Matrix Algebra and its Applications[1]
MA 322, Section 001, Fall 2007

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1 Class Location and Time:

CB 347, MWF, 8-8:50AM

2 Office Location and Hours:

831 Patterson Office Tower. Office hours are MWF, 9-10AM, or by appointment.

3 Text:

The primary text for this course will be *Linear Algebra and Its Applications, 3rd edition*, by David C. Lay. It is critical that you have a copy of the textbook because homework will include reading material not discussed in class. There will also be material presented in class that is not contained in the text.

4 Course Description:

The big question we need to answer is "what is Linear Algebra and why should we care about it?" It will take the entire semester to answer this question appropriately, but let me give a short answer now.

The majority of the time in our prior algebra courses is spent learning about polynomials. We begin these courses with numbers, which we add, subtract, multiply, and divide. If we are talking about integers alone then we can also factor them into primes. At some point, we stop caring about the exact value of the numbers we are dealing with and only care about the operations we can do with numbers. In these cases, instead of writing a number, we write a variable, say x or y, a placeholder where any number can go. When we add, subtract, and multiply with these variables, we get polynomials. We then learn how to add, subtract, multiply, and (sometimes) divide polynomials, how to find their roots, factor them into irreducible polynomials, etc., extending what we were able to do with numbers in the first place. Thus, your prior algebra experience can be considered "abstract arithmetic."

We also encounter geometry: x squared is the area of a square with side length x, while x cubed is the volume of a cube with side length x, etc. When we write the equation $(x+a)(x+a) = (x^2 + 2ax + a^2)$, we are finding two different but equal formulas for the area

[1] I reserve the right to change or amend this syllabus at any time for any reason.

of the square with side length $(x + a)$ (draw the picture!). The algebra we have studied up to now encompassed abstract arithmetic and the geometry of numbers. Hopefully it is clear to you that this investigation bears beautiful and useful fruit.

What then is Linear Algebra? What does it abstract and what geometry does it describe? The answer is all in the name. Three-dimensional space consists of a bunch of points; think of these as zero-dimensional objects. We also have lines through space that are intuitively one-dimensional. We also have different planes which lie in this space; these are all intuitively two-dimensional. We have already seen in calculus that one way to understand functions is to find the tangent line to a curve, or the tangent plane to a surface (i.e. the derivative of a function). We use these approximations because lines and planes are much easier to understand than the curves and surfaces they are tangent to. We want to study lines and planes in some detail, giving us the subject of Linear Algebra, the study of "line-like" objects (hence the "Line-ar" in Linear Algebra). What makes this subject difficult is that we don't just want to deal with one or two or three dimensions, we want to be able to understand what happens in 16- or 27- or 12,425-dimensional space too! In other words, we want to be able to understand the higher-dimensional analogues of lines and planes, objects called hyperplanes.

So, the answer to the first part of our original question is that Linear Algebra is the algebraic study of hyperplanes, the higher-dimensional analogue of lines and planes. In general, we will be studying the transformations of n-dimensional space which preserve hyperplanes (i.e. linear transformations), and we will be studying the ways that these hyperplanes can intersect each other (i.e. solving systems of linear equations). Specifically, we will be investigating six core topics:

- Systems of Linear Equations
- Linear Transformations
- Determinants
- Vector Spaces, Subspaces and Dimension
- Orthogonality
- Eigenvalues and Eigenvectors

Every first Linear Algebra course will introduce students to these topics. Further, these six topics are linked by the idea of a matrix. Matrices are fundamental objects in Linear Algebra and we will study them extensively. That is why this class is officially called Matrix Algebra.

As for why we should care about any of this? First, Linear Algebra has diverse applications. It is used by people in computer science, forestry and agriculture, physics, biology, chemistry, the social sciences, pure and applied mathematics, engineering, and more. The reason is because linear relationships are plentiful in the natural world, and when they are not present, often you can get a good approximation to what you are looking at by changing it to a linear problem (think about tangent lines and planes in calculus).

Second, Linear Algebra is a beautiful subject. It is filled with geometry and the key ideas are simple and elegant (though not easy). Just like in any other human activity, there is a tremendous aesthetic in mathematics and this subject is a masterpiece in many ways.

5 Course Goals:

We have two main goals in this course:

- Develop the ability to solve computational problems involving any of the six topics listed above, and
- Develop a conceptual understanding of why the computational procedures involved in those solutions actually work.

In other words, we will learn how to do Linear Algebra and learn why it works. Our mantra for this semester is "know how, and also know why."

6 Course Assessment:

You will demonstrate your progress to me with your homework, pop quizzes, and exams. Your solutions to problems should be written neatly in quality mathematical English; in other words, symbols alone won't suffice. Mathematics is about the communication of ideas, ideas which allow us one way of understanding the world. You should look on these assignments as an opportunity to develop your communication skills as well as your computational skills.

- *Pop Quizzes*
  - There will be pop quizzes given randomly throughout the semester. These will be given at the beginning of class.
  - Make-up quizzes will not be given except for excused abscenses as outlined in SR 5.2.4.2 found at http://www.uky.edu/StudentAffairs/Code/. Make-up quizzes must be arranged prior to the original quiz date and completed within 7 days of the original quiz date.
- *Homework*
  - Each week you will be given a set of questions about the course material. I will collect your answers at the beginning of class on the due date and grade selected problems. The problems I grade will be the same for everyone and will be determined in advance. You will get full credit for each problem where you have a clear, coherent write-up with a correct answer. If you are not able to finish a problem, please write up what you attempted so you can get partial credit for it.
  - Each homework assignment should have a completed cover sheet which will be distributed with the weekly list of questions. Your answers on the cover sheet will not be graded, but ***you must provide your completed cover sheet for each homework set. Homework turned in without a completed cover sheet will receive a "0" score.***
  - You are encouraged to work on homework problems with your classmates, as mathematics is a social endeavor. For each homework problem, list the people you worked with. ***You must write up your own answers to all the questions;***

do not let cooperation degenerate into one person solving the problem and other people copying their answers. The act of copying a written answer from another student and submitting it as your own will be considered cheating and will be dealt with according to the procedures referenced in Section 8.

  - I will drop the two lowest homework scores.
  - No late homework will be accepted.

- *Exams*

  - There will be two exams and one cumulative final. They will each be taken in class and will consist of questions similar in style and content to the homework questions. In cases of excused abscenses as outlined in SR 5.2.4.2 found at http://www.uky.edu/StudentAffairs/Code/, a make-up exam may be arranged. This must be arranged prior to the original exam date and completed within 7 days of the original exam date. Alternate exams may have different problems than those given originally.

7 Course Grades:

Your course grade will be determined by your exams, quizzes, and homework. The grading scale will be no stricter than the usual A>89.9, B>79.9, C>69.9, D>59.9, E otherwise, weighted as follows:

- Quizzes: 5%
- Homework: 25%
- Exams 1 and 2: 20% each
- Cumulative Final Exam: 30%

8 Academic Integrity and Classroom Demeanor:

All students are expected to follow the academic integrity standards as explained in the University Senate Rules, particularly Chapter 6, found at:

http://www.uky.edu/USC/New/SenateRulesMain.htm

Turn off all cell phones, pagers, etc. prior to entering the classroom. ***You are not to use your cell phones, pagers, or other electronic devices during class.*** An attitude of respect for and civility towards other students in the class and the instructor is expected at all times.

9 Attendance:

Class attendance is expected of all students.

10 Classroom and Learning Accommodations:

Any student with a disability who is taking this course and needs classroom or exam accommodations should contact the Disability Resource Center, 257-2754, room 2 Alumni Gym, jkarnes@uky.edu.

11 Tentative Plan:

- 1.1 Systems of Linear Equations
- 1.2 Row Reduction and Echelon Forms
- 1.3 Vector Equations
- 1.4 The Matrix Equation $Ax = b$
- 1.5 Solution Sets of Linear Systems
- 1.7 Linear Independence
- 1.8 Introduction to Linear Transformations
- 1.9 The Matrix of a Linear Transformation
- 2.1 Matrix Operations
- 2.2 The Inverse of a Matrix
- 2.3 Characterizations of Invertible Matrices
- 2.5 Matrix Factorizations
- 2.7 Applications to Computer Graphics
- **Exam 1: Oct. 1**
- 3.1 Determinants
- 3.2 Properties of Determinants
- 4.1 Vector Spaces and Subspaces
- 4.2 Null Spaces, Column Spaces, and Linear Transformations
- 4.3 Linearly Independent Sets; Bases

- 4.4 Coordinate Systems
- 4.5 The Dimension of a Vector Space
- 4.6 Rank
- 4.7 Change of Basis
- 4.9 Applications to Markov Chains
- **Exam 2: Oct. 31**
- 6.1 Inner Product, Length, and Orthogonality
- 6.2 Orthogonal Sets
- 6.3 Orthogonal Projections
- 6.4 The Gram-Schmidt Process
- 6.5 Least Squares Problems
- 5.1 Eigenvalues and Eigenvectors
- 5.2 The Characteristic Equation
- 5.3 Diagonalization
- 7.1 Diagonalization of Symmetric Matrices
- 7.2 Quadratic Forms
- 7.3 Constrained Optimization
- 7.4 The Singular Value Decomposition
- Additional Topics as Time Permits
- **Cumulative Final Exam: Monday, Dec. 10, 8-10AM**