# MA 113    3/29/17

$\boxed{1}$ REEF today

$\boxed{2}$ Today we are continuing to discuss optimization problems.

$\boxed{3}$ See Canvas announcements for assignments this week.

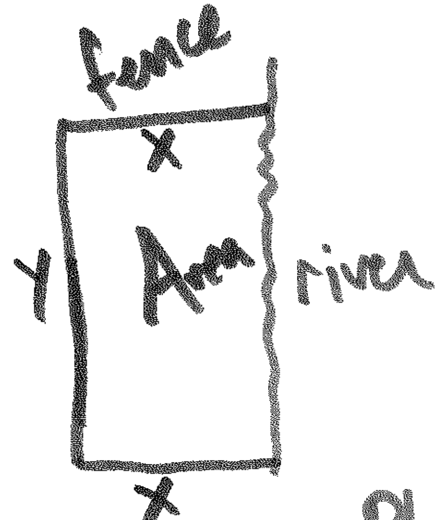
$\boxed{4}$ Lecture notes are up to date on my website.

$\boxed{5}$ First problem today; Start working on it!

A farmer has 100 meters of fence, and wants to enclose a rectangular field of max possible area bordering a river with a straight shore/bank. What $\text{dim}^{\underline{n}}\text{'s}$ should be used for the field?

Do three things to create math. model:

1. Draw a diagram.
2. Find/create/derive a one-variable function to max/min optimize.
3. Specify domain of that function for the model.




Maximize Area $= x \cdot y$ ← want only one variable.

where $2x + y = 100$.

Since $y = 100 - 2x$, Area $= A(x) = x(100-2x)$
$= 100x - 2x^2$.

Physical limits tell us

- $x > 0$
- $x < 50$ (since $2x < 100$).

Domain of model is $(0,50)$.

**GOAL**: Maximize $A(x)$ on $(0,50)$.

Find critical values: $A'(x) = 100 - 4x$.

So, $A'(x)$ is defined for all $x$, $A'(x) = 0$ when $x = 25$.

Check $A'(x)$ to verify $A'(x) < 0$ for $x > 25$,

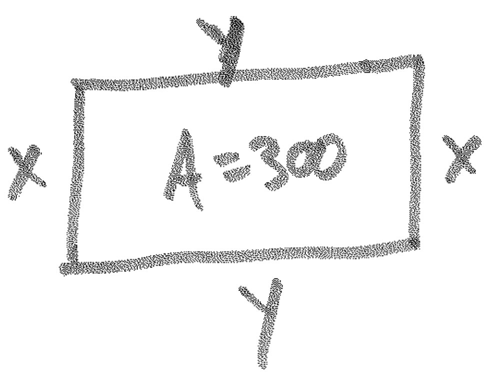
and $A'(x) > 0$ when $x < 25$.

By an 1st der. test for absolute maxes, we have an abs. max at $x = 25$.

If $x = 25$, $y = 100 - 2(25) = 100 - 50 = 50$.

So, dim's are $x = 25$m, $y = 50$m for max area.

**Ex:** Find the dim$^{\underline{n}}$s of a rectangle w/ area 300 cm$^2$ that has minimum perimeter.



Minimize $2x+2y$ = perimeter.

$300 = x \cdot y$ is area.

So, $y = \frac{300}{x} \Rightarrow \underset{\text{perimeter}}{P(x)} = 2x + 2 \cdot \left(\frac{300}{x}\right)$

$= 2x + \frac{600}{x}$.

Physical limits are $x, y > 0$.

So, domain is $(0, \infty)$.

**GOAL:** Minimize $P(x)$ on $(0, \infty)$.

Critical values: $P'(x) = 2 - \frac{600}{x^2}$ is defined on $(0, \infty)$.

$P'(x) = 0 \Rightarrow 2 - \frac{600}{x^2} = 0 \Rightarrow x^2 = 300 \Rightarrow x = \boxed{\sqrt{300}}$!

only use positive, since domain is $(0,\infty)$

Since only one critical value, this corresponds to an absolute max or min.

Since $P''(x) = \frac{1200}{x^3}$, we have $P''(\sqrt{300})$

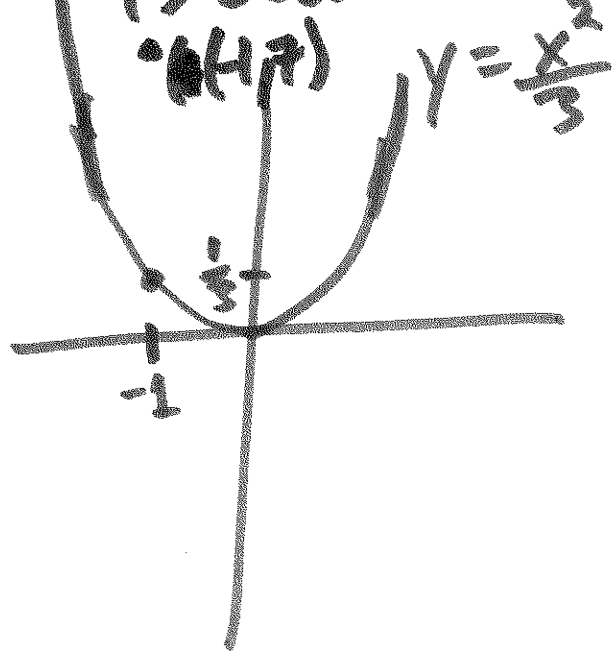
$$= \frac{1200}{(\sqrt{300})^3} > 0.$$

Thus, $2^{nd}$ der test tells us $x = \sqrt{300}$ yields a min on $(0, \infty)$.

Since $x = \sqrt{300}$, $y = \frac{300}{\sqrt{300}} = \sqrt{300}$. So, dim's are $\sqrt{300}$ by $\sqrt{300}$.

REEF: 5

<u>Ex</u>: Find point on parabola $3y = x^2$ that is closest to the point $(-1, 7)$.




<u>Idea</u>: minimize distance between $(-1,7)$ and pts $(x,y)$ on $3y = x^2$.

<u>General principle</u>: distances often involve square roots, which are messy. Instead of minimizing dist., minimize $(\text{distance})^2$.

Let $D$ = distance from $(-1,7)$ to $(x,y)$, and we have want to minimize.

$$D^2 = (x-(-1))^2 + (y-7)^2.$$

<u>NOTE</u>: $y = \frac{x^2}{3}$.

$$= (x+1)^2 + \left(\frac{x^2}{3} - 7\right)^2.$$

Domain is all $x$ in $(-\infty, \infty)$.

<u>GOAL</u>: Minimize $D^2(x) = \underbrace{(x+1)^2 + \left(\frac{x^2}{3} - 7\right)^2}_{\text{degree 4 polynomial.}}$ on $(-\infty, \infty)$.

Compute $(D^2)'(x) = \underbrace{2(x+1) + \frac{2x}{3} \cdot 2 \cdot \left(\frac{x^2}{3} - 7\right)}_{\text{cubic.}}$.

For this problem, we don't know how to solve exactly.

<u>Remark</u>: This shows how quickly exact answers elude us. This type of approximation of $(D^2)'(x) = 0$ is why Newton's method was invented.

# MA 113 3/31/17

1. Turn in your written assignments.
2. REEF ~~today~~.
3. Today: §4.9, Antiderivatives.
4. Q: What functions have $\cos(x)$ as a derivative?

   Q: What functions have $2x+1$ as a derivative?

Talk to your neighbor!

__Def^n__: $F(x)$ is an __antiderivative__ of $f(x)$ on $(a,b)$ if $F'(x)=f(x)$ on $(a,b)$.

__Thm__: If $F'(x)=f(x)$ on $(a,b)$, then every other antiderivative of $f$ on $(a,b)$ is of the form $F(x)+\underset{\uparrow \text{ constant}}{C}$.

__NOTATION:__ Write $\boxed{\int f(x)\,dx} = F(x)+C$ for an antiderivative of most general form.

__indefinite integral__.

**Ex:** Find $\int \cos x \, dx = \sin x + C$

**Ex:** Find $\int \sin x \, dx = -\cos x + C$

**Ex:** Find $\int e^x \, dx = e^x + C$.

domains $(-\infty, \infty)$

**Ex:** Find $\int x^3 \, dx = \frac{x^4}{4} + C$

domain of $\frac{1}{x}$ is $(-\infty, 0) \cup (0, \infty)$

**Ex:** Find $\int \frac{1}{x} \, dx = \ln|x| + C$

**Rmk:** Pay attention to domains!

__Ex__: If $F'(x) = f(x)$, $G'(x) = g(x)$ on $(a,b)$.

what is (on $(a,b)$) $\int f(x) + g(x)\, dx = F(x) + G(x) + C$

__Ex__: Find an antiderivative of $\left[3\cos x + \dfrac{3x^5 - 7\sqrt[3]{x}}{x^2}\right]$.

__Simplify__: $3\cos x + \dfrac{3x^5}{x^2} - \dfrac{7\sqrt[3]{x}}{x^2} = 3\cos x + 3x^3 - \dfrac{7\sqrt[3]{x}}{x^2}$

$3\cos x + 3x^3 - \dfrac{7x^{1/3}}{x^2} = \underbrace{3\cos x} + \underbrace{3x^3} \underbrace{- 7x^{-5/3}}$

Antider of $3\cos$ is $3\sin x$.

Antider of $3x^3$ is $\frac{3}{4}x^4$.

Antider of $-7x^{-5/3}$ is $\frac{-7x^{-2/3}}{-2/3}$

So, antider of original expression is

$$3\sin x + \frac{3}{4}x^4 + \frac{7x^{-2/3}}{2/3}.$$

$$\frac{21}{2\sqrt[3]{x^2}}.$$

REEF:

4

Ex: If $f''(x) = 12x^2 + 6x - 4$ w/ $f(0) = 4$, $f(1) = 1$, find $f$.

$$f'(x) = \int f''(x)\,dx = 4x^3 + 3x^2 - 4x + C.$$

$$f(x) = \int f'(x)\,dx = x^4 + x^3 - 2x^2 + Cx + D$$

another constant.

Know $f(x)$ is this polynomial w/ some undetermined coefficients.

$$f(0) = 0^4 + 0^3 - 2\cdot 0^2 + C\cdot 0 + D = 4. \Rightarrow D = 4.$$

$$f(1) = 1^4 + 1^3 - 2\cdot 1^2 + C\cdot 1 + 4 = 1$$

$$\Rightarrow 1 + 1 - 2 + C + 4 = 1 \Rightarrow C = -3.$$

Thus, $f(x) = x^4 + x^3 - 2x^2 - 3x + 4$.

Suppose you knew $f(1)=1$, $f(2)=5$.

$$f(1) = 1^4 + 1^3 - 2\cdot 1^2 + C\cdot 1 + D = 1$$

$$f(2) = 2^4 + 2^3 - 2\cdot 2^2 + C\cdot 2 + D = 5.$$

$\Rightarrow$ solve $2\times 2$ equation to find $C, D$.