# MA 113 1/30/17

1. Log into your REEF account, for attendance today.

2. See Canvas announcement for reminders about HW/Quiz this week.

3. Exam 1 on Tues, Feb 7. Exam location will be announced later this week.

REEF: 7

## Squeeze Thm Ex:

$$\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right).$$

Since $-x^2 \leq x^2 \sin\left(\frac{1}{x}\right) \leq x^2$ and

$\lim_{x \to 0} -x^2 = 0 = \lim_{x \to 0} x^2$, by the Squeeze Thm

we have $\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$.

This is a complete soln.

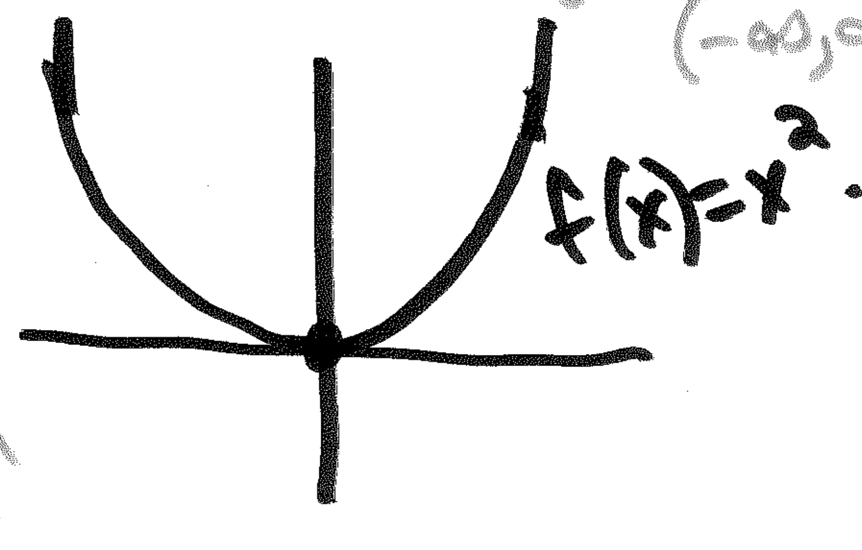
# §2.5: Continuity

**Defⁿ**: A function $f$ is <u>continuous at a number $a$</u> if

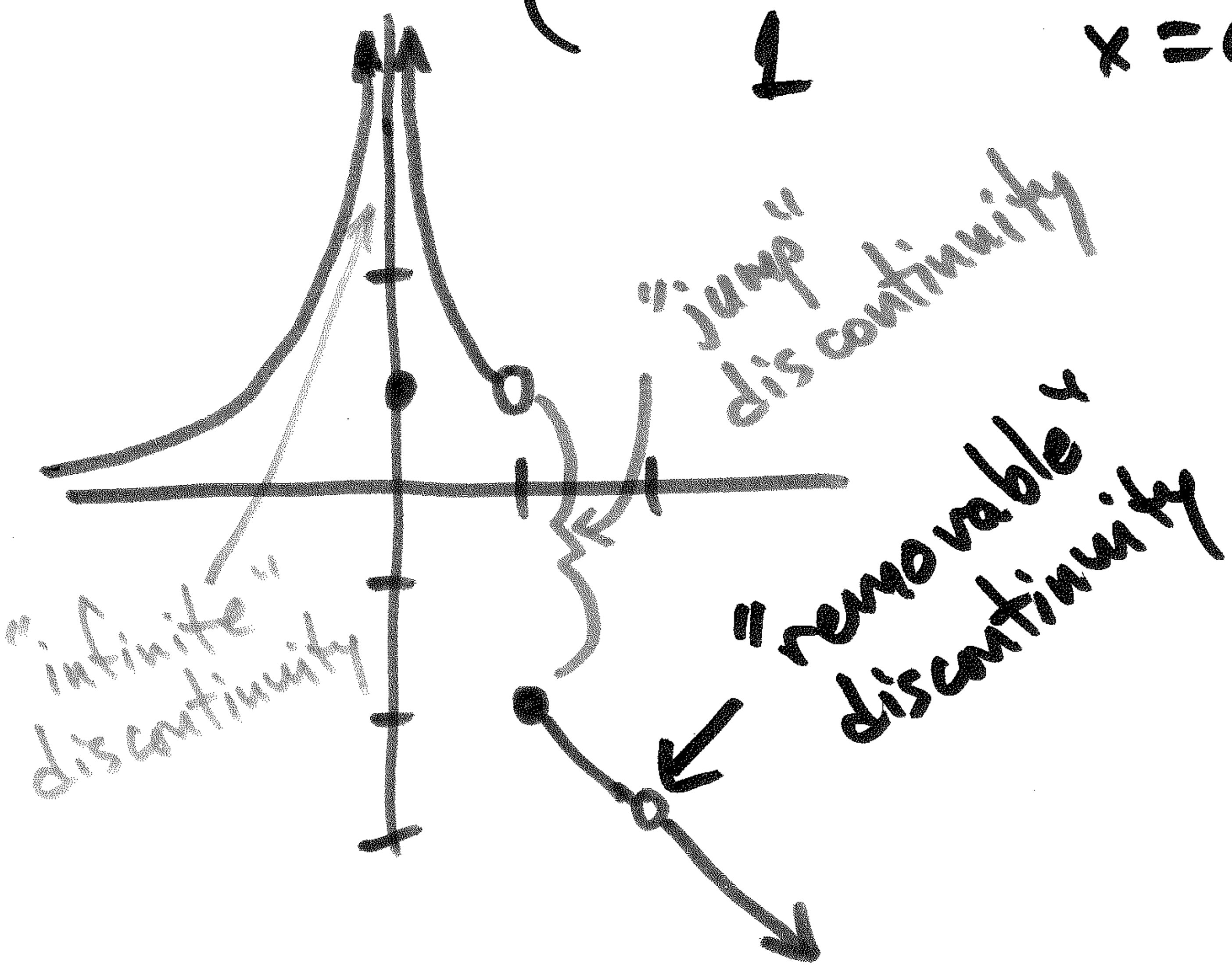
$$\lim_{x \to a} f(x) = f(a).$$

**Ex**: $f(x) = x^2$, then for every $a$ in $(-\infty, \infty)$, $f$ is cts at $a$.

"cts = continuous"



__Ex:__ $f(x) = \begin{cases} \dfrac{-x^2+x+2}{x-2} & x \geq 1 \\ 1/x^2 & x<0 \text{ and } 0<x<1 \\ 1 & x=0 \end{cases}$



Remark: Read §2.5 for def$^{\underline{n}}$ of one-sided continuity.

Def$^{\underline{n}}$: A function $f(x)$ is cts on an interval if $f$ is cts at every number in that interval.

Ex: $f(x) = \frac{1}{x^2}$ is not cts at $x=0$, but it is cts on $(-\infty, 0)$. since $\lim_{x \to a} f(x) = \frac{1}{a^2} = f(a)$ for any $a$ in $(-\infty, 0)$.

**Thm:** Continuity is preserved by $+, -, \times, \div$, and mult. by a constant.

**Ex:** $f, g$ both cts on $(a,b)$, then $f+g$, $f-g$, $f \cdot g$, etc are also cts on $(a,b)$.

**Note:** If $g(x) = 0$ for some $x$, then $f/g$ is not defined there, so cty fails.

**Thm:** The following functions are cts at every point in their domains:

- polynomials
- rational fns
- trig + inverse trig fns
- exponential + log fns.

__Ex:__ Find $\lim_{x \to 2\pi} \frac{\cos x + x^2}{3 - \sin x}$.

Since $\cos x$, $x^2$, $3$, $\sin x$ are all cts and fn is built from these using $+$, $-$, $\div$, and since $2\pi$ is in the domain of this fn, the fn is cts and we can use direct substitution.

$$\lim_{x \to 2\pi} \frac{\cos x + x^2}{3 - \sin x} = \frac{\cos(2\pi) + (2\pi)^2}{3 - \sin(2\pi)}$$

$$= \frac{1 + 4\pi^2}{3}.$$

**Thm:** If $f$ is cts at $g(a)$, and $g$ is cts at $a$, then $f \circ g$ is cts at $a$.

**Ex:** $\sin(\cos(x^4))$ is cts.

---

**Intermediate Value Thm:**

Suppose $f(x)$ is cts on $[a,b]$ with $f(a) \neq f(b)$ and some $N$ satisfies $f(a) < N < f(b)$. Then there is a "$c$" in $(a,b)$ so that $f(c) = N$.

# MA 113 2/1/17

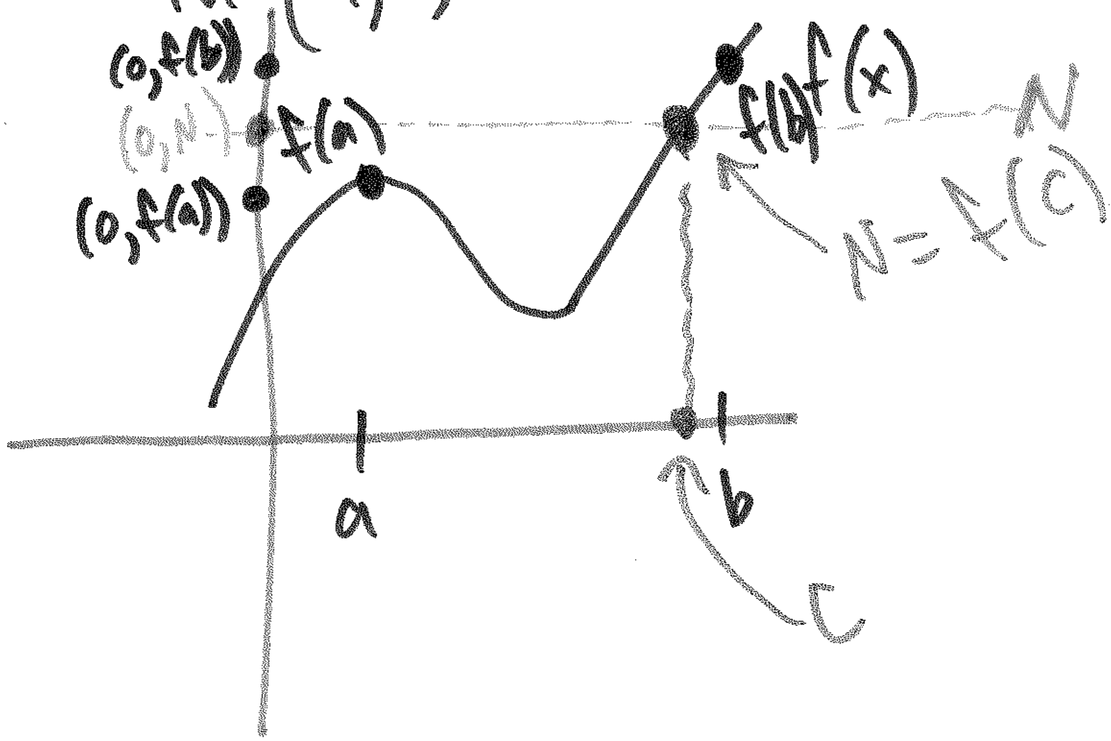
1. Log into your REEF account and join polling session.

2. See Canvas announcements about Exam 1.
   - → room locations
   - → review session
   - → Int. Value Theorem + exam structure

3. Graph $\frac{x^2-1}{x^2+1}$ and zoom out. What does the graph look like? Why? (discuss with your neighbors.)

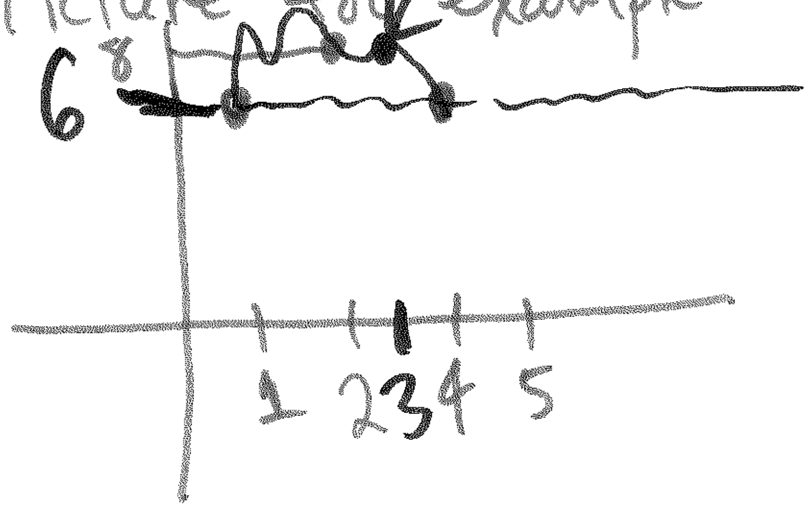
Reminder: Int. Value Theorem says

Suppose $f$ is cts on $[a,b]$, $f(a) \neq f(b)$, and $f(a) < N < f(b)$; then there is a "$c$" in $(a,b)$ where $f(c) = N$.




**Ex:** Suppose $f$ is cts on $[1,5]$. Suppose only solns to $f(x)=6$ are $x=1$, $x=4$. If $f(2)=8$, explain why $f(3)>6$.

Picture for example: $f(3)$ must be higher than 6.




Know $f(1)=6$, $f(4)=6$.

**Soln:** Since $f(1)=6$ and $f(4)=6$, and no other value of $x$ in $[1,5]$ has $f(x)=6$, then $f(x)$ is either $>6$ or $<6$ on $(1,4)$. This is because if some value $x_1$ had $f(x_1)>6$ and another value $x_2$ had $f(x_2)<6$, then between $x_1$ and $x_2$ we would have a third solution to $f(x)=6$, by I.V.T. Since $f(2)=8>6$, we must have $f(3)>6$ as well.

Claim about f.

explains claim using IVT

connects to solve problem.

REEF poll: 3

## §2.6: Limits at infinity

Ex: $f(x) = \dfrac{x^2-1}{x^2+1}$

Q: What happens when $x$ is very large (positive or negative)?

In this example, if $x$ is very large, we can divide numerator + denominator by $x^2$.

So, $f(x) = \dfrac{\frac{1}{x^2}\cdot(x^2-1)}{\frac{1}{x^2}(x^2+1)} = \dfrac{1-\frac{1}{x^2}}{1+\frac{1}{x^2}} \approx \dfrac{1}{1} = 1$ when $x$ is very large.

__Def$^{\underline{n}}$__: Let $f$ be defined on $(a, \infty)$. Then $\lim_{x \to \infty} f(x) = L$ means that the value of $f(x)$ can be made as close as we want to $L$ by requiring $x$ to be sufficiently large.

__NOTE:__ Similar def$^{\underline{n}}$ for $\lim_{x \to -\infty} f(x) = L$.

If $\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$, we say $y = L$ is a __horizontal asymptote__ for $f(x)$.

**Thm:** If $r>0$ is a rational number, then $\lim\limits_{x\to\infty} \frac{1}{x^r} = 0$.

**Think:** $\frac{1}{x^{1/2}} = \frac{1}{\sqrt{x}}$, or $\frac{1}{x^2}$, or $\frac{1}{x^7}$, or $\frac{1}{x^{3/4}} = \frac{1}{\sqrt[4]{x^3}}$.

If $r>0$ is rational and $x^r$ is defined for all $x$ in $(-\infty,\infty)$, then $\lim\limits_{x\to-\infty} \frac{1}{x^r} = 0$.

**Think:** Distinction between $\frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}}$ and $\frac{1}{\sqrt[3]{x}} = \frac{1}{x^{1/3}}$.

**Key Idea:** Use algebra to transform functions so we can take limits.

Ex: Find $\displaystyle\lim_{x\to\infty} \frac{4x^7 - x^6 + 1}{\frac{1}{5}x^7 - x + 3} = \star$

General technique: mult num. + den. by $\frac{1}{x^n}$ where $n =$ degree of denominator.

$$\star = \lim_{x\to\infty} \frac{\frac{1}{x^7}(4x^7 - x^6 + 1)}{\frac{1}{x^7}(\frac{1}{5}x^7 - x + 3)} = \lim_{x\to\infty} \frac{4 - \underbrace{\frac{1}{x}}_{\to 0} + \underbrace{\frac{1}{x^7}}_{\to 0}}{\frac{1}{5} - \underbrace{\frac{1}{x^6}}_{\to 0} + \underbrace{\frac{3}{x^7}}_{\to 0}}.$$

$$= \frac{4}{1/5} = 20.$$