MA 113   4/26/17

1 REEF today

2 I sent a Canvas announcement w/ info about the final exam

3 Today + Friday: Review for final

4 Please fill out course evaluations! Current response rates are 26% – 40% for various sections.

THANK YOU to those students who have completed their evaluations.

Ex: Find $\frac{1}{4}\int_0^6 4x\cos(2x^2)\,dx$. ← FTC. Pt2 [redacted] problem

$u = 2x^2$

$du = 4x\,dx$

$$= \frac{1}{4}\int_0^6 \underbrace{\cos(2x^2)}_{u} \cdot \underbrace{4x\,dx}_{du} = ⊛$$

$$⊛ \ \frac{1}{4}\int_{2\cdot 0^2}^{2\cdot 6^2} \cos u\,du = \frac{1}{4}\sin u\Big|_0^{72} = \frac{1}{4}(\sin 72 - \sin 0)$$

$$= \frac{1}{4}\sin 72.$$

Ex: Approximate $\frac{1}{\sqrt[3]{2}}$.

First thing to do: Select a fn $f(x)$ to use in our approximation.

Use $f(x) = \frac{1}{\sqrt[3]{1+x}}$ w/ $a=0$, and a 4^{th}-order approximation, i.e. $T_4(x)$.

Rewrite $f(x) = (1+x)^{-1/3}$, then $f(0) = 1^{-1/3} = 1$.

Also, $f(1) = (1+1)^{-1/3} = \frac{1}{\sqrt[3]{2}}$.

We need $T_4(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{3\cdot 2}x^3 + \frac{f^{(4)}(0)x^4}{4\cdot 3\cdot 2}$
at $a=0$

$f'(x) = \frac{-1}{3}(1+x)^{-4/3}$, $f''(x) = \frac{4}{\underset{\uparrow}{9}}(1+x)^{-7/3}$, $f'''(x) = \frac{-28}{27}(1+x)^{-10/3}$

$f^{(4)}(x) = \frac{280}{81}(1+x)^{-13/3}$.

$T_4(x) = 1 + \frac{-1}{3}x + \frac{4}{18}x^2 + \frac{-28}{27}\cdot\frac{1}{6}x^3 + \frac{280}{81}\cdot\frac{1}{24}x^4$

$\frac{1}{\sqrt[3]{2}} = f(1) \approx T_4(1) = 1 - \frac{1}{3} + \frac{4}{18} - \frac{28}{27}\cdot\frac{1}{6} + \frac{280}{81}\cdot\frac{1}{24}$

$= \frac{209}{243} \approx 0.8600\ldots \qquad \frac{1}{\sqrt[3]{2}} \approx 0.7937\ldots$

Ex: Express $\frac{dy}{dx}$ as a function of x & y for

$$x^3 + \underbrace{2x}_{}\cdot\underbrace{y}_{} + y^3 = \underbrace{7x^2}_{}\cdot\underbrace{y^2}_{} + 2.$$

Implicit diff: Apply $\frac{d}{dx}$ to each side & solve for $\frac{dy}{dx}$.

$$3x^2 + 2y + 2x\frac{dy}{dx} + 3y^2\cdot\frac{dy}{dx} = 14x\cdot y^2 + 7x^2\cdot 2y\cdot\frac{dy}{dx}$$

$$\Rightarrow \quad 2x\frac{dy}{dx} + 3y^2\frac{dy}{dx} - 7x^2\cdot 2y\frac{dy}{dx} = 14x\cdot y^2 - 3x^2 - 2y$$

$$\Rightarrow \frac{dy}{dx}\left(2x + 3y^2 - 7x^2\cdot 2y\right) = 14x\cdot y^2 - 3x^2 - 2y$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{14xy^2 - 3x^2 - 2y}{2x + 3y^2 - 7x^2\cdot 2y}$$

REEF:

3

Ex: If average velocity over $[-1,-1+h]$ is given by $\dfrac{\frac{1}{e}-e^{h-1}}{h}$, what is inst. vel at -1?

Av. Vel is $\dfrac{f(a+h)-f(a)}{h}$ for some $f(x)$.

It will be (for this problem) $\dfrac{f(-1+h)-f(-1)}{h}$.

Rewrite: $\dfrac{e^{-1}-e^{-1+h}}{h} = \dfrac{-e^{-1+h}+e^{-1}}{h} = \dfrac{-e^{-1+h}-(-e^{-1})}{h}$

$f(x)$ should be $-e^{x}$. Inst. velocity at -1 is given by $f'(-1)$. Since $f'(x)=-e^{x}$, $f'(-1)=-e^{-1}$ = inst. vel.

MA 113 4/28/17

1 REEF today

2 Final Exam Monday night → See Canvas announcements for details

3 Today: Review

4 Discuss w/ neighbors:

If $f(3)=7$, $f(5)=-2$, and f diff on $(-\infty,\infty)$, can $f'(x)\geq 2$ for all x?

5 MA 113 grades:
We will post final exam scores to Canvas. Your overall course grade will be entered into myuk by the end of the day on Monday, May 8.

Remember good strategies for taking final exam!

#4 above: Behavior of $f'(x)$, rather than computing a value, is usually influenced by the Mean Value Thm.

MVT says if know $f(a)$ + $f(b)$, + f is cts on $[a,b]$, diff on (a,b), then there is some c in (a,b) w/ $f'(c) = \frac{f(b)-f(a)}{b-a}$.

1st part of soln.

Is f diff on $(3,5)$? Yes, by condition given.
Is f cts on $[3,5]$? Yes, since diff implies cts.

2nd part of soln.

Then $\underbrace{\text{there}}_{\text{by MVT}}$ is a c in $(3,5)$ with $f'(c) = \frac{f(5)-f(3)}{5-3} = \frac{-2-7}{2} = \frac{-9}{2}$.
Question asked if $f'(x) \geq 2$ for all x.
Answer is no, since there is a point violating this.

Ex: Explain how the following sum is related to an integral.

$$\sum_{i=0}^{n-1} \underbrace{e^{\frac{2i}{n}+1} \cdot \sin\left(\frac{\pi n}{i}\right) \cdot \sqrt{1+\frac{i^3}{n^3}}}_{\text{should be in } f(x) \ldots} \cdot \overbrace{\frac{1}{n}}^{\text{candidate for } \Delta x.} .$$

Preprocess: This is a Riemann Sum. I need f(x) + Δx. specifically $f(x_i)$.
To do this, I need an interval $[a,b]$....

$\Delta x = \frac{b-a}{n} = \frac{1}{n}$. Let's go w/ $a=0, b=1$.
This will make things easier, since
$x_0 = a = 0$, so $x_i = x_0 + i\Delta x$
$= i\Delta x = \frac{i}{n}$.

So, $f(x_i) = e^{2x_i+1} \cdot \sin\left(\frac{\pi}{x_i}\right) \cdot \sqrt{1+(x_i)^3}$.

So, $f(x) = e^{2x+1} \cdot \sin\left(\frac{\pi}{x}\right) \cdot \sqrt{1+x^3}$.

Since $\sum$ is from 0 to $n-1$, we have a Left Riemann Sum.

REEF: 1

Answer: This is a left Riemann Sum for

$$\int_0^1 e^{2x+1} \sin\left(\frac{\pi}{x}\right) \sqrt{1+x^3}\, dx.$$

Ex: When is $f(t)=\int_{-1}^{t} x^3+3x^2+3x+1\,dx$ decreasing?

Hint: FTC Part 1.

↳ says $f(t)=\int_a^t g(x)\,dx$, then

$\frac{df}{dt}=g(t)$. So, for our example,

$$\frac{df}{dt}=t^3+3t^2+3t+1 . \quad \text{When is } \frac{df}{dt}<0?$$

$$=(t+1)^3.$$

So, $\frac{df}{dt}=0$ only at $t=-1$, and $\frac{df}{dt}$ is always defined.

So, $f(t)$ is decreasing on $(-\infty, -1)$.

Ex: Find $\lim_{x\to 0}\left[\frac{\tan x - x}{x^3}\right] \underset{\substack{L'H \\ \text{of form } \frac{0}{0}}}{=} \lim_{x\to 0}\left[\frac{\sec^2 x - 1}{3x^2}\right] \underset{\substack{L'H \\ \text{of form } \frac{0}{0}}}{=}$

$$\lim_{x\to 0}\left[\frac{2\sec^2 x \tan x}{6x}\right] = \underbrace{\lim_{x\to 0}\left[\frac{2\sec^2 x}{6}\right]}_{=\frac{1}{3}} \cdot \underbrace{\lim_{x\to 0}\left[\frac{\tan x}{x}\right]}_{\text{form } \frac{0}{0}\text{, use L'H}}$$

$$= \frac{1}{3}\cdot \lim_{x\to 0}\left[\frac{\sec^2 x}{1}\right] = \frac{1}{3}\cdot 1$$

$$= \frac{1}{3}.$$

since $\frac{d}{dx}\sec^2 x = \frac{d}{dx}(\cos x)^{-2}$

$\underset{\text{chain rule}}{=} -2(\cos x)^{-3}\cdot -\sin x = 2\,\frac{\sin x}{(\cos x)^3}$

$= 2\sec^2 x \tan x$