4/10/17 MA113

1. REEF TODAY
2. Review for exam today
   → For Exam 3, Newton's Method will not be covered.
3. Exam 3 is tomorrow (Tuesday) evening!

General reminders about taking exam:

1. Be strategic; i.e. pick problems in <u>your</u> order.

2. Make a cheat sheet in first 60 sec of exam.

3. Study w/ peers → work problems
   → map concepts.

4. write a positive comment on front page of Exam.

Ex: Find $\lim\limits_{x \to \infty} x \cdot \ln\left(1+\frac{1}{x}\right)$

As $x \to \infty$, $x \to \infty$ and $\ln\left(1+\frac{1}{x}\right)$ goes to...?

well, $1+\frac{1}{x} \to 1$ as $x \to \infty$. So, $\ln\left(1+\frac{1}{x}\right) \to 0$ as $x \to \infty$.

This limit is of form $\infty \cdot 0$. For this, I need L'Hopital.

$$\lim_{x \to \infty} x \cdot \ln\left(1+\frac{1}{x}\right) = \lim_{x \to \infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} \underset{L'H}{=} \lim_{x \to \infty} \frac{\frac{1}{1+\frac{1}{x}} \cdot \cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}}$$

$$= \lim_{x \to \infty} \frac{1}{1+\frac{1}{x}} = \frac{1}{1+0} = 1.$$

<u>Ex</u>: Find $\displaystyle\sum_{i=0}^{1}\left(\sum_{j=1}^{3}(i+j)\right) = \sum_{i=0}^{1}\left[(i+1)+(i+2)+(i+3)\right]$

"j"

"i"

$= (0+1)+(0+2)+(0+3)$
$\quad + (1+1)+(1+2)+(1+3)$

$= 6 + 9$

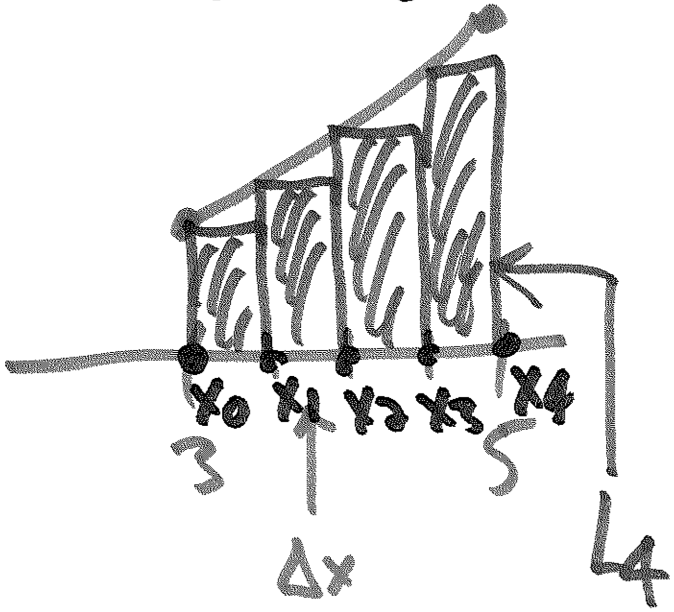
$= 15.$

A different example:

$\displaystyle\sum_{i=2}^{4} 2i = 2\cdot2+2\cdot3+2\cdot4$

"i"

Ex: Find L_4 for x^4 on $[3,5]$. Is this an over- or under-estimate of $\int_3^5 x^4\,dx$?



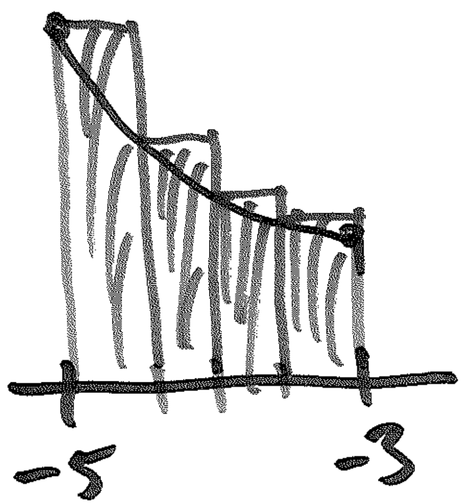
$L_4 \leftarrow$ 4 subintervals.

$\Delta x = \frac{5-3}{4} = \frac{1}{2}$.

<u>NOTE:</u> Underestimate. Since x^4 is increasing on $[3,5]$.

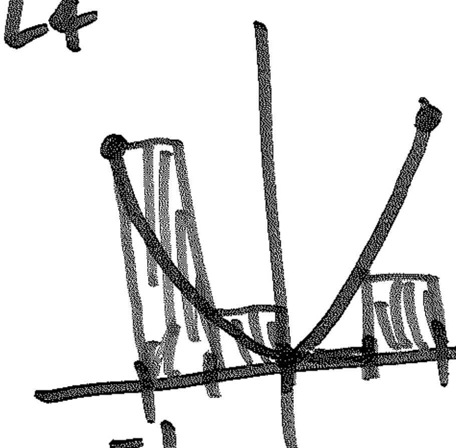
x^4 on $[-5,-3]$.

L_4.



x^4 on $[-1,1]$

L_4



$$L_4 = \sum_{i=0}^{3} (x_i)^4 \Delta x = x_0^4 \Delta x + x_1^4 \Delta x + x_2^4 \Delta x + x_3^4 \Delta x$$

$$= 3^4 \cdot \frac{1}{2} + 3.5^4 \cdot \frac{1}{2} + 4^4 \cdot \frac{1}{2} + 4.5^4 \cdot \frac{1}{2}$$

$$= 7177/16 \approx 448.56..$$

REEF: $\boxed{1}$

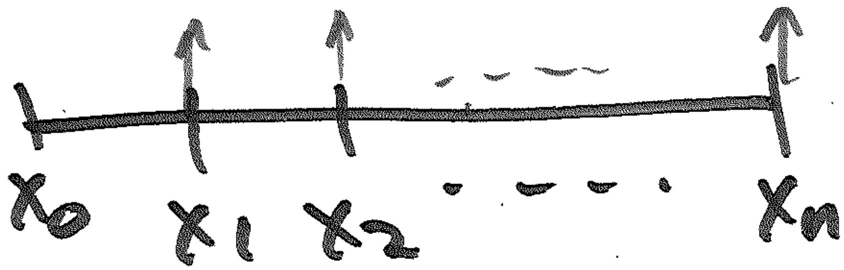
Ex: Which of these expresses R_n for $\sqrt{x} \cdot \sin(x)$ on $[0, \pi]$?

a) $\sum_{i=0}^{n-1} \left(\sqrt{\frac{i}{\pi}} \cdot \sin\left(\frac{i}{\pi}\right) \cdot \frac{\pi}{n} \right)$

b) $\sum_{i=1}^{n} \left(\sqrt{\frac{i\pi}{n}} \cdot \sin\left(\frac{i\pi}{n}\right) \cdot \frac{\pi}{n} \right)$

c) $\sum_{i=1}^{n} \left(\sqrt{\frac{i}{n}} \cdot \sin\left(\frac{i}{n}\right) \cdot \frac{i}{n} \right)$

ANS is (b)



$\sum_{i=1}^{n}$, $\Delta x = \frac{\pi - 0}{n} = \frac{\pi}{n}$.

MA113 4/12/17

1 ~~[illegible]~~

2 Today + Friday: §5.3, Fundamental Thm of Calc.

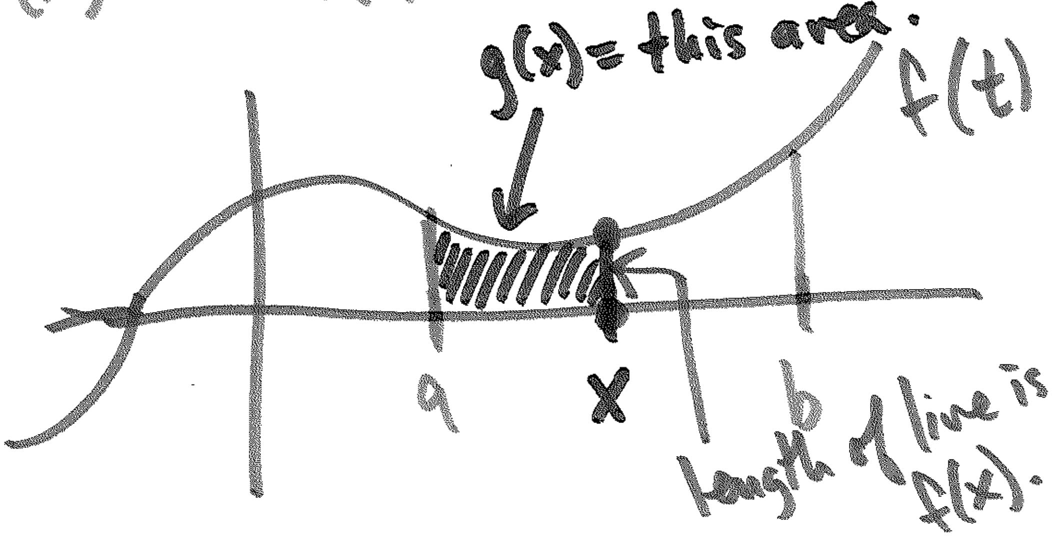
3 Next Webwork is D1, due Monday.

4 Exam 3 will not be returned until Tuesday, and curve will not be announced until early next week, due to administrative red tape on This Exam.

§5.3: F.T.C. = Fundamental Thm of Calculus.

F.T.C. says there are two ways to find area under a curve, & these ways agree.

F.T.C. Pt 1: Assume we define $\int_a^b f(t)\,dt$ using Riemann sums. If f is cts on $[a,b]$, then $g(x) = \int_a^x f(t)\,dt$ for $a \le x \le b$ is a cts fn on $[a,b]$. Further, $g(x)$ is diff on (a,b), with $g'(x) = f(x)$.

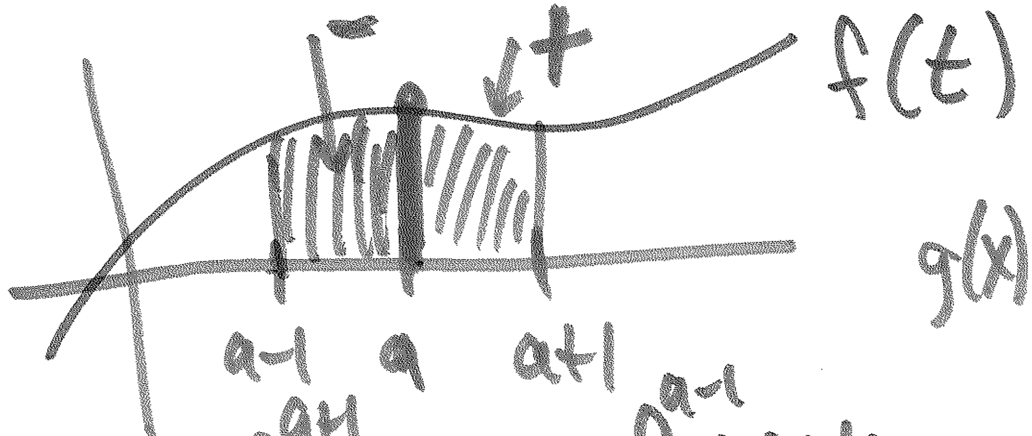


NOTE: For a proof, see §5.3.

F.T.C. Part 2: If f is cts on $[a,b]$, then for any antiderivative F such $F' = f$,

$$\underbrace{\int_a^b f(t)\,dt}_{\text{defined by Riemann Sums}} = F(b) - F(a).$$

antiderivative.



$$g(x) = \int_a^x f(t)\,dt$$

$$\int_a^{a+1} f(t)\,dt \qquad \int_a^{a-1} f(t)\,dt = -\int_{a-1}^{a} f(t)\,dt$$

$$x = a \Rightarrow \int_a^a f(t)\,dt.$$

F.T.C. pt 1 says these are both antiderivatives of $f(x)$.

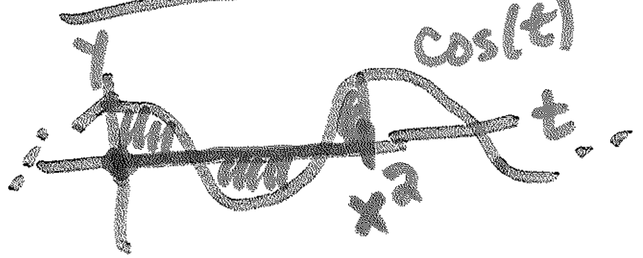
$$g(x)=\int_1^x f(t)\,dt = \text{area under } f(t) \text{ on } [1,x].$$

$$h(x)=\int_0^x f(t)\,dt = \text{area under } f(t) \text{ on } [0,x].$$

<u>Ex</u>: $\int_1^x \sqrt{2t^3}\,dt = g(x)$.

what is $g'(x)$? $\quad g'(x) = \sqrt{2x^3}$ by F.T.C. 1.

<u>Ex</u>: What is the derivative of $\int_0^{x^2} \cos t\,dt$?



$$x \longmapsto x^2 \longmapsto \int_0^{x^2} \cos t\,dt$$

chain rule!

$$u = x^2 \Rightarrow \int_0^{x^2} \cos t\,dt = \int_0^{u} \cos t\,dt.$$

$$\frac{d}{dx}\left(\int_0^{x^2} \cos t\,dt\right) = \underset{\uparrow \text{ inside}}{\frac{du}{dx}} \cdot \underbrace{\frac{d}{du}\left(\int_0^{u} \cos t\,dt\right)}_{\text{outside}} = 2x \cdot \cos u$$

$$= 2x \cdot \cos(x^2).$$

Ex: Find derivative $\frac{d}{dx}$ of $\int_{x^2}^{x^3} t^2\, dt$ for $x>0$.

Write $\int_{x^2}^{x^3} t^2 dt = \int_0^{x^3} t^2 dt - \int_0^{x^2} t^2 dt$.

Then $\frac{d}{dx}\left[\int_0^{x^3} t^2 dt - \int_0^{x^2} t^2 dt\right] = \frac{d}{dx}\int_0^{x^3} t^2 dt - \frac{d}{dx}\int_0^{x^2} t^2 dt$

$= 3x^2\cdot (x^3)^2 - 2x(x^2)^2 = 3x^8 - 2x^5.$

as before