9/9/16 12PM

1. Turn in Written Assignment

2. Next week: A5, A6, A7, Quiz 3

3. With neighbors: Explain the behavior of $\sin\left(\frac{\pi}{x}\right)$ as $x \to 0$.

→ Follow-up Q: Does $\sin\left(\frac{\pi}{x}\right)$ have any infinite limits? Why or why not?

Q: What is the behavior of $\sin(t)$ for $t = 10, 100, 1000, 10{,}000$, etc?

- oscillating
- between 1 + −1.
- 0 at $t=0$.

} we can't say much more. Powers of 10 don't interact well w/ period 2π.

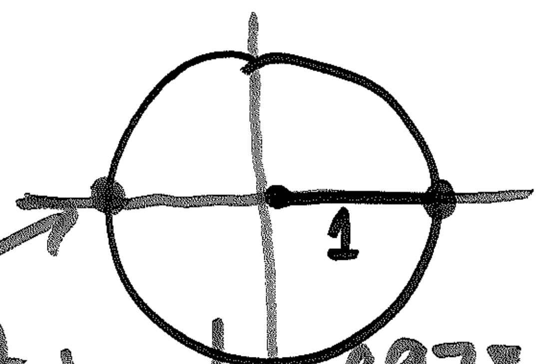
Q: What is the behavior of $\sin(\pi/x)$ when $x = \frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, \frac{1}{10{,}000}$, etc?

$\sin\left(\pi/\tfrac{1}{10}\right) = 0$, $\sin\left(\pi/\tfrac{1}{100}\right) = 0$, $\sin\left(\pi/\tfrac{1}{1000}\right) = 0$.

What is $\sin\left(\pi / 9{,}978{,}221\right)$

$$\left.\begin{array}{l} -1e^{-7} = 0.0000001 \\ -4e^{-7} = 0.0000004 \\ -4.7e^{-7} = 0.00000047 \\ -3.14e^{-7} = 0.000000314 \end{array}\right\} \text{For a calc, these are all } 0.$$

Picture:



$\sin(\underbrace{9{,}978{,}221 \cdot \pi}_{\text{in radians.}})$

This pt is at $9978221 \cdot \pi$ radians.
sin is 0.

If x is very small, $\frac{1}{x}$ is very large. So, $\frac{\pi}{x}$ is very large also.

As $x \to 0$, $\frac{\pi}{x} \to$ larger + larger values, either + or − depending on $x \to 0^+$ or $x \to 0^-$.

So, $\sin\left(\frac{\pi}{x}\right)$ cycles faster + faster as $x \to 0$. See desmos:

Since $\sin\left(\frac{\pi}{x}\right)$ does not approach a fixed value as $x \to 0$, $\lim_{x \to 0} \sin\left(\frac{\pi}{x}\right)$ does not exist.

Recall: Last time, $f(t) = \dfrac{\sqrt{4+t}-2}{t}$ gave us problems.

Clever idea: If you see $a-b$, try Mult. by $\dfrac{a+b}{a+b}$, since

$$(a-b)(a+b) = a^2 - b^2.$$

by dist. law.

$a = \sqrt{4+t} \Rightarrow a^2 = 4+t$

$b = 2 \Rightarrow b^2 = 4$

$$\frac{\sqrt{4+t}-2}{t} \cdot \frac{(\sqrt{4+t}+2)}{(\sqrt{4+t}+2)} = \frac{\cancel{4}+t\cancel{-4}}{\cancel{t}(\sqrt{4+t}+2)} = \frac{1}{\sqrt{4+t}+2}.$$

We were looking at

$$\lim_{t \to 0} \frac{\sqrt{4+t}-2}{t} = \lim_{t \to 0} \frac{1}{\sqrt{4+t}+2}.$$

Dream: Plug $t=0$ in to $\frac{1}{\sqrt{4+t}+2}$, get $\frac{1}{4}$.

This doesn't always work! You can't always plug in limiting value.

Limit laws allow us to determine when we can do this.

See §2.3 for list of many limit laws!

To use limit laws,

$$\lim_{x\to a} f(x) \quad \text{and} \quad \lim_{x\to a} g(x)$$

must both exist.

Ex: $$\lim_{t\to 0} \frac{\sqrt{4+t}-2}{t} = \lim_{t\to 0} \frac{1}{\sqrt{4+t}+2} = \frac{1}{\lim_{t\to 0}\left(\sqrt{4+t}+2\right)}$$

const. law in num
Quot. law if den limit ends up $\neq 0$.

$$= \frac{1}{\left(\lim_{t\to 0}\sqrt{4+t}\right)+2} = \frac{1}{\sqrt{\lim_{t\to 0} 4+t}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{4}.$$

sum law + const. law

root law

I Direct Subst. Thm:

If f is a polynomial or rational function, and a is in domain of f, then

$$\lim_{x \to a} f(x) = f(a).$$

Why? Since rat'l fns are built from $+, -, \times, \div$, + const. multiples of x.

II Squeeze Theorem:

If $f(x) \le g(x) \le h(x)$ for x near a, $x \ne a$,

and if $\lim\limits_{x \to a} f(x) = L = \lim\limits_{x \to a} g(x)$,

then $\lim\limits_{x \to a} g(x) = L$ as well.

<u>Ex</u>: What is $\lim\limits_{x \to 0} x^4 \sin\left(\frac{\pi}{x}\right)$?

<u>step 1</u>: get bounds for something.

$$-1 \le \sin(\pi/x) \le 1 \text{ for } x \ne 0.$$

<u>Step 2</u>: Modify those bounds, eg. mult. by $x^4 \ge 0$.

$$-x^4 \le x^4 \sin(\pi/x) \le x^4.$$

as $x \to 0$, $-x^4$ and $x^4 \to 0$. So, $\lim\limits_{x \to 0} x^4 \sin\left(\frac{\pi}{x}\right) = 0$

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$\boxed{1}$ Sign attendance

$\boxed{2}$ 3 WebWorks this week, 1 Quiz Thurs

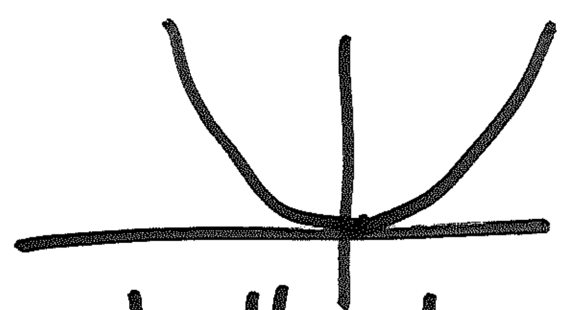
$\boxed{3}$ Exam info will be sent this week for all students.

Def$^{\underline{n}}$: A function f is <u>continuous</u> (cts) <u>at a number a</u> if

$$\lim_{x \to a} f(x) = f(a).$$

i.e. you can evaluate the limit using direct substitution.

Ex: $f(x) = x^2$



This is cts at all values.

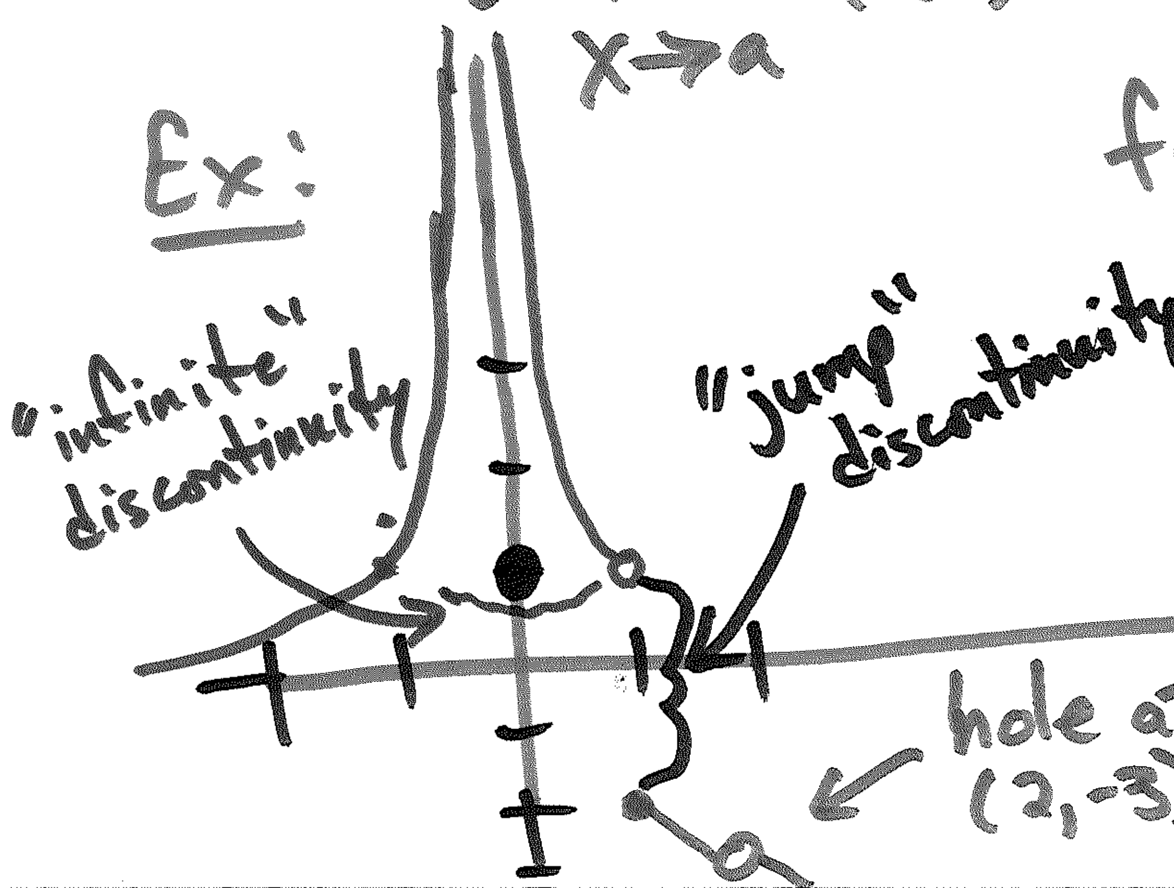
Cts fns need 3 things:

- $f(a)$ needs to be defined.
- $\lim\limits_{x \to a} f(x)$ exists.
- $\lim\limits_{x \to a} f(x) = f(a)$

Ex:

$$f(x) = \begin{cases} 1 & x = 0 \\ 1/x^2 & x < 0 \text{ or } 0 < x < 1 \\ \dfrac{-x^2+x+2}{x-2} & x \geq 1 \end{cases}$$

$$\underbrace{\dfrac{-x^2+x+2}{x-2}} = -x-1 \text{ if } x \neq 2.$$



hole at $(2,-3)$. ← "removable" discontinuity. ← Math causes this

<u>NOTE</u>:

- Read §2.5 for def$^{\underline{n}}$ of one-sided continuity ($\lim_{x \to a^+} f(x) = f(a)$, for example).
- A function f is <u>cts</u> if it is cts for each point in its domain.

<u>Thm</u>: Continuity is preserved by $+, -, \times, \div$, and mult. by a constant.

<u>Thm</u>: These fns are cts:

- Polynomials + rational fns
- trig + inverse trig fns
- exponential + log fns.

Ex: Find $\lim\limits_{x \to 2\pi} \left[\dfrac{\cos(x) + x^2}{3 - \sin x} \right]$

$$= \frac{\cos(2\pi) + (2\pi)^2}{3 - \sin(2\pi)} = \frac{1 + 4\pi^2}{3}$$

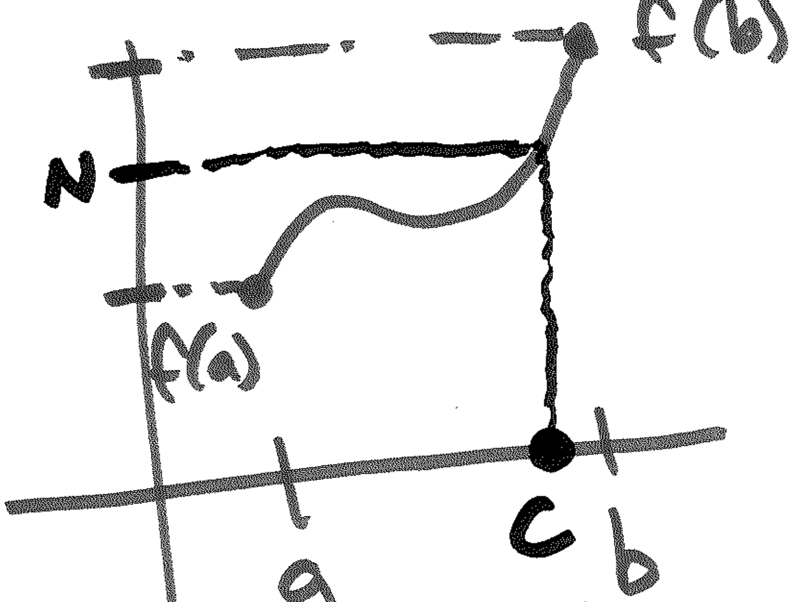
by continuity of $[\ldots]$ above.

Thm: If f and g are both cts, then $f \circ g$ is cts.

Ex: $\sin(\cos(x^4))$ is cts since

$\sin(x) \circ \cos(x) \circ x^4$.

Intermediate Value Thm:



find c where $f(c)=N$.
if f cts.

Suppose f is cts on $[a,b]$, with N satisfies $f(a) < N < f(b)$.
Then there is a c in (a,b) so that
$f(c)=N$.

Ex: If $f(x) = x^2 + 10\sin x$, is there an x where $f(x) = 1000$?

Step 1: Check that f is cts.
Here, "cts" + 10·"cts" $\Rightarrow$ cts fn.

Step 2: Find a w/ $f(a) < 1000$,
b w/ $f(b) > 1000$.

Here, guess + check.

eg $f(0) = 0^2 + 10 \cdot \sin 0 = 0 < 1000$. Set $a = 0$.

$f(100) = 100^2 + 10\sin(100)$

$= 10{,}000 + 10\sin(100)$

$\geq 10{,}000 - 10 > 1{,}000$. So, set $b = 100$.

I.V.T. $\Rightarrow$ There is a c in $(0, 100)$ w/ $f(c) = 1000$.

<u>9/14/16</u> 12PM

$\boxed{1}$ Quiz + WA solns posted to MA113 site.

$\boxed{2}$ Exam 1 <u>Review Session</u>
Monday, 9/19, 6-8PM FB 200

$\boxed{3}$ Graph $\dfrac{x^2-1}{x^2+1}$, $\dfrac{x-1}{x^2+1}$, $\dfrac{x^2-1}{x+2}$.

Why do they behave as they do for very large x-values?

Exam 1:

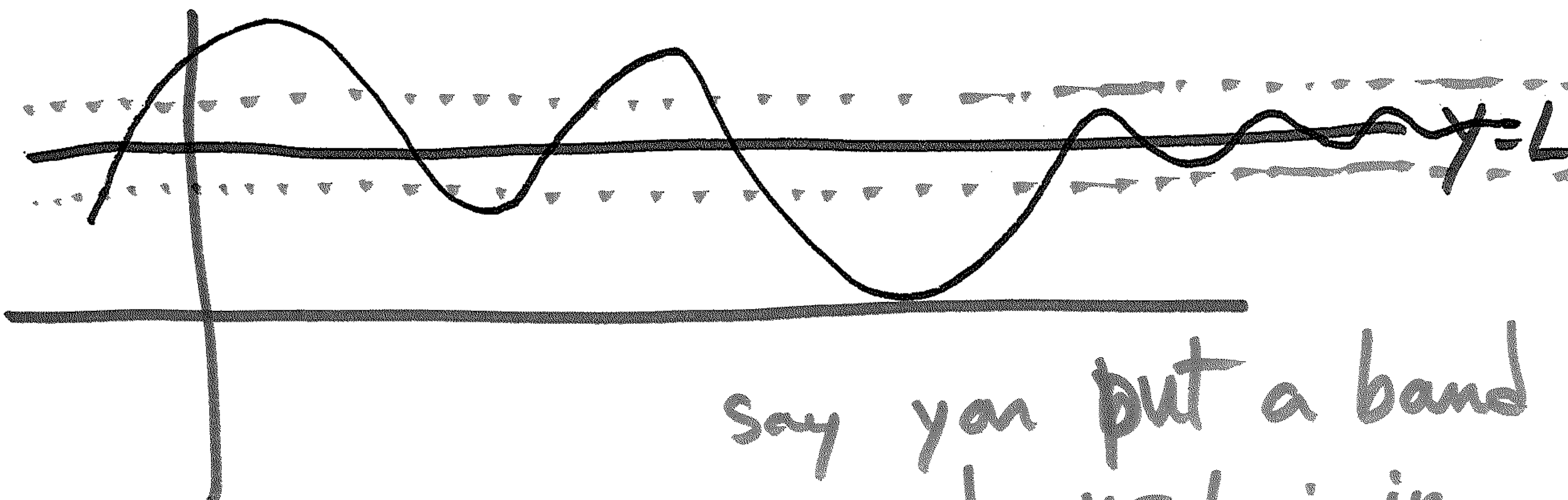
5 True/False

10 Mult. Choice

4 Free Response

Def$^{\underline{n}}$: Let $f(x)$ be defined on $[a,\infty)$ for some a. Then $\lim_{x\to\infty} f(x) = L$ means the value of $f(x)$ can be made as close as we want to L by requiring x to be sufficiently large.

Picture for this:



$\lim_{x\to\infty} f(x) = L$

Say you put a band around $y=L$; in orange.

Once you get far enough to right, graph of $f(x)$ is within orange band.

<u>Key</u>: This is true no matter how thin the band is.

__Note__: • Similar def^n holds for $\lim_{x \to -\infty} f(x) = L$.

• If $\lim_{x \to \bigcirc} f(x) = L$, we call

$+\infty$ or $-\infty$

$y = L$ a horizontal asymptote for f.

__Ex__: What is $\lim_{x \to \infty} \frac{1}{x^5}$? As $x \to \infty$, x^5 also goes to ∞. So, $\frac{1}{x^5} \to 0$ as $x \to \infty$.

So, $\lim_{x \to \infty} \frac{1}{x^5} = 0$.

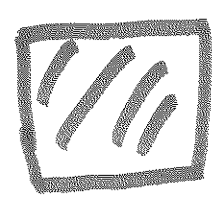
<u>Ex</u>: what is $\lim\limits_{x\to\infty} \frac{1}{\sqrt{x}}$?

<u>S's ideas</u>:

- $+\infty$
- 0
- 1

As $x \to \infty$, $\sqrt{x} \to \infty$.

why? $\sqrt{x}$ is increasing, e.g.



← Area of square is x.

← side length is $\sqrt{x}$.

Since length of square can become as long as we like, $\sqrt{x} \to \infty$ as $x \to \infty$.

So, $\frac{1}{\sqrt{x}} \to 0$ as $x \to \infty$.

<u>Thm</u>: • If $r > 0$ is a rational #, then $\lim_{x \to \infty} \frac{1}{x^r} = 0$.

• If $r > 0$ is a rational # and x^r is defined for all x in $(-\infty, \infty)$, then $\lim_{x \to -\infty} \frac{1}{x^r} = 0$.

<u>Key Idea</u>: Use algebra to be precise w/ these geometric observations.

<u>Ex</u>: $\displaystyle \lim_{x\to\infty} \frac{x^2-1}{x^2+1} \;=\; \lim_{x\to\infty} \frac{\frac{1}{x^2}(x^2-1)}{\frac{1}{x^2}(x^2+1)} =$

divide num + den by x^2

$$\lim_{x\to\infty} \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^2}} = \underbrace{\cdots\cdots}_{\text{string of limit laws}} = \frac{1-0}{1+0} = 1.$$

← show work on exam.

eg. $$\lim_{x \to \infty} \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^2}} = \frac{\lim_{x\to\infty}\left(1-\frac{1}{x^2}\right)}{\lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)} =$$

quot. law if den is $\neq 0$

diff and sum laws

$$\frac{\lim_{x\to\infty} 1 - \lim_{x\to\infty} \frac{1}{x^2}}{\lim_{x\to\infty} 1 + \lim_{x\to\infty} \frac{1}{x^2}} = \frac{1-0}{1+0} = 1.$$

const. law and our thm

Q: Why $\frac{1}{x^2}$? $\frac{x^2-1}{x^2+1} \approx \frac{x^2}{x^2}$ for large x.

Ex: What is $\lim_{x\to\infty} \frac{\sqrt{3x^2-x+1}}{2x+2}$?

Den: $2x+2 \approx 2x$ for large x.

Ignore coeff, x is key term.

Num: $\sqrt{3x^2-x+1} \approx \sqrt{3x^2}$ for large x.

$= \sqrt{3} \cdot x$.

Ignore coeff, x is key term.

$$\lim_{x\to\infty} \frac{\sqrt{3x^2-x+1}}{2x+2} = \lim_{x\to\infty} \frac{\frac{1}{x}\sqrt{3x^2-x+1}}{\frac{1}{x}(2x+2)} = \lim_{x\to\infty} \frac{\sqrt{3-\frac{1}{x}+\frac{1}{x^2}}}{2+\frac{2}{x}}$$

$$\underset{\text{limit laws}}{=} \cdots = \frac{\sqrt{3}}{2}.$$

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$\boxed{1}$ See 2 announcements on Canvas about Exam 1.

$\boxed{2}$ Sign Attendance.

A few more limits at ∞ problems.....

Ex: $\lim_{x \to \infty} \frac{x^2-1}{x+2}$. Q: What would you do *first* w/ this problem?

S answers:

- ~~Plug in 0.... My bad....~~
- Mult. by $\frac{1}{x}$
- compare degrees
- Plug in large #'s
- Graph it
- Mult by $1/x^2$.

Note: If I mult. by $\frac{1}{x}$, I get
num + den

$$\frac{x-\frac{1}{x}}{\underbrace{1+\frac{2}{x}}_{\text{as } x\to\infty,\ \text{den} \to 1.}}.$$

So, no division by 0.

In general, divide by highest order term in <u>denominator</u>.

Thus,

$$\lim_{x\to\infty}\frac{x^2-1}{x+2}=\lim_{x\to\infty}\frac{x-\frac{1}{x}}{1+\frac{2}{x}}\overset{\text{limit laws}}{=}\frac{\lim\limits_{x\to\infty}x-\frac{1}{x}}{1}=\infty.$$

Result is $\infty - \infty$, means nothing!
No conclusion!

<u>Algebra</u>: $$(a-b)\frac{(a+b)}{(a+b)} = \frac{a^2-b^2}{a+b}.$$

Here, $a = \sqrt{3x^2-x+1}$, $b = 2x+2$.

Applying this technique moves $\sqrt{\cdot}$ to denominator, which is better.

$$\sqrt{3x^2-x+1} - (2x+2) = \frac{(3x^2-x+1) - (2x+2)^2}{\sqrt{3x^2-x+1} + 2x+2}.$$

use algebra

$$= \frac{-x^2 - 9x - 3}{\sqrt{3x^2 - x + 1} + 2x + 2}.$$

So, our problem is now $\lim\limits_{x \to \infty} \dfrac{-x^2 - 9x - 3}{\sqrt{3x^2 - x + 1} + 2x + 2}$.

<u>Q:</u> What next? Think about "highest term" in denominator.

<u>Note</u>: If x large,

$$\sqrt{3x^2 - x + 1} \approx \sqrt{3x^2} = \sqrt{3} \cdot x.$$

So, den. $\approx \sqrt{3}x + 2x + 2 = (\sqrt{3} + 2)x + 2$.

So, x is "highest term".

Mult. num. + den by $\frac{1}{x}$.

$$\lim_{x\to\infty} \frac{\frac{1}{x}\left(-x^2-9x-3\right)}{\frac{1}{x}\left(\sqrt{3x^2-x+1}+2x+2\right)} = \lim_{x\to\infty} \frac{-x-9-\frac{3}{x}}{\sqrt{3-\frac{1}{x}+\frac{1}{x^2}}+2+\frac{2}{x}} = (*)$$

$$\boxed{\frac{1}{x} = \sqrt{\frac{1}{x^2}}}$$

use $\frac{1}{x}\sqrt{3x^2-x+1} = \sqrt{\frac{3x^2-x+1}{x^2}}$

$$(*) = \underset{\text{limit laws}}{\cdots} = \frac{\left(\lim_{x\to\infty} -x\right)-9-0}{\sqrt{3-0+0}+2+0}$$

$$= \frac{\left(\lim_{x\to\infty} -x\right)-9}{\sqrt{3}+2} = -\infty.$$

Study for Exam:

looking at trees in forest of Calc

- Review Webwork, Rec. Worksheets, Old Exams, odd numbered problems in book.

Trails through forest.

- Forming a study group, using Study + Mathskeller, TA + Prof Office Hours.

map

- Use Concept Map Creation to form a picture of your expectations of Exam + Calc.