Friday, Feb 5, 2016

1. See Announcements on Canvas about EXAM 1.
2. SIGN ATTENDANCE SHEET.
3. With your neighbors:

   Discuss your "concept maps" for the topic of sequences and series. What are your key examples? What are the most and least challenging topics for you?

Continue discussion about R. of C.

<u>Remark</u>: For $F(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$ with radius of convergence R, i.e. (For all x satisfying $|x-c| < R$, $F(x)$ converges), $F(x)$ has an <u>interval of convergence</u> that is one of the following:

$(c-R, c+R)$ $\quad$ $[c-R, c+R)$

$(c-R, c+R]$ $\quad$ $[c-R, c+R]$

So, we define the <u>Int. of Conv.</u> to be the choice above ....

of the interval containing x-values where F(x) converges.

Q: c is always in Int. of Conv. Why?

A: $F(c) = \sum_{n=0}^{\infty} a_n (\cancel{x} c - c)^n =$

$a_0(c-c)^0 + a_1(c-c)^1 + a_2(c-c)^2 + \dots$

$= a_0(0)^0 + \underbrace{a_1 0^1 + a_2 0^2 + \dots}_{\text{All are equal to } 0.}$

$= a_0 \cdot 1$

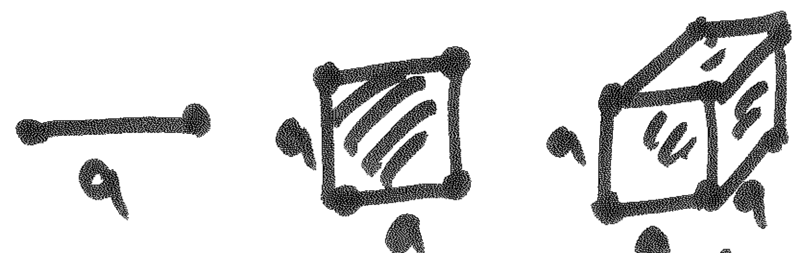
$= a_0.$

$\boxed{\text{Aside}}$ Why is $a^0 = 1$ for any a?

Two reasons why $a^0 = 1$.

#1: (Intuitive)

a^0	a^1	a^2	a^3	a^4	$\cdots$
$\boxed{1}$	a	$a \cdot a$	$a \cdot a \cdot a$	$a \cdot a \cdot a \cdot a$	$\cdots$



square　　cube

so I we define it to match this pattern.

#2: (Formal Reason)

$$a^x = e^{\ln(a^x)} = e^{x\ln(a)} =$$

use power series for e^z w/ $z = x\ln(a)$

$$1 + x\ln(a) + \frac{(x\cdot\ln(a))^2}{2!} + \frac{(x\cdot\ln(a))^3}{3!} + \cdots =$$

$$1 + \underbrace{\ln(a)\cdot x + \ln(a)^2\cdot\frac{x^2}{2!} + (\ln(a))^3\cdot\frac{x^3}{3!} + \cdots}_{\text{all zero when } x=0.}$$

For $x=0$, you get 1.

<u>Ex</u>: Use ratio test to find int. of conv.

for $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{\sqrt{n^2+1}}$.

$(-1)^n$ will cancel

x^n will cancel

<u>Ratio test</u>: $\displaystyle\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{\dfrac{(-1)^{n+1} \cdot x^{n+1}}{\sqrt{(n+1)^2+1}}}{\dfrac{(-1)^n x^n}{\sqrt{n^2+1}}}\right| =$

consider x as a fixed value.

$$\lim_{n\to\infty} \left|\frac{x\sqrt{n^2+1}}{\sqrt{n^2+2n+2}}\right| = |x| \cdot \lim_{n\to\infty} \underbrace{\left|\frac{\sqrt{n^2+1}}{\sqrt{n^2+2n+2}}\right|}_{\text{exercise}} = |x| \cdot 1 =$$

$|x| < 1$. ← This will yield abs. convergence by ratio test.

This gives us $|x|<1$, i.e. $x \in (-1,1)$.

But, we need to check endpoints, $x=1,-1$.

For $x=-1$, we get $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(-1)^n}{\sqrt{n^2+1}} = \underbrace{\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}}_{*}$.

<u>Exercise:</u> $*$ diverges by comparison test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$. So, $x=-1$ is not in Int. of Conv.

For $x=1$, we get $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(1)^n}{\sqrt{n^2+1}} = \underbrace{\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}}}_{\circledast}$.

<u>Exercise:</u> $\circledast$ converges by alt. series test.

So, Int. of Conv. is $(-1,1]$.

Stop Exam 1 material

$$\int (1 + x + x^2 + x^3 + \cdots)\, dx = C + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$$

$$= \int \frac{1}{1-x}\, dx$$

Remember

$$\int \frac{1}{x}\, dx = \underline{\ln|x|} + C$$

closely related