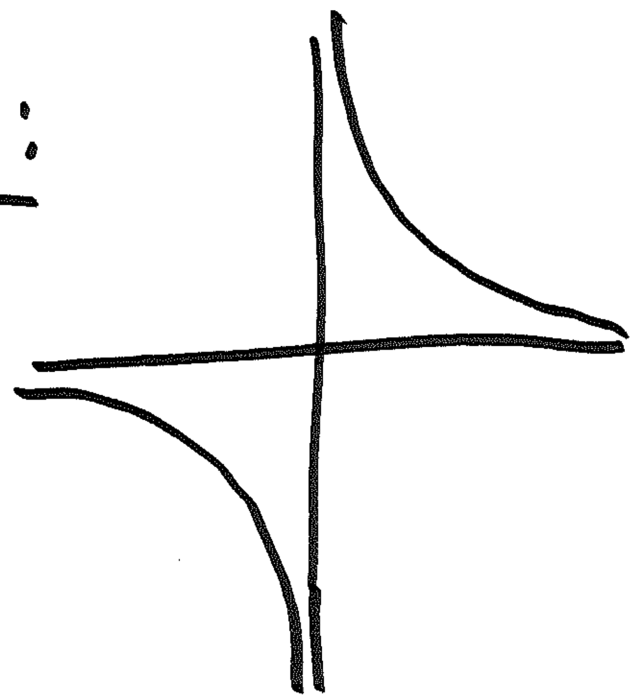
$$\lim_{x\to 0^-} f(x) = 1$$

$$\lim_{x\to 0^+} f(x) = 0$$

$\boxed{2}$ Read in §2.2 about infinite limits

$$\lim_{x \to a} f(x) = \pm\infty.$$

Ex:



$$f(x) = \frac{1}{x}$$

$$\lim_{x \to 0^-} f(x) = -\infty$$

$$\lim_{x \to 0^+} f(x) = \infty$$