

**MA 330 HOMEWORK**  
**DUE MONDAY, MARCH 3**

Your answers to the following questions should be written in quality English, i.e. use complete sentences, not unexplained pictures and computations.

- (1) Given a complex number  $z = a + bi$ , define the *length* of  $z$  to be

$$|z| = \sqrt{a^2 + b^2}.$$

Provide a geometric explanation for why we define the length of  $z$  using this formula.

- (2) Define the *conjugate* of  $z$  to be  $\bar{z} = a - bi$ . Provide a geometric interpretation of how  $z$  and  $\bar{z}$  are related.
- (3) Show that

$$|z| = \sqrt{z\bar{z}}.$$

- (4) Prove that for complex numbers  $z_1$  and  $z_2$ ,

$$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2.$$

- (5) Let

$$f(x) = \sum_{j=0}^n r_j x^j$$

be a polynomial of degree  $n$  with real coefficients  $r_j$ .

Show that if a complex number  $z$  satisfies

$$f(z) = 0,$$

then

$$f(\bar{z}) = 0.$$

In other words, for polynomials with real coefficients, non-real roots occur in conjugate pairs.