MA 113 3/3/17

$\boxed{1}$ Log in to REEF

$\boxed{2}$ Exam 2 will be over all material covered since Exam 1 → see course calendar on course website.

$\boxed{3}$ Today: Review

$\boxed{4}$ w/ your neighbors, discuss topics you think are most challenging for Exam 2.

<u>Ex</u>: Find $\frac{dy}{dx}$ for $y = \underbrace{e^{3x}}_{\text{chain rule}} \cdot \underbrace{2^{\cos(x^2+1)}}_{\text{chain rule two times.}}$

Product Rule

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{3x}\right) \cdot 2^{\cos(x^2+1)} + e^{3x} \cdot \frac{d}{dx}\left(2^{\cos(x^2+1)}\right)$$

$$= 3e^{3x} \cdot 2^{\cos(x^2+1)} + e^{3x} \cdot \ln(2) \cdot 2^{\cos(x^2+1)} \cdot \frac{d}{dx}\left(\cos(x^2+1)\right)$$

$$= 3e^{3x} \cdot 2^{\cos(x^2+1)} + e^{3x} \cdot \ln(2) \cdot 2^{\cos(x^2+1)} \cdot \left(-\sin(x^2+1)\right) \cdot 2x$$

$$= 3e^{3x} \cdot 2^{\cos(x^2+1)} - e^{3x} \cdot \ln(2) \cdot 2^{\cos(x^2+1)} \cdot \sin(x^2+1) \cdot \overset{1}{2}x$$

wolfram alpha combines these.

<u>Note</u>: $\frac{d}{dx} 2^x = \ln(2) \cdot 2^x$.

Because $2^x = e^{\ln(2^x)} = e^{\ln(2) \cdot x}$

<u>Ex</u>: Find equation to tangent line at $(1,1)$ for curve $y^3+2xy+x^3=4$.

<u>Step 1</u>: Find $\frac{dy}{dx}$ using implicit diff.

<u>Step 2</u>: evaluate $\frac{dy}{dx}$ at $x=1, y=1$, use pt-slope form to find line.

$\boxed{1}$:

$$\frac{d}{dx}\left(y^3+2xy+x^3\right)=\frac{d}{dx}(4)$$

$$\Rightarrow 3y^2\cdot\frac{dy}{dx}+2y+2x\frac{dy}{dx}+3x^2=0$$

← You should take several lines to compute this.

$\Rightarrow$ Solve $\frac{dy}{dx}=-\left(\frac{3x^2+2y}{3y^2+2x}\right)$

$\boxed{2}$: $\frac{dy}{dx}$ at $(x,y)=(1,1)$ gives $-\frac{5}{5}=-1$.

So, eqn of my line is $y-1=-1(x-1)$.

REEF: 9

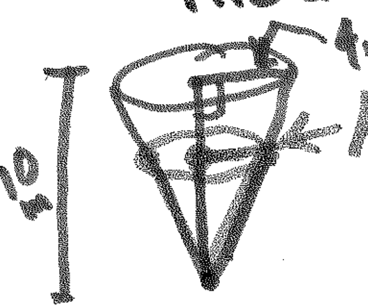
Concept Map ← create one w/ you study group.

In addition to working problems, thinking "big picture" about what you do + don't know, + how it all fits together, is important.

Ex: Suppose you have a conical tank, with water pouring in at $6m^3/min$; height of tank is 10M, radius at top is 4m. Find rate of H_2O-level increase at height 5m.

Recall: $V = \frac{1}{3}\pi \cdot h \cdot r^2$.

Given: $\frac{dV}{dt}$, need $\frac{dh}{dt}$ at $h=5$.



At time t, r(t) is radius of top of water, h(t) is height.

Need: Volume formula w/ only h, not r.

$V(t) = \frac{1}{3}\pi h(t) \cdot r(t)^2$

By similar triangles, $\frac{r(t)}{h(t)} = \frac{4}{10} \Rightarrow r(t) = .4 \cdot h(t)$.

So, $V(t) = \frac{1}{3} \cdot \pi h(t) \cdot (0.4 \cdot h(t))^2 = \frac{0.16}{3} \pi \cdot h(t)^3$

Apply $\frac{d}{dt}$ here, $\frac{dV}{dt} = \frac{0.16}{3} \cdot \pi \cdot 3 \cdot h(t)^2 \cdot \frac{dh}{dt}$.

we want $\frac{dh}{dt}$ when $h=5$, given $\frac{dV}{dt} = 6$.

So, $6 = \frac{0.16}{3} \cdot \pi \cdot 3 \cdot 5^2 \cdot \frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} \approx 0.48$ m/min.

MA 113 3/6/17

1. Log in to REEF.
2. Exam 2 tomorrow.
   → you must know the formal defn of a derivative. (look at textbook)
3. Today: Review.

Ex: Let $g = f^{-1}$, $f(2) = 4$, $f'(2) = -5$. Find $g(4)$, $g'(4)$.

Domain(f) Range(f)

$$\begin{tikzcd} 2 \arrow[r, bend left, "f"] & 4 \arrow[l, bend left, "f^{-1}"] \end{tikzcd}$$

Know $f^{-1}(4) = g(4) = 2$.

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$ ← Result we showed.

$$g'(4) = (f^{-1})'(4) = \frac{1}{f'(f^{-1}(4))} = \frac{1}{f'(2)} = \frac{1}{-5}.$$

NOTE: Remember our formula for $(f^{-1})'(x)$ is obtained by chain rule on $\underbrace{(f \circ f^{-1})(x)}_{f(f^{-1}(x))=x} = x$.

Ex: Use the def$^{\underline{n}}$ of derivative to find $f'(x)$ where $f(x) = \frac{2}{x^2}$.

By def$^{\underline{n}}$, $f'(x) = \lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \frac{\frac{2}{(x+h)^2} - \frac{2}{x^2}}{h}$

$$= \lim_{h\to 0} \frac{\frac{x^2 \cdot 2}{x^2(x+h)^2} - \frac{2\cdot(x+h)^2}{x^2(x+h)^2}}{h} = \lim_{h\to 0} \frac{\frac{2x^2 - 2(x+h)^2}{x^2(x+h)^2}}{h}$$

$$= 2 \cdot \lim_{h \to 0} \frac{x^2 - (x+h)^2}{h \cdot x^2 (x+h)^2} = 2 \cdot \lim_{h \to 0} \frac{\cancel{x^2} - \cancel{x^2} - 2xh - h^2}{h \cdot x^2 (x+h)^2}$$

$$= 2 \cdot \lim_{h \to 0} \frac{-2x\cancel{h} - h^{\cancel{2}}}{\cancel{h} \cdot x^2 (x+h)^2} = 2 \lim_{h \to 0} \frac{-2x - h}{x^2 (x+h)^2} \underset{\substack{\uparrow \\ \text{by limit} \\ \text{laws}}}{=}$$

$$2 \cdot \left(\frac{-2x - 0}{x^2 (x+0)^2} \right) = \frac{-4}{x^3}$$

<u>Check work</u>: use power rule on $f(x) = 2 \cdot x^{-2}$

or quotient rule on $f(x) = \frac{2}{x^2}$.

$\underline{\text{Ex}}$: Evaluate $\lim\limits_{x \to 0} \dfrac{\sin^3(12x)}{\pi \cdot x^3}$.

$\underline{\text{REEF}}$: 8

$$= \lim_{x\to 0} \frac{1}{\pi} \cdot \frac{12\sin(12x)}{12\cdot x} \cdot \frac{12\sin(12x)}{12\cdot x} \cdot \frac{12\sin(12x)}{12\cdot x}$$

$$= \frac{12\cdot 12\cdot 12}{\pi}\left(\lim_{x\to 0} \frac{\sin(12x)}{12x}\right)^3$$

Reminder:

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

came up when computed $\frac{d}{dx}\sin x$

$$= \frac{12^3}{\pi}\cdot 1^3 = \frac{12^3}{\pi} \approx 550 \text{ ish}$$

$\underline{\text{Check}}$: $\dfrac{\sin^3(12\cdot 0.01)}{\pi(0.01)^3} \approx 546$ ish ....

NOT A SOLN! Only a check of your careful work.

Ex: When does this have a hor. tangent line?

$$f(x) = 2\cos x + \cos^2 x$$

on $[0, 2\pi]$

Note: Hor. tangent line $\Rightarrow f'(x) = 0$.

Q: When is $f'(x) = 0$ for x in $[0, 2\pi]$?

Let's look at $f'(x) = -2\sin x + 2 \cdot \cos x \cdot -\sin x$

$= -2\sin x - 2\cos x \sin x$

$= -2\sin x(1 + \cos x)$.

for $f'(x)$ to be 0, I need *either* $\sin x = 0$ or

$1 + \cos x = 0$.

To finish solving problem, find where

$$\left.\begin{aligned} &\sin x = 0 \text{ on } [0, 2\pi] \\ \text{and } &\cos x = -1 \text{ on } [0, 2\pi]. \end{aligned}\right\} \text{Exercise.}$$