MA 113 4/14/17

$\boxed{1}$ Today FTC part II (§5.3)

$\boxed{2}$ Next Webwork D1 due Monday

$\boxed{3}$ Exams will be handed back <u>Tuesday</u>. Curve (if applicable) will be announced next week.

$\boxed{4}$ Reef

Find the derivative $\frac{d}{dx}$ of

$$\int_{x^2}^{x^3} t^2 \, dt \quad \text{for } x > 0.$$

$$= \int_0^{x^3} t^2 \, dt - \int_0^{x^2} t^2 \, dt.$$

$$\frac{d}{dx}\left[\int_0^{x^3} t^2 \, dt\right] - \frac{d}{dx}\left[\int_0^{x^2} t^2 \, dt\right]$$

$$= (x^3)^2 \cdot \frac{d}{dx}[x^3] - (x^2)^2 \cdot \frac{d}{dx}[x^2]$$

$$3x^2(x^6) - 2x(x^4) = \boxed{3x^8 - 2x^5}$$

FTC Part 2:

If f is cts on $[a,b]$, then for **any** antiderivative F so that $F' = f$,

$$\underbrace{\int_a^b f(t)\,dt}_{\text{Rieman Sum}} = \underbrace{F(b) - F(a)}_{\text{antiderivatives}}.$$

Proof of FTC 2:

If $g(x) = \int_a^x f(t)\,dt$ FTC part 1,

$g'(x) = f(x)$. This means that g is an antiderivative of f.

Also $g(b) - g(a) = \int_a^b f(t)\,dt - \underbrace{\int_a^a f(t)\,dt}_{=0}$

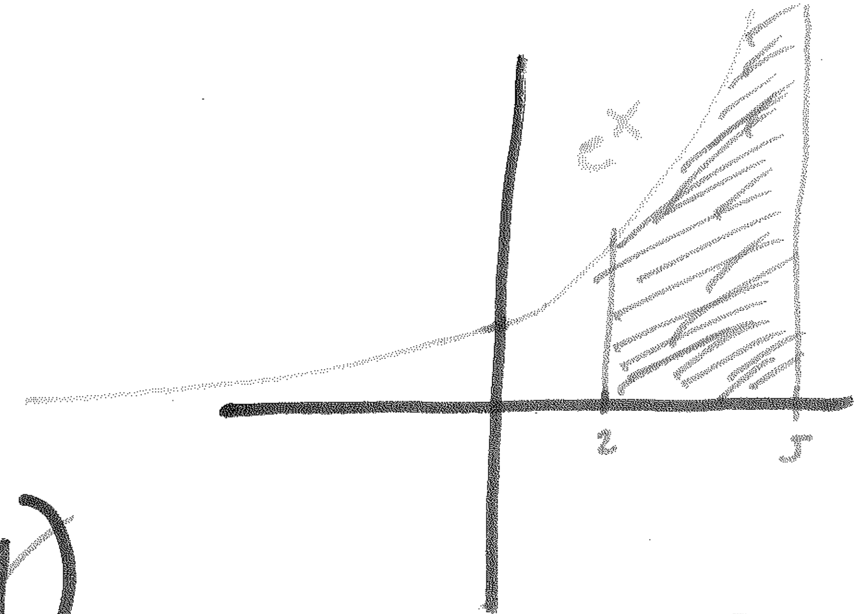
$= \int_a^b f(t)\,dt.$

If $F(x) = g(x) + c$. (This is any antiderivative of $f(x)$)

$$F(b) - F(a) = g(b) + \cancel{c} - (g(a) + \cancel{c})$$
$$= g(b) - g(a)$$
$$= \int_a^b f(t)\,dt. \quad \square$$

$$\int_2^5 e^x \, dx = \left(e^x + 1\right)\Big|_2^5$$

any antiderivative will do



$$= \left(e^5 \cancel{+1}\right) - \left(e^2 \cancel{+1}\right)$$

$$= e^5 - e^2 = \boxed{e^2\left(e^3 - 1\right)}$$

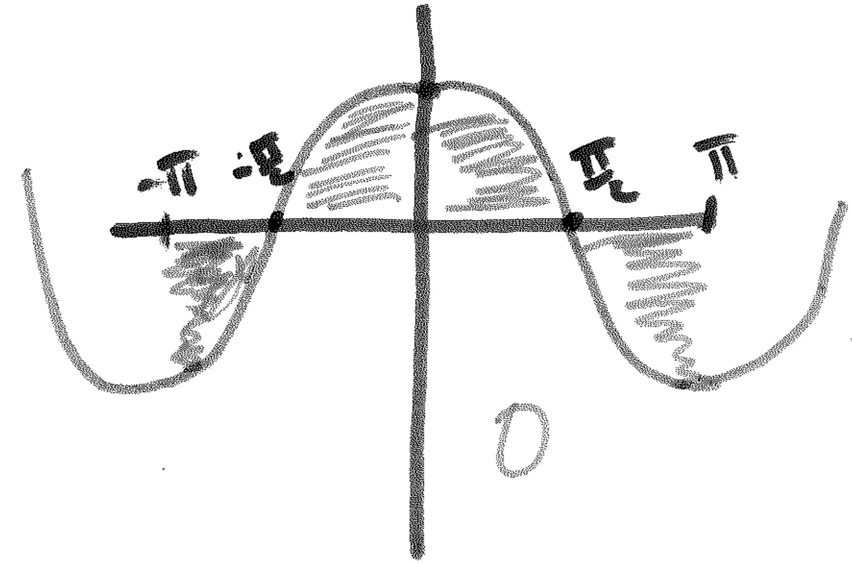
Ex

$$\int_{-\pi}^{\pi} \cos(x)\, dx$$

$$= \sin(x) \Big|_{-\pi}^{\pi}$$

$$= \sin(\pi) - \sin(-\pi)$$

$$= 0 - 0 = \boxed{0}$$



$$\int_0^2 x^2\,dx = \frac{x^3}{3}\Big|_0^2 = \frac{8}{3} - 0 = \boxed{\frac{8}{3}}$$

$$\int_3^6 \frac{1}{x}\,dx = \ln|x|\Big|_3^6$$

[3,6]

$$= \ln(x)\Big|_3^6$$

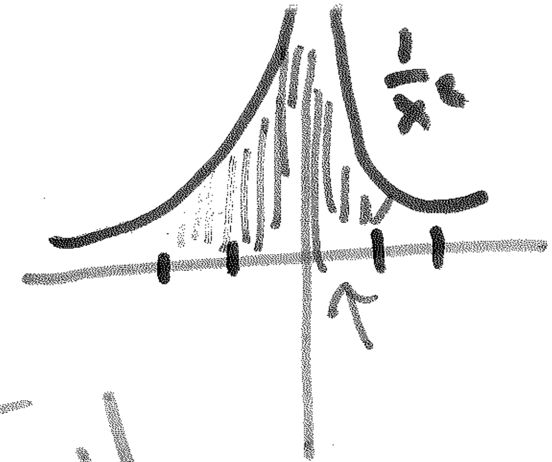
$$= \ln(6) - \ln(3)$$

$$= \ln\left(\frac{6}{3}\right) = \boxed{\ln(2)}$$

$$\int_{-2}^{2} \frac{1}{x^2}\,dx$$ ← Not continuous on $[-2,2]$

FTC part 2

DOES NOT Apply!!



FALSE Solution:

$$\int_{-2}^{2} \frac{1}{x^2}\,dx = -\frac{1}{x}\Big|_{-2}^{2} = -\frac{1}{2} - \left(-\frac{1}{-2}\right)$$

$$= -\frac{1}{2} - \frac{1}{2} = \boxed{-1}$$

FTC Part II

DOES NOT APPLY!

$$\int_0^4 e^{x+1}\,dx \qquad e^{x+1} = (e^x)(e^1)$$

$$= \int_0^4 e \cdot e^x = e \int_0^4 e^x\,dx$$

$$= e\left[e^x \Big|_0^4\right]$$

$$= \boxed{e^5 - e} \text{ OR}$$

$$= \boxed{e(e^4 - 1)}$$

$$\int_0^{\pi} \sin(x)\,dx = -\cos(x)\Big|_0^{\pi}$$

$$= -\cos(\pi) + \cos(0)$$

$$= -(-1) + 1$$

$$= \boxed{2}$$

Return to $\int_{x^2}^{x^3} t^2\,dt$.

Find $\int_{x^2}^{x^3} t^2\,dt = G(x)$.

By previous work using FTC 1:

$$\frac{d}{dx}\left[\int_{x^2}^{x^3} t^2\,dt\right] = 3x^8 - 2x^5 = \underline{f(x)}$$

so

$$F(x) = \frac{3x^9}{9} - \frac{2x^6}{6} = \boxed{\frac{x^9}{3} - \frac{x^6}{3}} \qquad \boxed{F'(x) = f(x)}$$

We know that $F(x) = G(x) + C$ for some C.

Note: $\underline{G(1) = 0}$

why? $G(1) = \int_{(1)^2}^{(1)^3} t^2\,dt = \int_1^1 t^2\,dt = 0$

$$G(x) = F(x) - C$$

so

$$0 = G(1) = F(1) - C = \frac{1}{3} - \frac{1}{3} - C = -C$$

Conclusion:

$$\boxed{G(x) = \frac{x^9}{3} - \frac{x^6}{3}}$$

Also

$$G(x) = \int_{x^2}^{x^3} t^2\,dt = \left.\frac{t^3}{3}\right|_{x^2}^{x^3} = \boxed{\frac{x^9}{3} - \frac{x^6}{3}}$$

FTC 1 & FTC 2 agree!

MA 113 4/17/17

$\boxed{1}$ REEF today

$\boxed{2}$ Today is §5.4 on net change

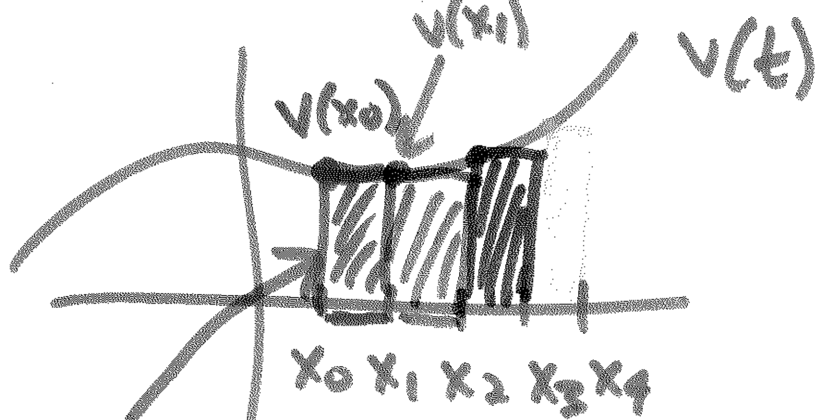
$\boxed{3}$ As soon as the Exam 3 curve is computed (if there is one) I'll send a canvas announcement.

$\boxed{4}$ See Canvas announcements for assignments this week.

$\boxed{5}$ Discuss w/ your neighbors: If $v(t)$ is velocity, <u>why</u> is $\int_0^x v(t)\,dt$ = position at time x?

<u>Hint:</u> There are <u>two</u> answers, one using F.T.C. Pt 1 and the other using F.T.C. Pt 2.

Answer 1:



distance traveled in interval $[x_0, x_1]$.

Sample velocity often, at $x_0, \dots, x_4$.
Use $v(x_i)$ as a "constant" velocity on $[x_i, x_{i+1}]$.

Result:
distanced traveled is sum of these rectangular areas, each representing approximate distance traveled during $[x_i, x_{i+1}]$.

F.T.C. Pt 1

Answer 2:

we know that $s(t)$ is position at time t, then

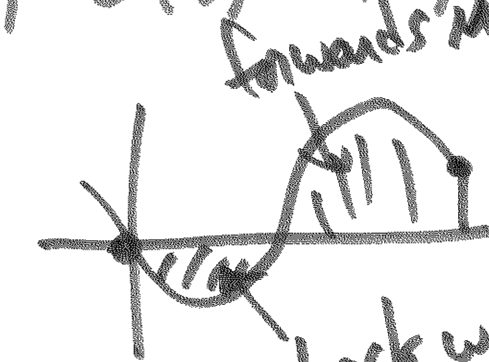
$$\int_0^x v(t)\,dt = \underbrace{S(x) - S(0)}_{\text{distance traveled}}.$$

F.T.C. Pt 2

Integrals measure net change!

① $v(t)$ is velocity, $s(t)$ = position, $s'(t) = v(t)$

then $\int_a^b v(t)\,dt = \underbrace{s(b)-s(a)}_{\text{displacement}}$.



② To calculate total distance traveled, we compute

$$\int_a^b |v(t)|\,dt$$

changes graph above to this.

③ If $V(t)$ = volume of water at time t, $V'(t)$ = rate of water flow, and $\int_a^b V'(t)\,dt = V(b)-V(a)$

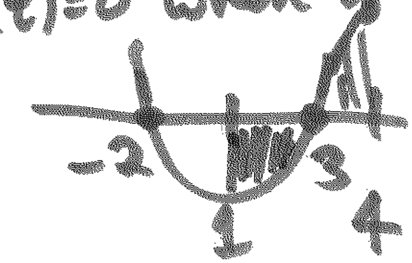
Ex: A particle ~~[illegible]~~ moves in a line w/ velocity
$v(t) = t^2 - t - 6$ m/s.
Find displacement + total distance traveled over $1 \le t \le 4$.

ⓐ Displacement: $s(4) - s(1) \overset{\text{FTC 2}}{=} \int_1^4 v(t)\,dt = \int_1^4 t^2 - t - 6\,dt$

$= \left[\frac{t^3}{3} - \frac{t^2}{2} - 6t\right]_1^4 = -\frac{9}{2}$ m.

ⓑ Total distance: $\int_1^4 |v(t)|\,dt =$ (*)

when is $v(t)$ pos + neg?
$v(t) = t^2 - t - 6$
$= (t-3)(t+2)$.
$v(t) = 0$ when $t = 3$ and -2.



(*) $= \int_1^3 -v(t)\,dt + \int_3^4 v(t)\,dt$

$$= \int_1^3 -t^2+t+6\,dt + \int_3^4 t^2-t-6\,dt$$

$$= \left[-\frac{t^3}{3}+\frac{t^2}{2}+6t\right]_1^3 + \left[\frac{t^3}{3}-\frac{t^2}{2}-6t\right]_3^4 = \frac{61}{6}\,m.$$

$$\left(-\frac{3^3}{3}+\frac{3^2}{2}+6\cdot 3\right)-\left(-\frac{1^3}{3}+\frac{1^2}{2}+6\cdot 1\right)+\left(\frac{4^3}{3}-\frac{4^2}{2}-6\cdot 4\right)-\left(\frac{3^3}{3}-\frac{3^2}{2}-6\cdot 3\right)$$

<u>Remark:</u> $F(b)-F(a) = F\Big|_a^b \qquad F\Big]_a^b$

$$= [F]\Big|_a^b$$

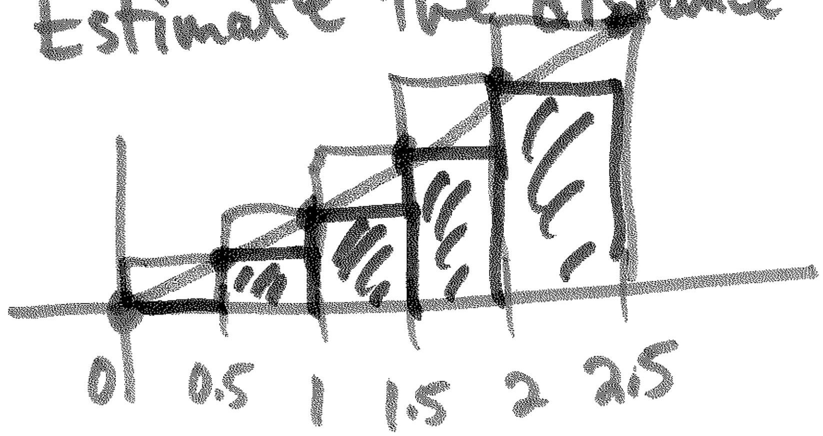
$$= [F]_a^b$$

$$\cancel{= F_a^b} \leftarrow \text{bad!}$$

<u>Ex</u>: The velocity of a car is recorded as:

sec t	0	0.5	1.0	1.5	2.0	2.5
m/s $v(t)$	0	1	3	7	17	26

Estimate the distance traveled by the car on interval $[0, 2.5]$



$$R_5 = \frac{1}{2}\cdot 1 + \frac{1}{2}\cdot 3 + \frac{1}{2}\cdot 7 + \frac{1}{2}\cdot 17 + \frac{1}{2}\cdot 26 = 27\,m$$

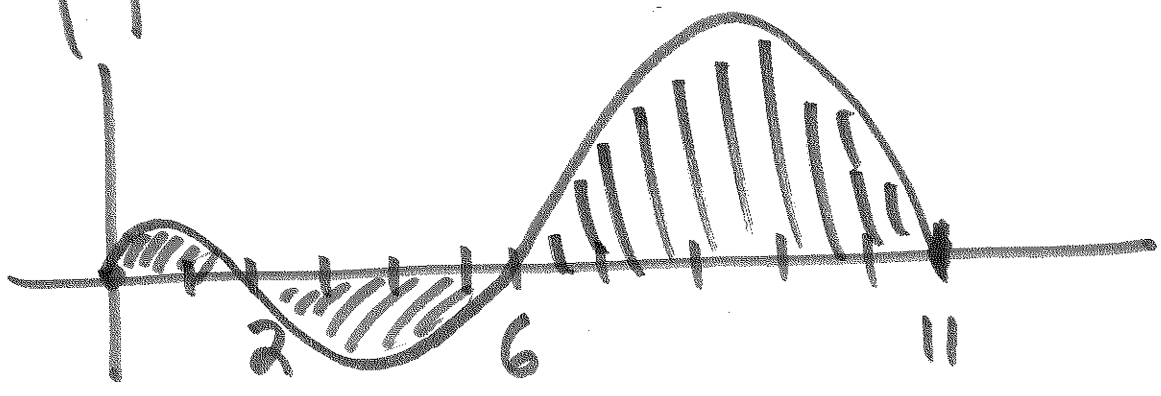
$$L_5 = \frac{1}{2}\cdot 0 + \frac{1}{2}\cdot 1 + \frac{1}{2}\cdot 3 + \frac{1}{2}\cdot 7 + \frac{1}{2}\cdot 17 = 14\,m.$$

Good approximation is

$$\frac{R_5 + L_5}{2} = 20.5\,m.$$

Reef:
5

Ex: Suppose the rate of change of a fish population over 12 months is given by:



Q: When is population at highest + lowest?

[0,6] has most negative area. ← at 6 months is lowest.

[0,11] has most positive area. ← at 11 months is highest.