MA 113 3/8/17

$\boxed{1}$ Log in to REEF

$\boxed{2}$ No homework or quiz the rest of this week.

$\boxed{3}$ Today: §3.8, Exponential Growth

$\boxed{4}$ If $5 = e^{k \cdot 5}$, what is k?

Compare your soln w/ neighbors.

§3.8: If $\underbrace{\frac{dy}{dt} = ky}$ for some k a constant.

inst. rate of change is proportional to function value...

In calc II you learn:

<u>Thm</u>: The only solns to $\frac{dy}{dt} = ky$ are

$$y = y_0 \cdot e^{kt} \text{ for some constants } y_0, k.$$

If $k>0$, we say y models <u>growth</u>.

If $k<0$, we say y models <u>decay</u>.

<u>Ex</u>: Population Growth. If $P(t)$ models population at time t, say $k = \frac{1}{P} \cdot \frac{dP}{dt}$ is the <u>relative</u> <u>growth</u> <u>rate</u>.

(Assuming $\frac{dP}{dt} = kP$).

Q: What is relative growth rate if $P(0) = 2000$ and $P(5) = 10{,}000$?

$$P(t) = 2000\, e^{kt} \Rightarrow P(5) = 10000 = 2000 \cdot e^{k \cdot 5}$$

$$\Rightarrow 5 = e^{k \cdot 5} \Rightarrow k = \frac{\ln(5)}{5}.$$

Ex: Radioactive decay:

Experiments show that mass $m(t)$ of a radioactive substance at time t satisfies

$$\frac{dm}{dt} = K \cdot m(t)$$ for same constant K.

From this and our ~~Cal~~ Calc II thm, we know

That $m(t) = m(0) \cdot e^{kt}$ where $m(0)$ is initial mass.

Half-life of substance is the time t where

$$m(t) = \frac{1}{2} m(0).$$

This is equivalent to knowing k. Why?

If t_h is time for half-life, then

$$m(0) \cdot e^{kt_h} = m(t_h) = \frac{1}{2} m(0)$$

$$\Rightarrow e^{k \cdot t_h} = \frac{1}{2}.$$ Solve for k, done.

Similarly, if you know k, half-life can be found.

Ex: Half-life of radium 226 is 1590 yrs. If $m(0) = 1\,kg$, how much is left after 100 yrs?

First Find k: $m(1590) = \frac{1}{2} = 1 \cdot e^{k \cdot 1590}$

$$\Rightarrow e^{1590} = \frac{1}{2} \Rightarrow k = \frac{\ln(\frac{1}{2})}{1590} = \frac{\ln(2^{-1})}{1590} = \frac{-\ln(2)}{1590} < 0.$$

So, $m(t) = e^{-\ln(2)/1590 \cdot t}$

$$\Rightarrow m(100) = e^{\frac{-\ln(2)\cdot 100}{1590}} \text{ kg.}$$

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Ex: Newton's Law of Cooling

Let $T(t)$ = temp of an object at time t.

Let T_s = temp of surroundings, constant.

Law of Cooling: $$\frac{dT}{dt} = k(T - T_s).$$

We need a substitution: Set $y = T(t) - T_S$.

$\underbrace{\frac{dy}{dt} = K \cdot y}$ since $\frac{dy}{dt} = \frac{d}{dt}(T - T_S) = \frac{dT}{dt}$

$\Rightarrow y = y(0) \cdot e^{kt}$

constant so derivative is 0.

Thus, $T = y + T_S \Leftrightarrow T - T_S = y$

by then for y $T(t) = \underbrace{(T(0) - T_S)}_{y(0) = T(0) - T_S} e^{kt} + T_S$. $\leftarrow$ function arises from Newton's law involving derivatives.

<u>Ex</u>: If temp outside is 20°C & coffee is 98°C, w/ temp 30min later 70°C, find $T(t)$.

$T_S = 20$, $T(0) = 98$.

$$T(t) = (98-20)\cdot e^{kt} + 20 \quad \text{by our formula.}$$

$$= 78e^{kt} + 20.$$

Know $T(30) = 70 = 78\cdot e^{k\cdot 30} + 20$

$$\Rightarrow \quad 50 = 78e^{k\cdot 30} \Rightarrow k = \frac{\ln(50/78)}{30}.$$

So, $T(t) = 78e^{t\cdot \ln(50/78)/30} + 20.$ °C

Read in §3.8 the example on Compound Interest.

MA 113 3/10/17

$\boxed{1}$ Log in to REEF

$\boxed{2}$ Today: § 4.1, Min/Max Values.

$\boxed{3}$ Exam 2 average was a 63. We have added a 7-pt curve to this, bringing the average to 70.

$\boxed{4}$ Midterm grades have been assigned based on you current total % in Canvas.

§4.1: In Ch4, we will start looking ~~at~~ at extreme behaviors of fns.

Ex: For $v(x) = 3x^4 - 16x^3 + 18x^2$, what is the minimal value attained by v?

w/out calc, look at graph: desmos

w/ Calculus, we see that locally max + min values seem to happen where derivative is 0. (b/c of horizontal tangent line)

Check: $v'(x) = 12x^3 - 48x^2 + 36x$

$= 12x(x-3)(x-1)$

This is $=0$ when $x = 0, 1, 3$ as we observed.

Def$^{\underline{n}}$: Say c is in domain of f. Then $f(c)$ is the

- <u>absolute</u> <u>max</u> of f if $f(c) \geq f(x)$ for all x in domain of f.

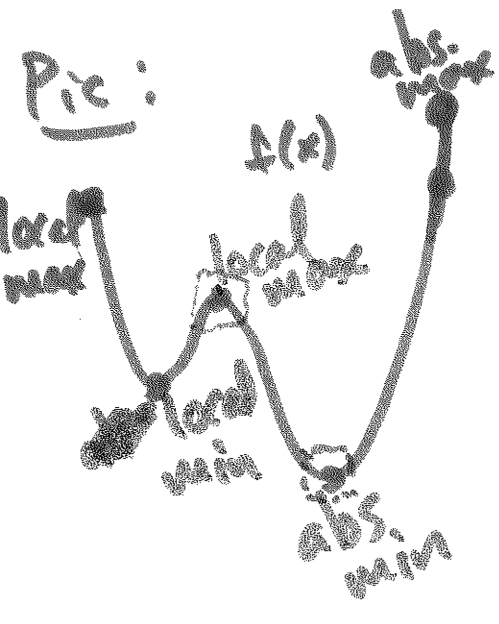
- <u>absolute min</u> of f if $f(c) \leq f(x)$ for all x in domain of f.

We say $f(c)$ is a

- <u>local max</u> if $f(c) \geq f(x)$ when x is near c.

- <u>local min</u> of f if $f(c) \leq f(x)$ when x is near c.

<u>Pic</u>:



<u>Note</u>: abs max/min are local max/min values.

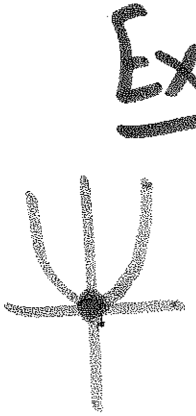
<u>Def$^{\underline{n}}$</u>: A <u>critical number</u> of f is a number c in domain of f such that either

$f'(c) = 0$ <u>or</u> $f'(c)$ does not exist.

<u>Ex</u>: $f(x) = x^2$,

then 0 is a critical #

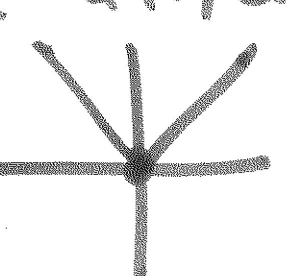
since $f'(0) = 2 \cdot 0 = 0$.



$f(x) = |x|$

then 0 is a critical #

since $f'(0)$ does not exist.



<u>Theorem (Fermat's Thm)</u>: If f has a local max or a local min at c, then c is a critical # of f.

<u>Key idea</u>: To find local max/mins, find critical #'s and check if these are max/mins.

Extreme Value Theorem: If f is cts on $[a,b]$, then f attains both an abs max and an abs min on $[a,b]$.

Note: This seems obvious, but took several hundred years for people to figure out why it is true. Actually hard to prove.

Today's Goal: Develop method to find abs ~~of~~ max/min values.

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Method to find abs min/max on closed intervals:

f defined on $[a,b]$.

1: Use f' to find critical #s of f on $[a,b]$.

2: Find value of f at endpts a, b.

3: Find largest + smallest values of f at the critical #s + end pts. These are your abs. max + min.

Ex: $v(x) = 3x^4 - 16x^3 + 18x^2$ on $[-1,5]$.

1: Find crit. values: $v'(x) = 12x(x-3)(x-1) = 0$

$\Rightarrow x = 0, 1, 3$ are crit. values in $[-1,5]$.

2: endpts are $-1, 5$.

3: $v(-1) = 37$

$v(0) = 0$

$v(1) = 5$

$v(3) = -27 \leftarrow$ min

$v(5) = 325. \leftarrow$ max

So, abs. min is -27
abs. max is 325.

Ex: $f(t) = 3t + \sin(4t) + 100$. Find abs. min/max on $[0,4]$.

1: $f'(t) = 3 + 4\cos(4t)$

So, $f'(t) = 0 \Rightarrow 3 + 4\cos(4t) = 0$

$\Rightarrow \cos(4t) = \frac{-3}{4}$.

we need all solns to $\cos(4t) = -\frac{3}{4}$ on $[0,4]$.

This is hard. At best, we get approximate solns.

Using computer: on $[0,4]$, we need to check

0.6047, 0.9661, 2.1755, 2.5369,

3.7463, and endpts.

<u>get:</u> $f(0) = 100$ is min, $f(3.7463) \approx$ some value max.