

Mon, Feb 8, 2016:

First item: For HW 3, not an exam: diff. + integration of power series.

Thm 2, §10.6: If  $F(x) = \sum_1 a_n(x-c)^n$  with radius of convergence  $R$ , then  $F'(x)$  and  $\int F(x)dx$  can be obtained by using the power rule term-by-term on  $F(x)$ . Further,  $F'(x)$  and  $\int F(x)dx$  both have R of Conv.  $R$ .

Ex: In calc 1, you learned that

$$\ln(1+x) = \int_0^x \frac{1}{1+t} dt. \text{ Let's integrate!}$$

$$\int_0^x \frac{1}{1+t} dt = \int_0^x \frac{1}{1-(t)} dt =$$

$$\int_0^x (1-t+t^2-t^3+t^4-\dots) dt =$$

$$\left( t - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4} + \frac{t^5}{5} - \dots \right) \Big|_0^x =$$

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots = \ln(1+x).$$

Because of the integral...

Result:  $x=1$

gives

$$\ln(2) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$

Alt. Harm. Series.

## EXAM Review:

1. Find the radius of convergence

$$\text{for } F(x) = 1 - x - x^2 + x^3 - x^4 + x^5 + x^6 - \dots$$

pattern:  $- , - , + , - , + , \dots$



Def<sup>n</sup> of power series:

$$\sum a_n (x-c)^n$$

Example of Power Series:  $\sum \frac{x^n}{3^n}$

$$\sum \frac{x^n}{n!}$$

$$\sum x^n$$

$$\sum_{n=0}^{\infty} \frac{x^n}{3^n} = \frac{x^0}{3^0} + \frac{x^1}{3^1} + \frac{x^2}{3^2} + \frac{x^3}{3^3} + \dots$$

$$= 1 + \frac{x}{3} + \frac{x^2}{9} + \frac{x^3}{27} + \dots$$

start here  
in your  
mind.

Recall a power series is:

$$a_0 + a_1(x-c) + a_2(x-c)^2 + a_3(x-c)^3 + \dots$$

$$= \sum_{n=0}^{\infty} a_n(x-c)^n$$

For  $1 - x - x^2 + x^3 - x^4 + x^5 + x^6 - \dots$

$$c = 0 \quad 1 - (x-0) - 1 \cdot (x-0)^2 + 1 \cdot (x-0)^3 - 1 \cdot (x-0)^4 + \dots$$

Ratio test ignores signs, so let's

use it + not worry about sign pattern.

$$1 - x - x^2 + x^3 - x^4 + x^5 - x^6 + \dots$$

Ratio:  $\lim_{n \rightarrow \infty} \left| \frac{x^{n+1}}{x^n} \right|$

4<sup>th</sup> term is  $-x^4$

6<sup>th</sup> term is  $x^6$

7<sup>th</sup> term is  $-x^7$

8<sup>th</sup> term is  $x^8$

9<sup>th</sup> term is  $-x^9$

10<sup>th</sup> term is  $x^{10}$

11<sup>th</sup> term is  $-x^{11}$

12<sup>th</sup> term is  $x^{12}$

13<sup>th</sup> term is  $-x^{13}$

14<sup>th</sup> term is  $x^{14}$

15<sup>th</sup> term is  $-x^{15}$

16<sup>th</sup> term is  $x^{16}$

17<sup>th</sup> term is  $-x^{17}$

18<sup>th</sup> term is  $x^{18}$

19<sup>th</sup> term is  $-x^{19}$

20<sup>th</sup> term is  $x^{20}$

21<sup>th</sup> term is  $-x^{21}$

22<sup>th</sup> term is  $x^{22}$

23<sup>th</sup> term is  $-x^{23}$

24<sup>th</sup> term is  $x^{24}$

25<sup>th</sup> term is  $-x^{25}$

26<sup>th</sup> term is  $x^{26}$

27<sup>th</sup> term is  $-x^{27}$

28<sup>th</sup> term is  $x^{28}$

29<sup>th</sup> term is  $-x^{29}$

30<sup>th</sup> term is  $x^{30}$

31<sup>th</sup> term is  $-x^{31}$

32<sup>th</sup> term is  $x^{32}$

33<sup>th</sup> term is  $-x^{33}$

34<sup>th</sup> term is  $x^{34}$

35<sup>th</sup> term is  $-x^{35}$

36<sup>th</sup> term is  $x^{36}$

37<sup>th</sup> term is  $-x^{37}$

38<sup>th</sup> term is  $x^{38}$

39<sup>th</sup> term is  $-x^{39}$

40<sup>th</sup> term is  $x^{40}$

41<sup>th</sup> term is  $-x^{41}$

42<sup>th</sup> term is  $x^{42}$

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49<sup>th</sup> term is  $-x^{49}$

50<sup>th</sup> term is  $x^{50}$

51<sup>th</sup> term is  $-x^{51}$

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89<sup>th</sup> term is  $-x^{89}$

90<sup>th</sup> term is  $x^{90}$

91<sup>th</sup> term is  $-x^{91}$

92<sup>th</sup> term is  $x^{92}$

93<sup>th</sup> term is  $-x^{93}$

94<sup>th</sup> term is  $x^{94}$

95<sup>th</sup> term is  $-x^{95}$

96<sup>th</sup> term is  $x^{96}$

97<sup>th</sup> term is  $-x^{97}$

98<sup>th</sup> term is  $x^{98}$

99<sup>th</sup> term is  $-x^{99}$

100<sup>th</sup> term is  $x^{100}$

$$= \lim_{n \rightarrow \infty} |x| < 1 \text{ ensures by ratio test}$$

abs convergence.

So,  $|x| < 1$  yields  $R=1$  as R.of. Conv.

Find the int. of conv. for

$$\sum_{n=0}^{\infty} \frac{n^6}{n^{3+1}} (x-3)^n.$$

$$C=3. \quad 0 + \frac{1}{2}(x-3) + \frac{64}{257}(x-3)^2 + \frac{3^6}{3^{2+1}}(x-3)^3 + \dots$$

Use ratio test:  $\lim_{n \rightarrow \infty} \left| \frac{n^{1st} \text{ term}}{n^{th} \text{ term}} \right| < 1$

$$\begin{aligned} \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}(x-c)^{n+1}}{a_n(x-c)^n} \right| &= \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} (x-c) \right| \\ &= |x-c| \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1 \quad \text{converges.} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left| \frac{\frac{(n+1)^6 (x-3)^{n+1}}{(n+1)^{2+1}}}{\frac{n^6 (x-3)^n}{n^{2+1}}} \right| = \lim_{n \rightarrow \infty} \left| \frac{(n+1)^6 (x-3)^{n+1}}{(n+1)^3 \cdot n^6 (x-3)^n} \right|$$

$$= |x-3| \cdot \lim_{n \rightarrow \infty} \left| \frac{(n+1)^6}{(n+1)^3 \cdot n^6} \right| = \frac{n^3+1}{n^6} \cdot \frac{n^6}{n^3+1} =$$

$$|x-3| \cdot \lim_{n \rightarrow \infty} \left| \frac{n^3+1}{(n+1)^3+1} \right| < 1$$

converges by ratio test.

$$\Leftrightarrow |x-3| < 1$$

~~$x \in (2, 4)$~~

Since  $c=3$ ,  $R=1$ , we get  $(2, 4)$

↑ ↑ ↑  
Check endpoints!

When  $x=2$ :

$$\sum_{n=0}^{\infty} \frac{n^6}{n^{k+1}} (2-3)^n = \sum_{n=0}^{\infty} \frac{n^6}{n^{k+1}} (-1)^n$$

←  $k=5$  series  
←  $k=6$  series

When  $x=4$ :

$$\sum_{n=0}^{\infty} \frac{n^6}{n^{k+1}} (4-3)^n = \sum_{n=0}^{\infty} \frac{n^6}{n^{k+1}}$$

←  $k=5$  series  
←  $k=6$  series

So,  $I_{\mathcal{C}}$  is  $[2, 4]$ .