MA 113 2/3/17

$\boxed{1}$ Log in to REEF for poll mid-class.

$\boxed{2}$ Reminder: EXAM 1 on Tues, Feb 7.

Canvas announcement → Room assignments
→ Exam format.

$\boxed{3}$ With your neighbors: Find

$$\lim_{x \to \infty} \frac{\sqrt{3x^2 - x + 1}}{2x + 2}.$$

__Observation:__ For large values of x,

$$x^n \approx \sqrt{x^{2n} + \text{error}}$$

__Ex:__ $\sqrt{3}\cdot x = \sqrt{3x^2} \approx \sqrt{3x^2 - x + 1}$. for $x > 0$.

$$\lim_{x\to\infty} \frac{\sqrt{3x^2-x+1}}{2x+2} = \lim_{x\to\infty} \frac{\frac{1}{x}\sqrt{3x^2-x+1}}{\frac{1}{x}(2x+2)} \underset{\substack{\uparrow \\ \text{since} \\ x>0}}{=} \lim_{x\to\infty} \frac{\sqrt{\frac{1}{x^2}(3x^2-x+1)}}{\frac{1}{x}(2x+2)}$$

$$\boxed{\frac{1}{x} = \sqrt{\frac{1}{x^2}} \text{ if } x>0}$$

$$= \lim_{x\to\infty} \frac{\sqrt{3 - \underbrace{\tfrac{1}{x}}_{\to 0} + \underbrace{\tfrac{1}{x^2}}_{\to 0}}}{2 + \underbrace{\tfrac{2}{x}}_{\to 0}} = \frac{\sqrt{3}}{2}.$$

degree of den is 1, so divide by x^1.

Ex: $\lim_{x \to \infty} \frac{\sqrt{2x^3+3}}{3x+7}$

<u>Thought:</u> What should the answer be?

$\sqrt{2x^3+3} \approx \sqrt{2x^3}$ if $x >> 0$.

$3x+7 \approx 3x$. if $x >> 0$.

So, if $x >> 0$, $\frac{\sqrt{2x^3+3}}{3x+7} \approx \frac{\sqrt{2x^3}}{3x} = \frac{\sqrt{2}\cdot\sqrt{x^3}}{3x} = \frac{\sqrt{2}\cdot \cancel{x}\sqrt{x}}{3\cancel{x}}$

$= \frac{\sqrt{2}}{3}\cdot\sqrt{x}$.

<u>Expect:</u> when $x >> 0$,

$\frac{\sqrt{2x^3+3}}{3x+7} \approx \frac{\sqrt{2}}{3}\cdot\sqrt{x}$. $\longrightarrow$ goes to ∞ as $x \to \infty$.

Soln: $\lim\limits_{x\to\infty} \dfrac{\sqrt{2x^3+3}}{3x+7} = \lim\limits_{x\to\infty} \dfrac{\frac{1}{x}\sqrt{2x^3+3}}{\frac{1}{x}(3x+7)} =$

$$\lim_{x\to\infty} \frac{\sqrt{\frac{1}{x^2}(2x^3+3)}}{\frac{1}{x}(3x+7)} = \lim_{x\to\infty} \frac{\sqrt{\overset{\infty}{2x} + \overset{\to 0}{\frac{3}{x^2}}}}{\underbrace{3}_{\text{constant}} + \underset{\to 0}{\frac{7}{x}}} = \infty.$$

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Ex: Observation: $(a-b)(a+b) = a^2 - b^2$.

Eg. $x^2 - y^2 = (x-y)(x+y)$.

$16 - 9 = 4^2 - 3^2 = (4-3)(4+3) = 1 \cdot 7 = 7$.

$117 - 27 = (\sqrt{117})^2 - (\sqrt{27})^2 = (\sqrt{117} - \sqrt{27}) \cdot (\sqrt{117} + \sqrt{27})$.

$$\lim_{x\to\infty} \underbrace{\sqrt{3x^2-x+1}}_{a} - \underbrace{(2x+2)}_{b}$$

$$a \quad - \quad b$$

~~$\infty - \infty = 0$~~ WRONG

Use $(a-b)\dfrac{(a+b)}{a+b} = \dfrac{a^2-b^2}{a+b}$ as our technique.

To solve, consider

$$\lim_{x\to\infty}\left[\sqrt{3x^2-x+1} - (2x+2)\right]\left[\frac{\sqrt{3x^2-x+1}+(2x+2)}{\sqrt{3x^2-x+1}+(2x+2)}\right] =$$

$$\lim_{x\to\infty}\left[\frac{(3x^2-x+1)-(2x+2)^2}{\sqrt{3x^2-x+1}+(2x+2)}\right] =$$

$$\lim_{x\to\infty} \frac{-x^2-9x-3}{\underline{\sqrt{3x^2-x+1}+(2x+2)}}.$$

"highest degree" is $x=\sqrt{x^2}$, so divide num + den by x.

Soln is an exercise for you to finish:
you should get this equal to $-\infty$.

<u>Q</u>: How to study? Focus on 2 things.

<u>1</u>: Identify your strengths & weaknesses

<u>2</u>: Work problems w/out looking at youtube or going to a tutor too soon.

<u>At exam</u>: • On front page, write "I can do this!".
• Create a strategy / work flow for your exam.

→ Study individually <u>+</u> w/a study group.

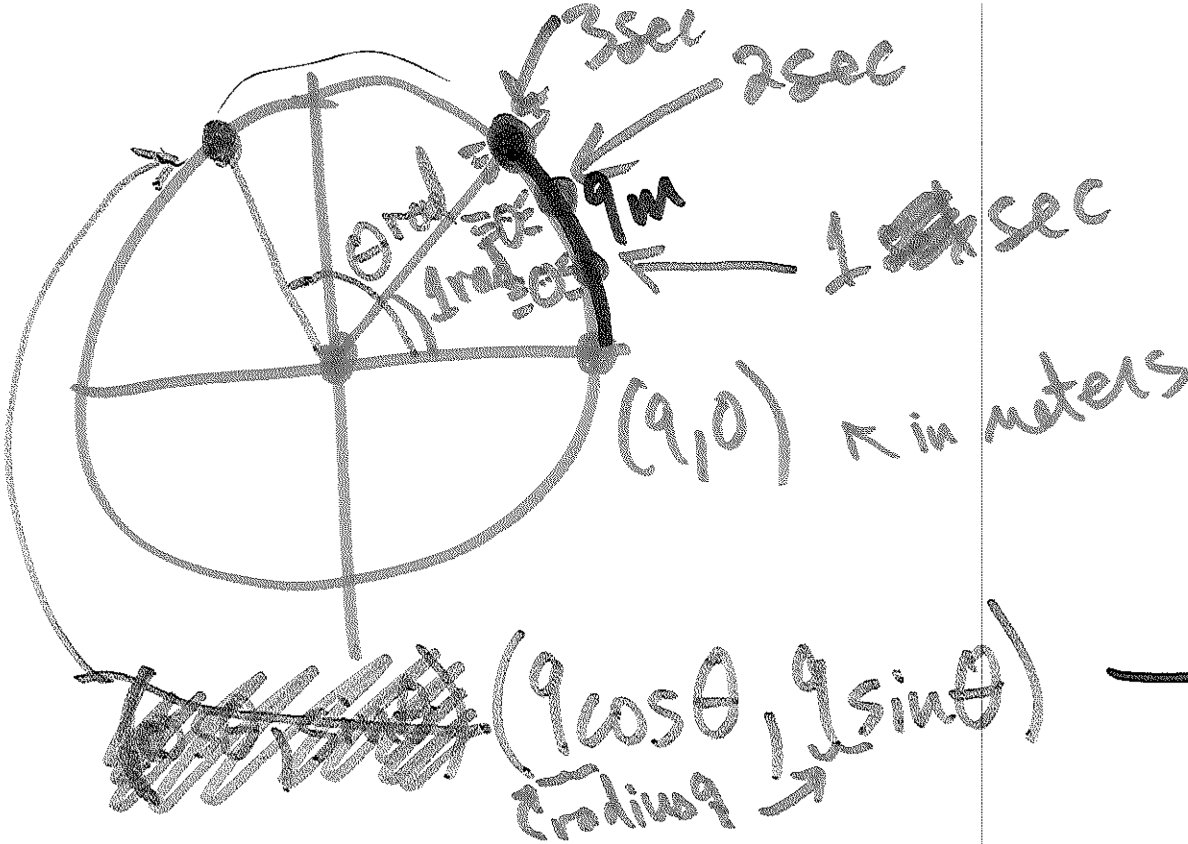
MA 113 2/6/17

$\boxed{1}$ Reminder: [EXAM 1 tomorrow, 5PM. See past Canvas announcements for room + format info.

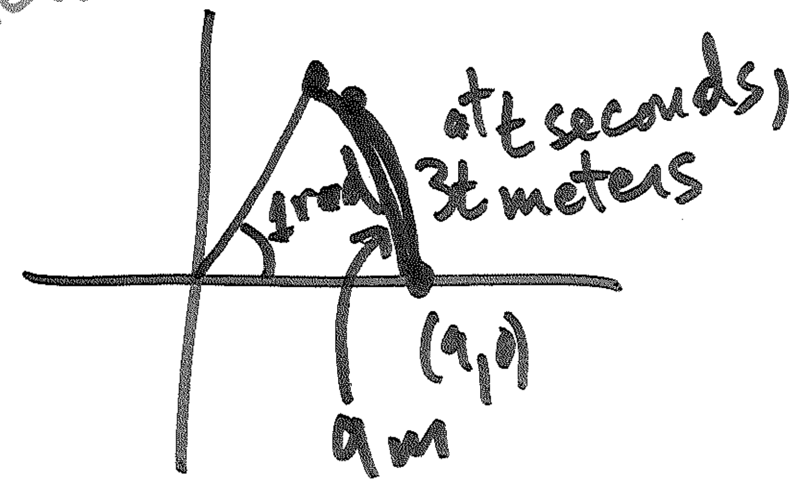
$\boxed{2}$ Log in to REEF.

$\boxed{3}$ A bug is at the point $(9,0)$, (in meters) at $t=0$ seconds. It crawls at 3m/sec counterclockwise along radius 9 circle. What are the x-y coordinates of bug after t seconds?

[Work on this w/ your neighbors].



Q: After t seconds, how many radians has the bug traveled?



After t seconds, bug has traveled $\frac{3t}{9}$ rad.

So, coords of bug are

$$\left(9\cos\left(\frac{3t}{9}\right), 9\sin\left(\frac{3t}{9}\right)\right).$$

coming from velocity.

both coming from radius

<u>Ex</u>: Suppose $\lim_{x\to -3} f(x) = 7$,

$\lim_{x\to -3} g(x) = -4$.

Find $\lim_{x\to -3} \dfrac{3f(x)-g(x)}{(f(x))^2-(g(x))^2} = \circledast$

<u>Solve</u>: $\circledast = \dfrac{\lim_{x\to -3} 3f(x)-g(x)}{\lim_{x\to -3} f(x)^2-g(x)^2}$ ← Quot. law, if den $\neq 0$.

diff. law → $= \dfrac{\lim_{x\to -3} 3f(x) - \lim_{x\to -3} g(x)}{\lim_{x\to -3} f(x)^2 - \lim_{x\to -3} g(x)^2} =$ ← const. law, power law

$= \dfrac{3\lim_{x\to -3} f(x) - \lim_{x\to -3} g(x)}{\left(\lim_{x\to -3} f(x)\right)^2 - \left(\lim_{x\to -3} g(x)\right)^2} = \dfrac{3\cdot 7-(-4)}{7^2-4^2}$.

Ex: Find $\displaystyle\lim_{h\to 0} \frac{\frac{1}{(2+h)^2} - \frac{1}{4}}{h}$.

Remark: What is the inst. velocity at 2 sec of a particle whose position is given by $\frac{1}{x^2}$ meters at x sec?

You get that the inst. vel $\underline{\underline{\text{is}}}$ this limit.

$\frac{1\cdot b}{a\cdot b} - \frac{1\cdot a}{b\cdot a} = \frac{b-a}{ab}$

Sol'n: $\displaystyle\lim_{h\to 0} \frac{\boxed{\frac{1}{(2+h)^2} - \frac{1}{4}}}{h} = \lim_{h\to 0} \frac{\frac{4-(2+h)^2}{4\cdot(2+h)^2}}{h}$

Find common den. in num.

$$= \lim_{h\to 0} \frac{4-(4+4h+h^2)}{h\cdot 4\cdot(2+h)^2} = \lim_{h\to 0} \frac{\cancel{4}-\cancel{4}-4h-h^2}{h\cdot 4\cdot(2+h)^2}$$

simplify

NOTE: $\dfrac{\frac{a}{b}}{c} = \dfrac{a}{b\cdot c}$

$$= \lim_{h \to 0} \frac{-4h-h^2}{h \cdot 4(2+h)^2} = \lim_{h \to 0} \frac{\cancel{h}(-4-h)}{\cancel{h} \cdot 4(2+h)^2} = \lim_{h \to 0} \frac{-4-h}{\underbrace{4(2+h)^2}_{\text{cts at } h=0.}}$$

$$= \frac{-4-0}{4(2+0)^2} = \frac{-4}{4 \cdot 4} = \frac{-1}{4}.$$

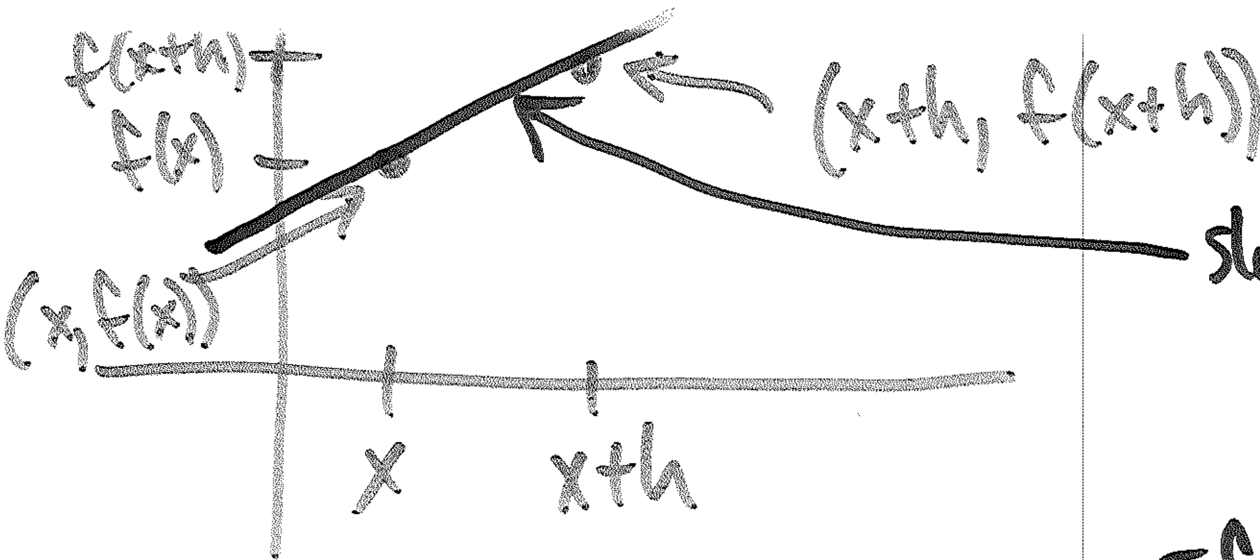
$\boxed{\underline{\text{REEF}}: 4}$

MA 113 | 2/8/17

1. Log in to REEF.
2. Webwork B1 due Friday.
3. Exams will be returned in recitation tomorrow. If there is a curve, it will be announced on Friday.
4. Discuss w/ your neighbors: why is

$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$$

the slope of a line? What line is it the slope of?



slope $\dfrac{f(x+h)-f(x)}{h}$.

If f is position of a particle, this is average velocity.

$\boxed{\text{See Desmos}}$

Def$^{\underline{n}}$: Given points $P=(a, f(a))$ and $Q=(a+h, f(a+h))$, on graph of f, slope of <u>secant</u> <u>line</u> between P + Q is $\dfrac{f(a+h)-f(a)}{h} := m_{PQ}$.

The <u>tangent line</u> to graph of f at P is the line through $P = (a, f(a))$ with slope

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}, \quad \underline{\underline{\text{if}}} \text{ limit exist.}$$

(This is section 2.7 in book).

<u>Note</u>: Two ways to represent average + inst. velocity / ~~sec~~ secant + tangent lines.

$$[a, a+h] \longrightarrow \frac{f(a+h) - f(a)}{h} = \text{average velocity} = \text{slope of secant line.}$$

$$[a, x] \longrightarrow \frac{f(x) - f(a)}{x - a} = \text{same.}$$

Dictionary

Object w/ position given by $f(x)$	$\longleftrightarrow$	graph of $f(x)$.
Average velocity on $[a,x] = [a,a+h]$ is $\dfrac{f(x)-f(a)}{x-a} = \dfrac{f(a+h)-f(a)}{h}$	$\longleftrightarrow$	slope of secant line between $(a, f(a))$ and $(a+h, f(a+h)) = (x, f(x))$.
Inst. velocity on $[a,x] = [a,a+h]$ is $\lim\limits_{h\to 0} \dfrac{f(a+h)-f(a)}{h} = \lim\limits_{x\to a} \dfrac{f(x)-f(a)}{x-a}$	$\longleftrightarrow$	Slope of tangent line to $(a, f(a))$ on graph of $f(x)$.

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Def$^{\underline{n}}$: The <u>derivative</u>, if it exists, of $f(x)$ at a is

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}.$$

<u>Ex</u>: Let $f(x) = x^2+x-3$. Compute $f'(a)$ using limits.

By def$^{\underline{n}}$, $f'(a) = \lim_{h\to 0} \frac{(a+h)^2+(a+h)-3-(a^2+a-3)}{h}$

$$= \lim_{h\to 0} \frac{\cancel{a^2}+2ah+h^2+\cancel{a}+h\cancel{-3}\cancel{-a^2}\cancel{-a}\cancel{+3}}{h}$$

$$= \lim_{h\to 0} \frac{2a\cancel{h}+h^{\cancel{2}}+\cancel{h}}{\cancel{h}} = \lim_{h\to 0} 2a+h+1 = 2a+1.$$

Start §2.8:

If you let a vary, $f'(a)$ is a function.