MA 113 1/13/17

$\boxed{1}$ Introduce yourself to the students near you.

$\boxed{2}$ Discuss with your neighbors:

why is $2^0 = 1$? why is $3^0 = 1$?

Can you give an explanation for this that a middle-school student would understand?

(Saying "my teacher told me" does not count as an explanation. 😊)

Reminders

A. See my canvas announcement from today.

B. Webwork login

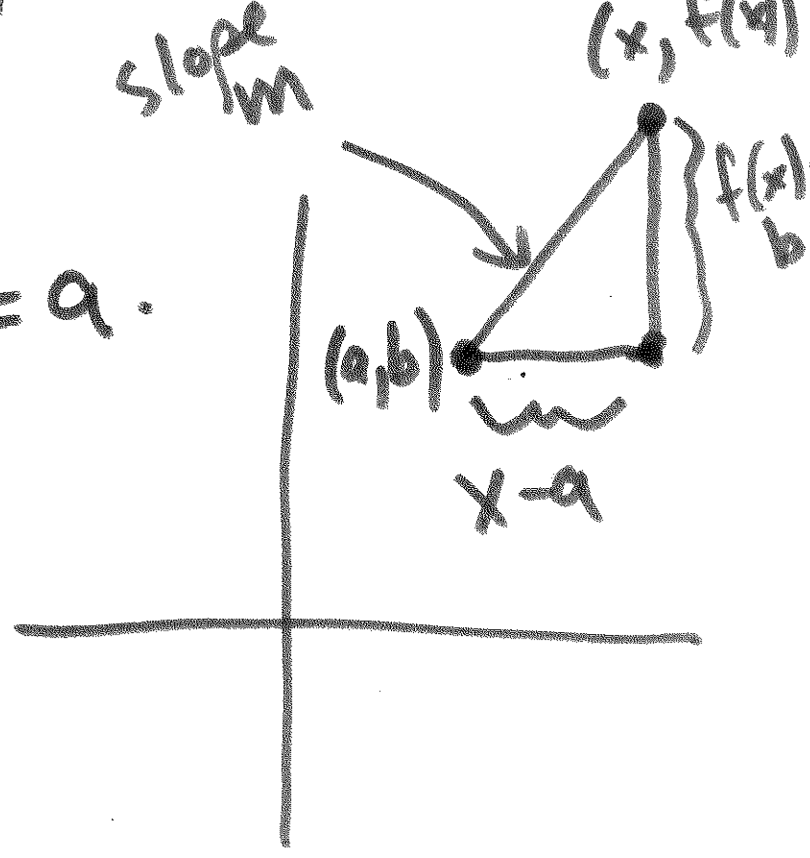
C. HW due next week
  - Webworks A1, A2
  - Written Assignment #1

D. We will use REEF polling for attendance.
More info on this next week.

Q: What is a line? (in terms of equations)

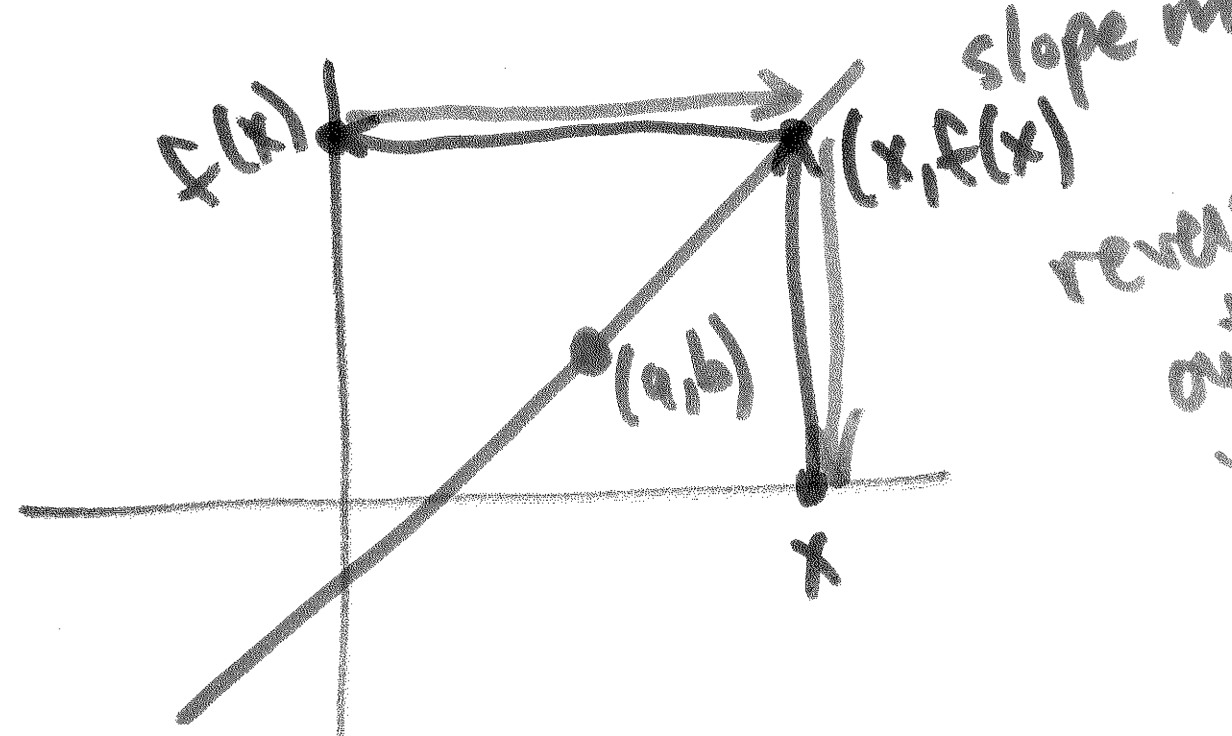
Def$^{\underline{n}}$: A function $f(x)$ is linear if there is a point (a,b) and a slope m so that

$$\underbrace{\frac{f(x)-b}{x-a}}_{} = m \quad \text{for } x \neq a.$$



"secretly" point-slope form, cross-multiply

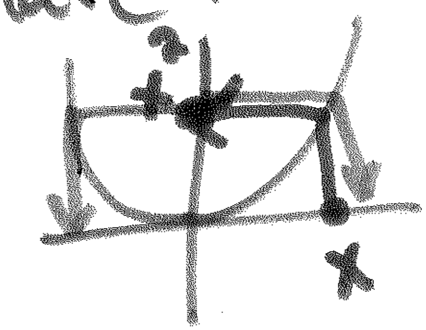
<u>Note:</u> Linear functions are invertible if slope is not 0.



reversing input and output creates inverse.

Inverses don't have to exist!

$f(x) = x^2$



when I try to invert, I get two inputs for the output.

(§1.4, 1.5)

<u>Q:</u> a) If a population of bacteria doubles every hour and has a starting population of 162 bacteria, how many will there be after 5 hours? 5.25 hours?

After 1 hour, 2×162 bact.

After 2 hours, $2 \times 2 \times 162$ bact.

After 3 hrs, $2 \times 2 \times 2 \times 162$ bact.

$\vdots \qquad \vdots$

After t hours, $2^t \cdot 162$ bact.

So, after 5 hours, we have

$$2^5 \cdot 162 \text{ bact.}$$

after 5.25 hours, we have

$$2^{5.25} \cdot 162 \text{ bact.}$$

We are studying the function

$$P(t) = 162 \cdot 2^t.$$

$$P(5.25) = 162 \cdot 2^{5.25} = 162 \cdot 2^{5+\frac{1}{4}}$$

$$= \underbrace{162 \cdot 2^5}_{\text{we like!}} \cdot \underbrace{2^{1/4}}_{\text{this is hard}}$$

Q: why is $2^{1/4} = \sqrt[4]{2}$?

$\sqrt[4]{2}$ satisfies $\sqrt[4]{2} \cdot \sqrt[4]{2} \cdot \sqrt[4]{2} \cdot \sqrt[4]{2} = 2.$

What about $2^{1/4}$? $2^{1/4} \cdot 2^{1/4} \cdot 2^{1/4} \cdot 2^{1/4} =$

$2^{1/4+1/4+1/4+1/4} = 2^1 = 2.$

We need a technique to evaluate $2^{1/4} = \sqrt[4]{2}$ to estimate $162 \cdot 2^{5.25}$. These come from calc.

Q: How do we estimate 2^{π}?

well, $\pi \approx 3.14 = \frac{314}{100}$.

So, $2^{\pi} \approx 2^{3.14} = 2^{314/100} = \sqrt[100]{2^{314}}$.

Exponentials:

Pick a base $b>0$, usually not $b=1$.

Define $f(x) = b^t$. This makes sense for integers (see Tuesday recitation). When t is a fraction, approximate using calc. When t is irrational, approx. t using fraction, then approx b^t using calc. This lets you define b^t for any real number t.

Note: $f(t) = b^t$, if $b > 0$ and $b \neq 1$, is one to one, so it has an inverse.

Ex: $f(t) = 2^t$. $\longrightarrow$ graph w/ desmos.

Def$^{\underline{n}}$: Inverse of $f(t) = b^t$ is written $\log_b(x)$ and satisfies

$$b^t = x \iff \log_b(x) = t.$$

NOTE: Figuring out what $\log_b(x)$ *actually* is and computing with it is learned in Calc II.

Note: We've seen e^x and $\ln(x)$.

Q: what is e? It is a value that makes the <u>derivative</u> (later) of e^x nice.

Alternatively, it is the value that makes

$$e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{3\cdot 2} + \frac{x^4}{4\cdot 3\cdot 2} + \frac{x^5}{5\cdot 4\cdot 3\cdot 2} + \frac{x^6}{6\cdot 5\cdot 4\cdot 3\cdot 2} + \cdots + \frac{x^n}{n\cdot \cdots \cdot 2}$$

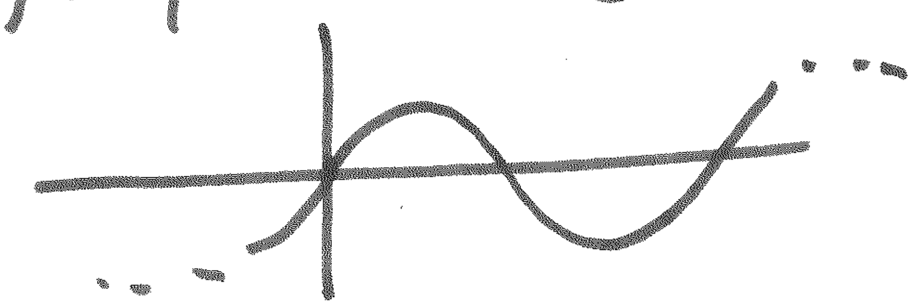
MA113 1/18/17

$\boxed{1}$ With your neighbors, discuss the following:

a) Why are there 360° in a circle? Why not 100°? or 300°?

b) Why are there 2π radians in a circle? What is 1 radian?

c) Why does the graph of $\sin x$ look like

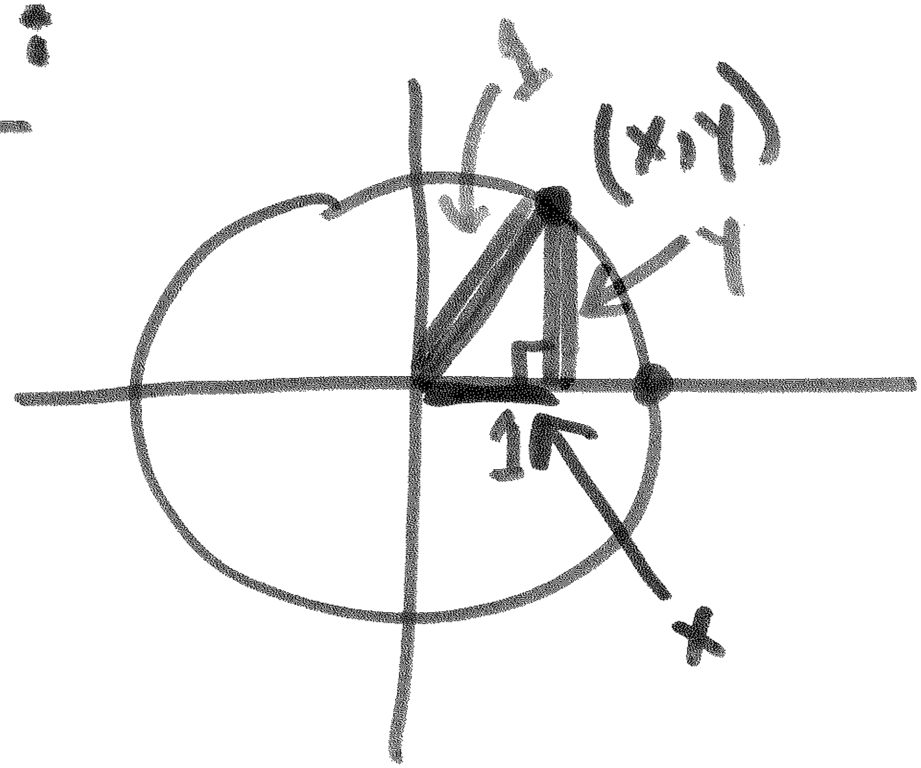


Reminders

- Webwork A1 due tonight
  A2 due Friday
- Quiz 1 at end of recitation tomorrow.
  (2 problems, 5 points, 10 minutes)
- Written Assignment #1 due Friday at beginning of lecture.
  → WA#1 available on the course website, see my announcement on Canvas about this.
- Attendance starts Monday using REEF -- I'll send info via Canvas.

Trig Review:

Unit Circle:



Pyth. Thm says
$$x^2+y^2=1.$$

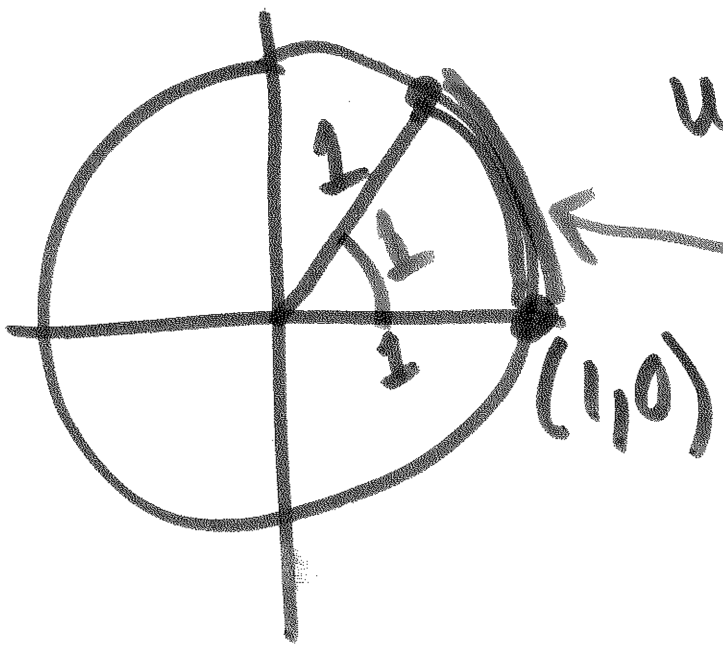
Issue: We want to measure how far around the circle we have gone.

Ancient method: Use 360° as total measurement around circle.

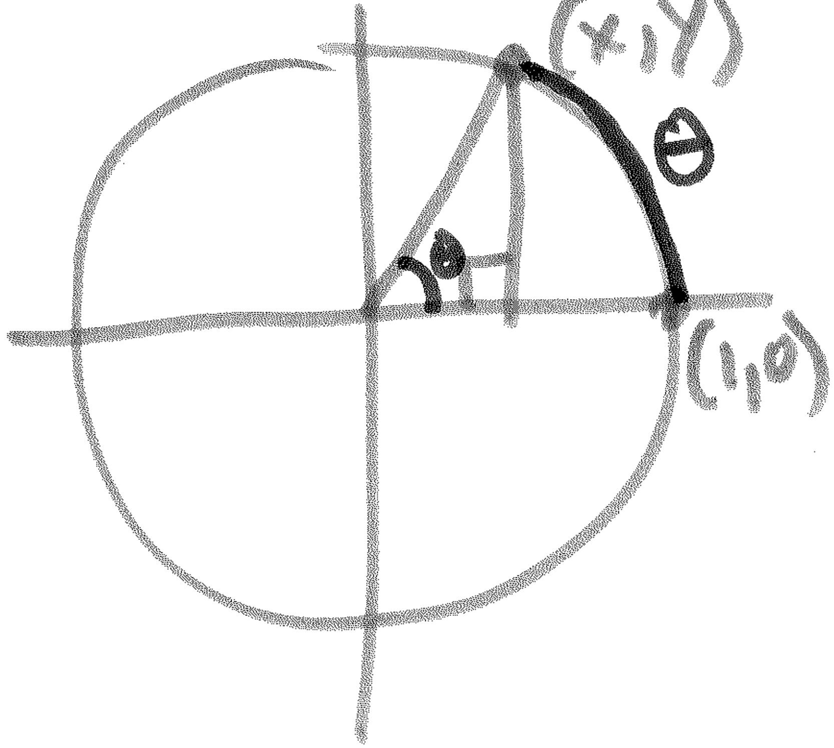
Problem: This isn't sensical mathematically.

<u>Solution:</u> Measure angles w/ radius of your circle.

$\longrightarrow$ see wikipedia unit circle



1 radian is angle where this arc length is equal to 1 (on unit circle).



Q: How can I determine x and y based on length of arc from $(1,0)$ to (x,y) counter-clockwise?

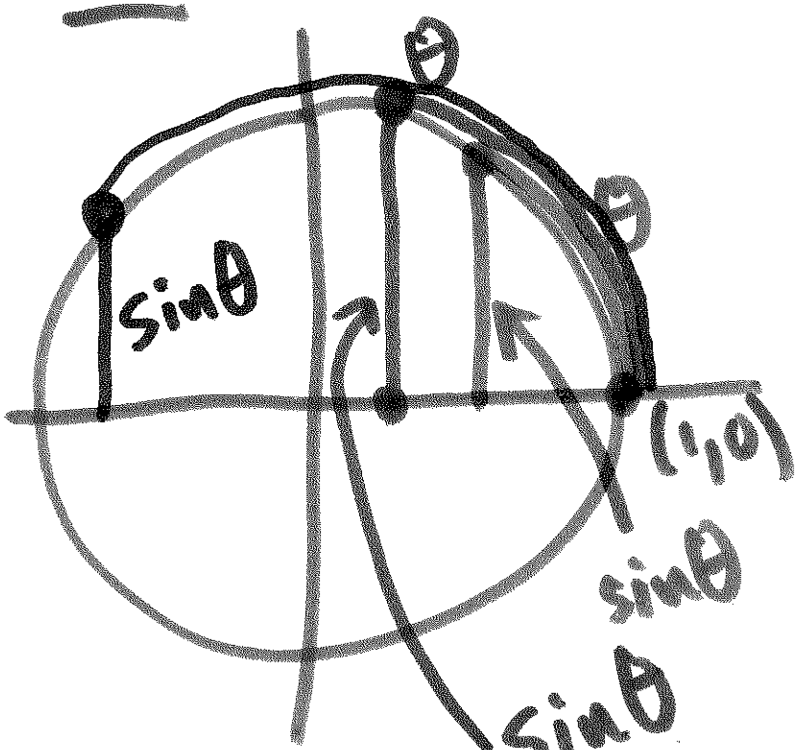
A: This is *very* difficult!

What we do, since we don't know the answer, is give these values a name.

Call $x = \cos\theta$, call $y = \sin\theta$.

Note: At end of MA113, we will *see* good approximations of these with polynomials.

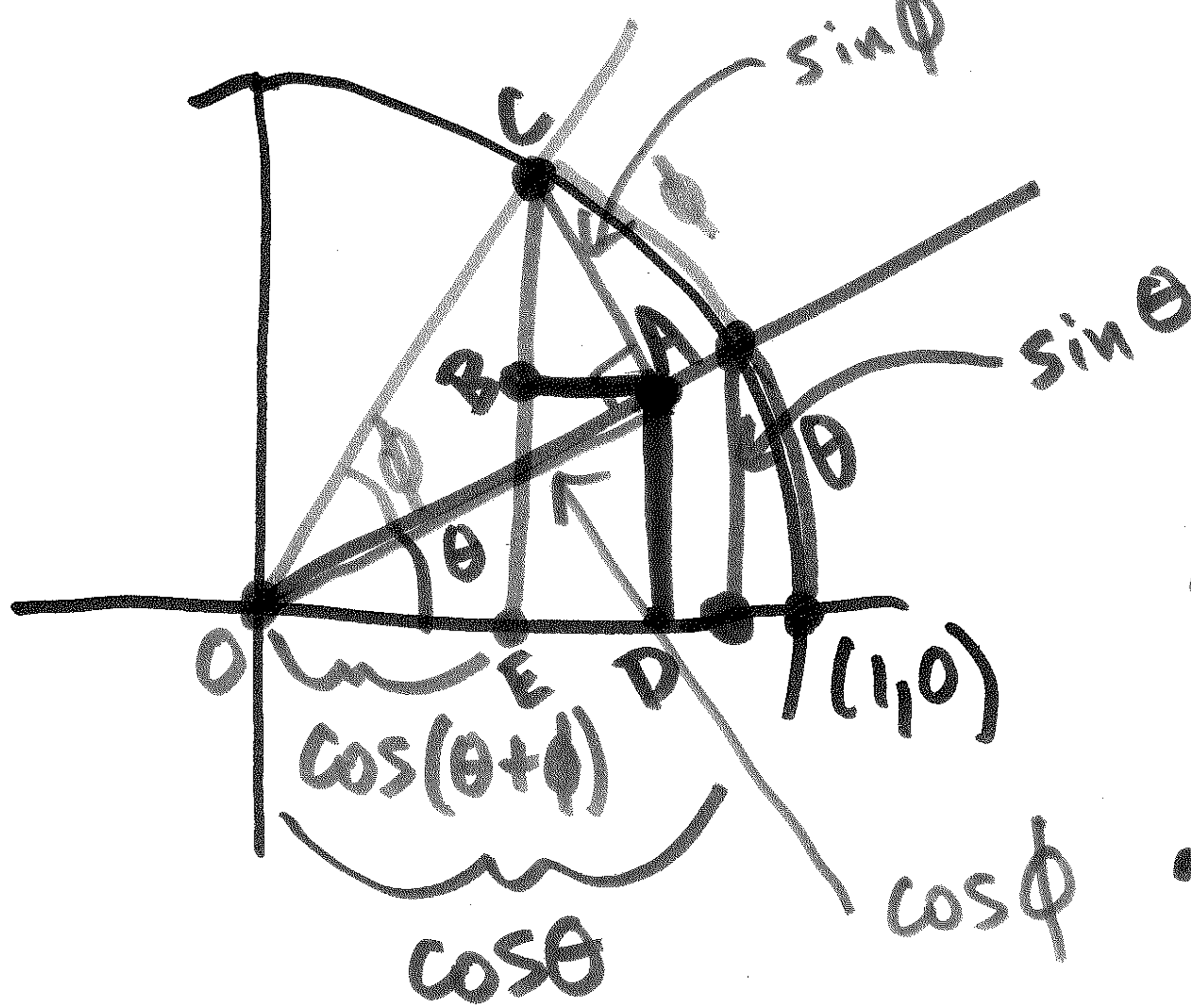
Q: How does $\sin\theta$ vary as θ varies?



→ see wikipedia.
Graph is result of circular motion.

How does unit circle help beyond motivation?

$$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi.$$



We want OE.

$= \cos(\theta+\phi)$

$OE = OD - \underbrace{AB}_{=ED}$

- $OA = \cos\phi$, so

$OD = \cos\theta\cos\phi$ by scaling

- $AC = \sin\phi$, so

$AB = \sin\theta\sin\phi$ by similar triangles

BCA similar to triangle formed by angle θ.

so, $OE = OD - AB$

$\Rightarrow \cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi.$