9/2/16 $\boxed{\text{9AM}}$

$\boxed{1}$ Turn in your written assignment on front table **WITH YOUR SECTION.**

$\boxed{2}$ Discuss with neighbors:

(a) What was something interesting you learned in 1st week of Calc?

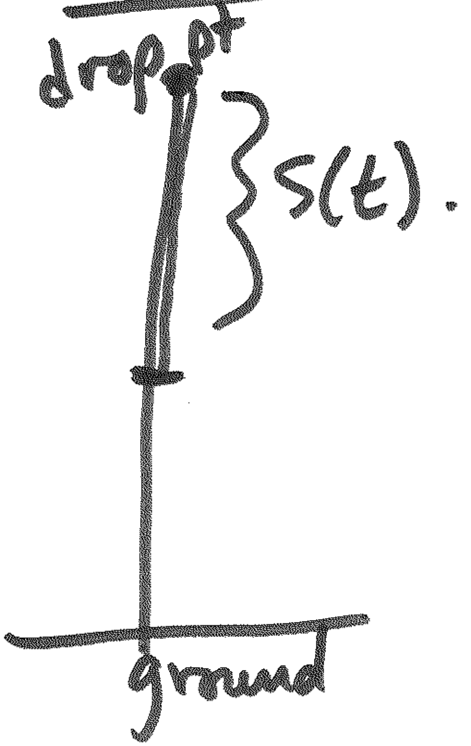
(b) What is difference between speed and velocity?

Ex: Suppose we drop an object from a high drop point. Ignoring physical reality, Galileo's Law applies:

If $s(t)$ = distance in meters traveled after t seconds, then

$$s(t) = 4.9 \cdot t^2 \text{ m/s} \leftarrow \text{units!}$$

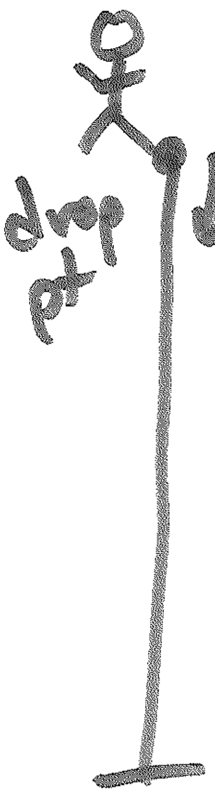
Picture:



Q: Should $s(t)$ be a positive or negative value?

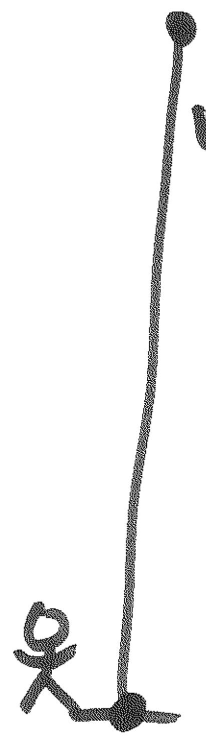
A: It depends on your observation point.

<u>Your book</u> says $s(t) = +4.9 \cdot t^2$.



watching it get farther from you.

<u>BUT</u>:



watching it move towards you.

$$\underbrace{s(t) = -4.9 \cdot t^2}_{\text{usual physical convention.}}$$

<u>Moral</u>: Different mathematical models can describe same situation.

For today, use $s(t) = 4.9 \cdot t^2$.

Q: Given a position function $f(t)$, where $f(t)$ is in meters, t in seconds, how can we determine a velocity of the object in motion?

Student responses:

- Integrate something ← we don't know...
- Power rule for first derivative
- find tangent line at a point
- limit as $\Delta t \to 0$
- * change in m / change in sec

- ~~Power~~ Quotient

Difference

- Derivative of position.

<u>What is velocity?</u> Two things

- how fast something is going (speed)
- direction it is going.

In Calc I, things move in a straight line.

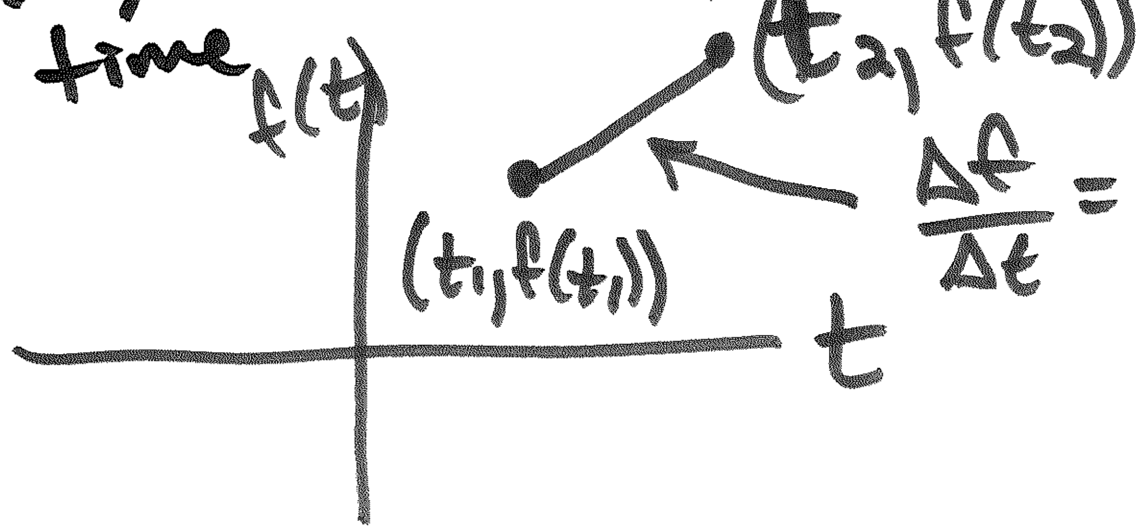
e.g. drop an object straight down.

→ 2 ways to think about velocity:

- at a moment in time (instantaneous)
- over an interval of time (average)

<u>Def$^{\underline{n}}$</u>: Given t seconds & $f(t)$ meters measuring the position of an object, the <u>average</u> <u>velocity</u> of the object between time t_1 and t_2 is

$$\frac{\text{change in pos}}{\text{change in time}} = \frac{\Delta f}{\Delta t} = \frac{f(t_2)-f(t_1)}{t_2-t_1}\ \frac{m}{s}.$$



$\frac{\Delta f}{\Delta t}$ = slope of this line.

Q: If position is $s(t) = 4.9\,t^2$, which $\underbrace{\text{interval}}_{\text{time}}$ has greatest average velocity?

(a) $[0,1] \longrightarrow \dfrac{s(1)-s(0)}{1-1} = 4.9 \text{ m/s}$

(b) $[1/2, 3/4] \longrightarrow \dfrac{s(3/4)-s(1/2)}{3/4-1/2} = 6.125 \text{ m/s}$

(c) $[3, 3.1] \longrightarrow \dfrac{s(3.1)-s(3)}{3.1-3} = 29.89 \text{ m/s}$

For $s(t) = 4.9\,t^2$, choose intervals close to $t = 3.2$ s.

$[3.2, 3.21] \longrightarrow \dfrac{\Delta s}{\Delta t} = 31.409 \text{ m/s}$

$[3.199, 3.2] \longrightarrow \dfrac{\Delta s}{\Delta t} = 31.3551 \text{ m/s}$

$[3.15, 3.25] \to \frac{\Delta s}{\Delta t} = 31.36$ m/s.

At 3.2 s, object is moving at ≈ 31.4 m/s

This leads to idea of <u>instantaneous</u> <u>velocity</u>. For t_0 = time in sec, $f(t)$ = position in meters, the <u>inst. velocity</u> of object at time t_0 is the limiting value of $\frac{\Delta f}{\Delta t}$ for time intervals close to t_0. e.g. $\frac{f(t_0+t) - f(t_0)}{t_0+t - t_0}$ for $[t_0, t_0+t]$.

<u>Q</u>: Suppose f and t satisfy:

t sec	1.0	1.5	1.8	1.9	2.1	2.2	2.5	3.0
f m	2.0	2.75	3.44	3.71	4.31	4.64	5.75	8.0

What is **best** interval to use to estimate inst. velocity at $t=2$?
What is you estimate?

I don't know...

If you use $[1.9, 2.1]$, get $\frac{\Delta f}{\Delta t} \approx 3\, m/s$

9/7/2016 9AM

0 Sign attendance on table!

1 Check Canvas Announcement for Webwork, Quiz, Written Assign. info

2 MATH CLUB Organizing Meeting Sept 8 (tomorrow) 5-6PM, POT 745

3 With neighbors: Estimate inst. velocity of $f(x)=\sqrt{x}$ at $x=4$. Obtain accurate results to 2 significant digits.

On interval $[4, 4+t]$, with t small, average velocity is

$$\frac{f(4+t)-f(4)}{4+t-4} = \frac{\sqrt{4+t}-2}{t}.$$

<u>Estimates:</u> (with wolframalpha)

$$t = 0.01 \Rightarrow \frac{\Delta f}{\Delta t} = 0.249844$$

$$t = 0.0001 \Rightarrow \frac{\Delta f}{\Delta t} = 0.249998$$

$$t = 0.00000001 \Rightarrow \frac{\Delta f}{\Delta t} = 0.25$$

$$= 10^{-8}$$

Try again: get rid of negative powers of 10. Set $t = 10^{-8}$.

$$\frac{\sqrt{4+10^{-8}}-2}{10^{-8}} = 10^{8}\sqrt{4+10^{-8}} - 10^{8}\cdot 2$$

$$= \sqrt{4\cdot 10^{16}+10^{8}} - 2\cdot 10^{8}.$$

since $10^{8} = \sqrt{10^{16}}$

Graph $\frac{\sqrt{4+t}-2}{t}$ on desmos, zoom in on $(0, 1/4)$, it gets weird...

The issue here is that we want to know the "limiting value" of

$$\frac{f(4+t)-f(4)}{4+t-4}$$ for small t-values.

This leads to the theory of <u>limits</u>.

<u>Def^{n}</u>: Suppose $f(x)$ is defined on an open interval containing the number a. We write $\lim_{x \to a} f(x) = L$ if we can make the value of $f(x)$ as close to L as we like by restricting x to be close enough to a (but not $x=a$). Say limit of $f(x)$ as $x \to a$ is L.

Q: Why not $x = a$? (Hint: Think about $\dfrac{f(4+t) - f(4)}{4+t-4}$)

We might want to rename

$$f(x) = \frac{\sqrt{4+x} - 2}{x} \quad \text{with } a = 0.$$

In this example, we consider what the value of $f(x)$ is when x is near $a = 0$.

So, what if $x = 0$?

We can't divide by 0!

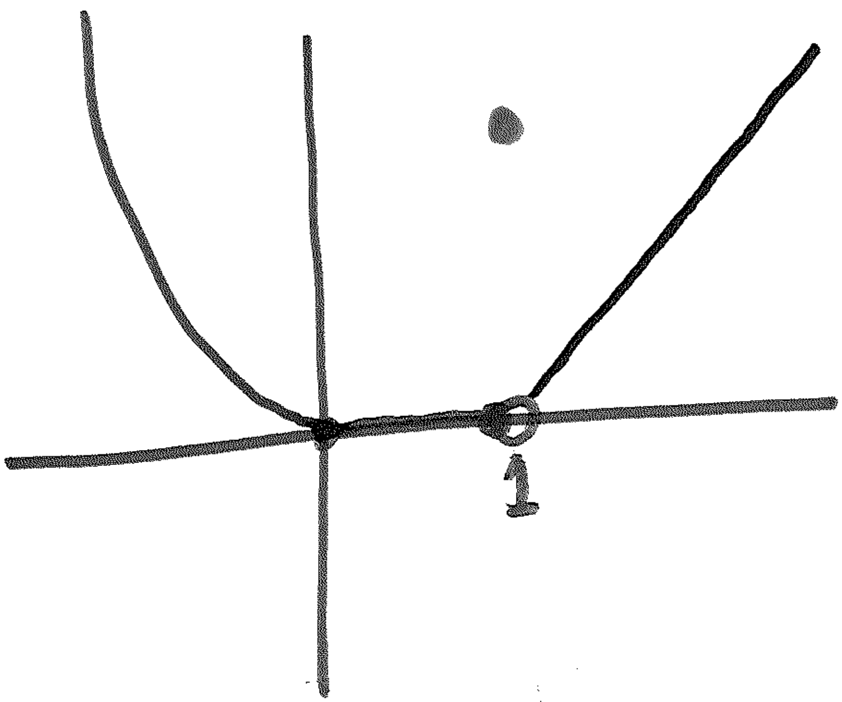
Think about estimating inst. velocity: We want

$$\frac{f(a+x) - f(a)}{a+x-a} = \frac{f(a+x) - f(a)}{x} \quad \text{for interval } [a, a+x].$$

If $x = 0$, den. is a problem!

For today and A4 on Webwork, trust your calculator. (Ha).
On Friday, limit laws will allow us to be precise.

Ex: Let $f(x) = \begin{cases} x^2 & x \leq 0 \\ 0 & 0 \leq x < 1 \\ 2 & x = 1 \\ x-1 & x > 1. \end{cases}$



Q: $\lim_{x \to 1} f(x) = 0$ **NOT** $f(1)$

Q: $\lim_{x \to -1} f(x) = 1$ This time, it is $f(-1)$.

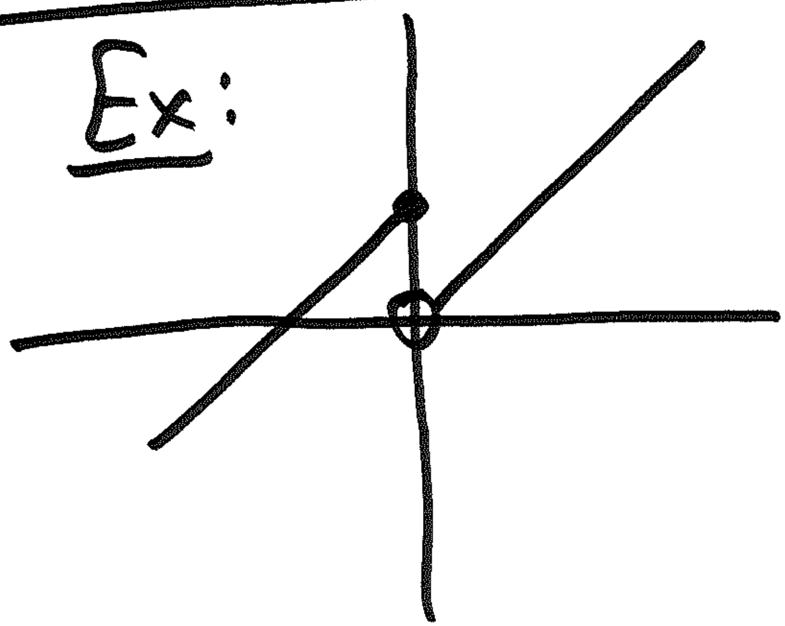
Note: $f(-1) = (-1)^2 = 1$.

NOTE: Look up def$^{\underline{n}}$s of

$$\lim_{x \to a^-} f(x) \quad \text{and} \quad \lim_{x \to a^+} f(x).$$

"one-sided limits" §2.2

Ex:



$$f(x) = \begin{cases} x+1 & x \le 0 \\ x & x > 0 \end{cases}$$

$$\lim_{x \to 0^-} f(x) = 1$$

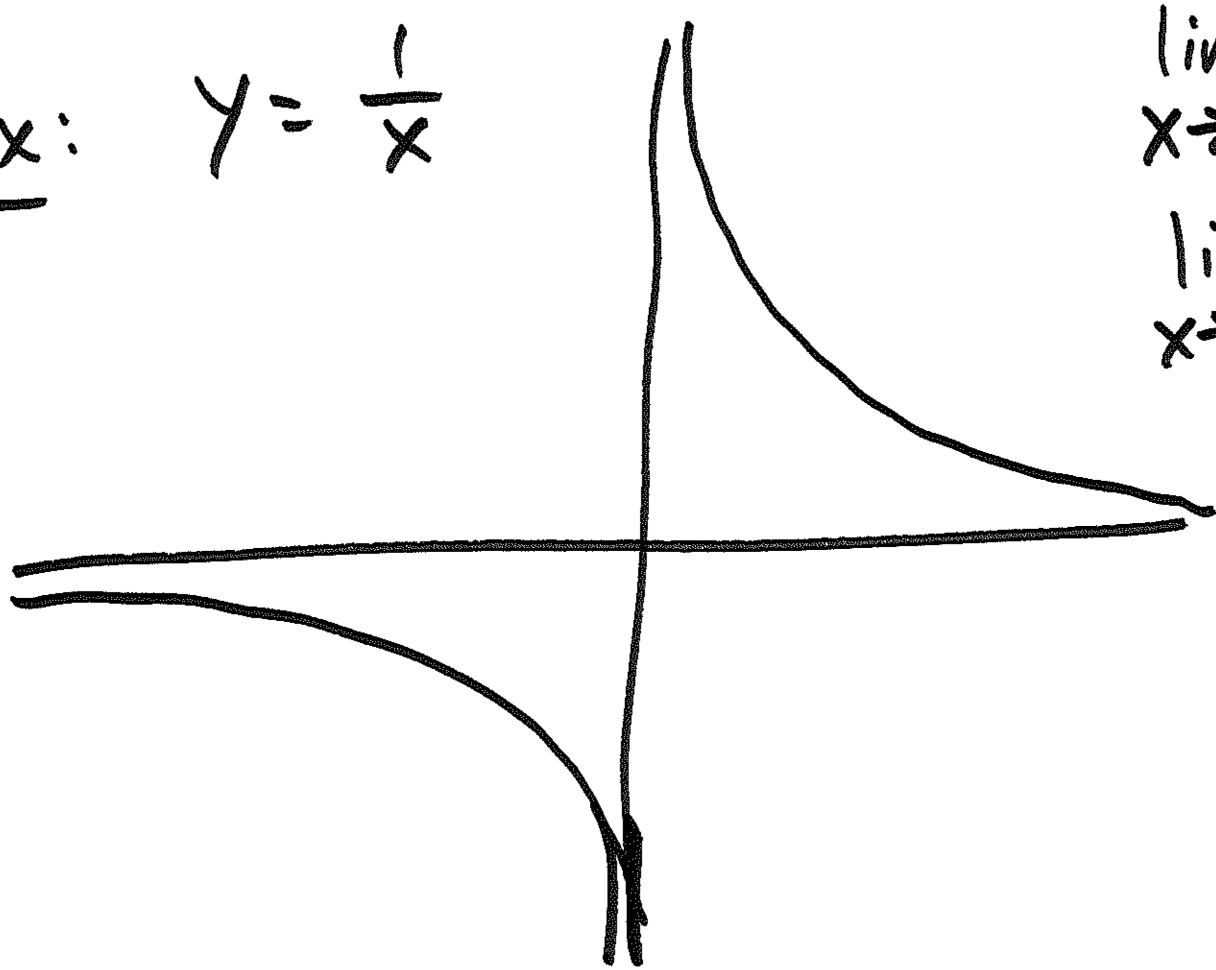
$$\lim_{x \to 0^+} f(x) = 0$$

Look up def$^{\underline{n}}$ in §2.2 of

$$\lim_{x \to a} f(x) = \pm\infty .$$

"infinite limits", i.e. asymptotes.

Ex: $y = \frac{1}{x}$



$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$

$$\lim_{x \to 0^+} \frac{1}{x} = \infty$$