# Creating a Toolbox for Student-Centered Assessment

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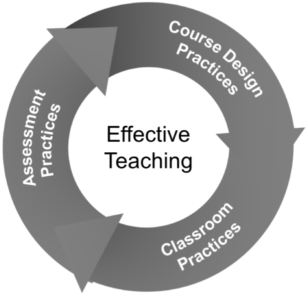
27-28 July 2017

This session is based on the forthcoming *MAA Instructional Practices Guide*.




I am a member of the IP Guide Steering Committee and served as one of two lead writers for the chapter on Assessment.

# Introductions

Please share your:

- Name
- Institution
- Level of Experience (e.g. new PhD, 1 year past PhD, tenured faculty, MAA Staff, etc)

# Our Internal Definitions

Without any discussion, spend three minutes writing a paragraph or two that begins with:

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Whole-group discussion: are your responses all consistent and aligned?

# The Problem With Definitions

What is the black king in chess? This is a strange question, and the most satisfactory way to deal with it seems to be to sidestep it slightly. What more can one do than point to a chessboard and explain the rules of the game, perhaps paying particular attention to the black king as one does so? What matters about the black king is not its existence, or its intrinsic nature, but the role that it plays in the game... The abstract method in mathematics, as it is sometimes called, is what results when one takes a similar attitude to mathematical objects. This attitude can be encapsulated in the following slogan: a mathematical object *is* what it *does*.

— Timothy Gowers, Mathematics: A Very Short Introduction

# Assessment Is What It Does

**Definition:** Effective assessment occurs when we

1. clearly state high-quality goals for student learning,
2. give students frequent informal feedback about their progress toward these goals, and
3. evaluate student growth and proficiency based on these goals.

The purpose of this session is to elaborate on these three characteristics of effective assessment.

# There Are Many Existing Resources

- MAA Assessment Practices in Undergraduate Mathematics (1999)
- MAA Supporting Assessment in Undergraduate Mathematics (2006)
- American Statistical Association Guidelines for Assessment and Instruction in Statistics Education (GAISE, 2016)
- Society for Industrial and Applied Mathematics Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME, 2016)
- MAA CUPM Curriculum Guide (2004 and 2015)
- CBMS Mathematical Education of Teachers II (2012)
- NCTM Principles to Actions (2014)

# Remember Equity Considerations

Gutiérrez' Four Dimensions of Equity details four key aspects of the educational process that require attention.

1. **Access:** Ability to gain intellectual and physical access to mathematical ideas and mathematical teaching and learning spaces
2. **Achievement:** Success in mathematics as traditionally measured
3. **Identity:** Who students are and who they become through their participation in mathematics
4. **Power:** Attending to the distribution of power between instructor and student, between students, and between students and mathematics

Introductions and Definitions

## Student Learning Outcomes

Formative Assessment

Summative Assessment

Formative Assessment Focused on Emotional and Behavioral Domains

Working Groups

# Small-Group Activity

We will refer to the learning goals for students in a course as *Student Learning Outcomes (SLOs)*. There are many other names for these!

With your small group, create a comprehensive set of 8-10 SLOs for students in a College Algebra course that prepares students for Calculus (no trig).

Time: 10 minutes
Reminder: Introduce yourselves to each other again.

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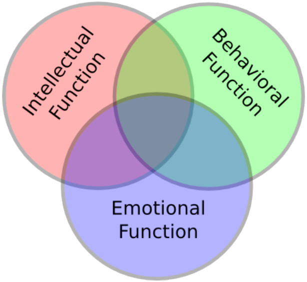
Time: 10 minutes
Reminder: Introduce yourselves to each other again.

Whole-group discussion question: What does "comprehensive" mean?

# Three Psychological Domains

Modern psychology provides a basic framework of the human psyche with three domains.




Many math courses focus primarily on "Intellectual" aspects of student learning. Effective teaching requires course design, classroom practices, and assessment practices that address all three domains.

This three-domain framework is reflected in contemporary descriptions of mathematical proficiency.

# Mathematical Proficiency at the K-12 level

The 2001 NRC report *Adding It Up* emphasized a five "strand" model of proficiency, focused at the K-8 level:

- **Conceptual understanding:** comprehension of mathematical concepts, operations, and relations
- **Procedural fluency:** skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- **Strategic competence:** ability to formulate, represent, and solve mathematical problems
- **Adaptive reasoning:** capacity for logical thought, reflection, explanation, and justification
- **Productive disposition:** habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy

I believe this framework also works well for courses in high school and the first two years of postsecondary mathematics education.

# CCSSM and MAA Curriculum Guide





Many other prominent standards/reports/guides make recommendations that reflect a broad vision of mathematical proficiency, e.g.:

- The Common Core State Standards for Mathematics includes both Standards for Mathematical Practice and Grade-Level Content Standards.
- The 2015 MAA CUPM Curriculum Guide to Majors in the Mathematical Sciences provides four "Cognitive" goals for major programs and nine "Content" goals for major programs.

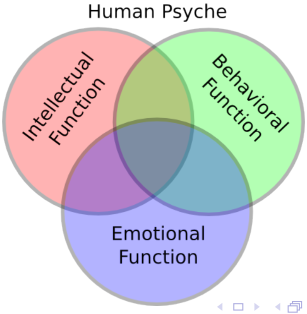
# Selected MAA Goals for Major Programs in Mathematical Sciences

- Students should develop effective thinking and communication skills.
  - e.g. state problems carefully, use and compare analytical, visual, and numerical perspectives in exploring mathematics, assess the correctness of solutions, read mathematics with understanding, communicate mathematical ideas clearly and coherently both verbally and in writing, approach mathematical problems with curiosity and creativity and persist in the face of difficulties
- Students should learn to link applications and theory.
- Students should learn to use technological tools.
- Students should develop mathematical independence and experience open-ended inquiry.
- Students should learn to read, understand, analyze, and produce proofs at increasing depth as they progress through a major.

# Putting Theory Into Practice

**Take-away:** A "comprehensive" set of SLOs needs to address all three of the intellectual, behavioral, and emotional domains of human functioning.

If you want to and are able to write SLOs that are fine-tuned to align with an existing framework for mathematical proficiency (such as those just given), go for it! If not, you can still design thoughtful and effective SLOs by keeping in mind the three-domain framework.



Questions?

Questions?
Possible break time.

# Small-Group Activity

Consider the list of learning outcomes you generated previously, and label each of them with one of the three domains: intellectual, behavioral, emotional.

If you need, add a few SLOs to cover "missing" domains in your list.

Time: 5-10 minutes

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Time: 5-10 minutes

Next we consider a set of 10 SLOs for College Algebra that the IP Guide writers feel are reasonably comprehensive. It is based in part on the MAA CRAFTY Guidelines for College Algebra (CRAFTY = Curriculum Renewal Across the First Two Years).

# College Algebra SLOs, Part I

Students will:

- Use multiple perspectives (symbolic, numeric, graphic, and verbal) to explore elementary functions
- Algebraically solve linear, quadratic, exponential, logarithmic, and power equations
- Sketch polynomial and rational functions using a graphing calculator.
- Identify and algebraically find important characteristics of these graphs such as intercepts, vertical asymptotes, and horizontal asymptotes.
- Recognize and use standard transformations with graphs of elementary functions
- Use and solve systems of equations to model real world situations

# College Algebra SLOs, Part II

Students will have opportunities to engage in the following mathematical practices:

- Being persistent, working through perceived failure
- Collaborating productively with a team
- Developing a personal framework of problem solving techniques (e.g. to make sense of problems, sketch and label diagrams, restate and clarify questions, identify variables and parameters, and use analytical, numerical, and graphical solution methods)
- Creating, interpreting, and revising real-world models and solutions of problems

Pause for whole-group questions/discussion about SLOs.

Possible break time.

# What's Next?





**Definition:** Effective assessment occurs when we

1. clearly state high-quality goals for student learning,
2. give students frequent informal feedback about their progress toward these goals, and
3. evaluate student growth and proficiency based on these goals.

The second bullet is handled by formative assessment utilizing active learning and by assessment targeting emotional and behavioral domains. We will discuss the first of these next.

Introductions and Definitions

Student Learning Outcomes

## Formative Assessment

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Working Groups

# An Informal Definition

Schoenfeld summarized previous literature, leading to a definition of formative assessment as

> *examinations or performance opportunities the primary purpose of which is to provide student and teachers feedback about the student's current state, while there are still opportunities for student improvement.*

Key point: feedback is provided prior to final (summative) evaluation of student performance/achievement.

# Formal Characteristics

Black and Wiliam identified five “key strategies” for instructors implementing formative assessments:

1. clarifying and sharing learning intentions and criteria for success;
2. engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding;
3. providing feedback that moves learners forward;
4. activating students as instructional resources for one another; and
5. activating students as the owners of their own learning.

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Small-group discussion: for each of these five strategies, identify a situation where one of you has seen this strategy "brought to life" in a classroom (possibly with you as a teacher, or as a student, or as an observer).

Time: 10 minutes

Often, in-class formative assessment is implemented using active learning techniques.

# Active Learning

In a given course, an *active learning method* is a classroom teaching technique in which *students complete a task or activity* directly supporting development in

1. one or more student learning outcomes, and
2. one or more psychological domains.

Each course should include a balance of direct instruction and active learning techniques that collectively support development across all of the course SLOs.

# AL Example #1: Think-Pair-Share in small-lecture Number Theory

**Technique:** Ask students to use Euclid's proof of the infinitude of primes to produce as many new prime numbers as possible starting with only the prime 5. Students have three minutes to compute independently, then three minutes spent comparing their results with one or two of their neighbors in class, discussing the reason for why their lists are the same or different. A subset of the students are then asked to share the results of their conversations in order to start a whole-class discussion.

# AL Example #2: IBL-style small group activity

**Technique:** Assign students to small groups. Give each group a theorem (or problem) with a 15-line proof (or solution) where each line is separately cut out and mixed together, where the proof (or solution) has one fixable error. Students must first collaboratively reconstruct the proof, then identify and correct the error. The instructor spends the class time circling the room, listening to student conversations, offering clarification and answering questions, etc.

# Unexpected Formative Assessments Exist

An example of formative assessment that is often not recognized as formative: online homework systems where students have unlimited attempts to answer the problem. This provides feedback to students (correct or incorrect) and can be used to support items #4 and #5 from Black and Wiliam's key strategies:

4. activating students as instructional resources for one another; and
5. activating students as the owners of their own learning.

It is a useful habit to reflect on your regular teaching practices to identify "secret" formative assessment you do. Viewing these practices explicitly as formative assessment techniques can make them more effective.

Pause for whole-group questions/discussion.

In response to questions, the following blog articles I wrote were referenced:
http://blogs.ams.org/matheducation/2016/05/16/believing-in-mathematics/
http://blogs.ams.org/matheducation/2015/05/01/famous-unsolved-math-problems-as-homework/
http://blogs.ams.org/matheducation/2015/09/01/the-secret-question-are-we-actually-good-at-math/
http://blogs.ams.org/matheducation/2017/02/06/aspirations-and-ideals-struggles-and-reality/

End Day 1 material, begin Day 2 material with following slide.

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# Growth and Proficiency

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In courses for STEM majors, such as calculus, linear algebra, and differential equations, it is reasonable for proficiency to be the primary goal. In courses for non-STEM majors, such as general education or quantitative literacy courses, it is equally reasonable for growth to be the focus. Sometimes, growth and proficiency might be given equal weight when assessment methods are chosen.

For any given course, a clear and well-articulated decision of how to balance the assessment of growth and proficiency should be made.

# Small-Group Activity

Discuss with your small group:

- In your courses, do you typically value growth or proficiency more?
- Does this vary from course to course?
- Have you reflected on this distinction before?
- If so, how has it affected your teaching?

Time: 10-12 minutes
REMINDER: Introduce yourself to your new small group!

# Defining Summative Assessment

*Summative assessment* is conducted with the purpose of evaluating student growth and/or proficiency with regard to one or more learning outcomes.

Most of the assessments that we typically see in math courses, such as exams, quizzes, and homework (when graded after only one attempt) fall within this context.

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Two major challenges are:

- to design a meaningful overall evaluation scheme (i.e. course grading system).
- to create and select problems/tasks with a clear sense of what the problem/task:
  - is *intended* to assess (this can be done)
  - *actually* assesses (this is usually difficult to determine)

# Course Grading Systems



There are several established course grading systems for postsecondary math courses:

- Point-Based Systems
- Specifications Grading, Standards-Based Grading, Mastery-Based Grading, Points-Free Grading, Etc
- Portfolio Grading

An in-depth discussion of these would take more time than we have, and there are many excellent resources available online describing these methods, so we will only briefly mention how a points-based grading system might look.

# A Points-Based Example

Using a points-based system is a reasonable approach, as long as the average is created in a thoughtful manner that reflects the SLOs for the course.

Here is an example of a points-based scheme for a course where proficiency comprises 68% of the summative course grade.

- Three Exams: 45% of course grade
- Weekly Quizzes: 15% of course grade
- Two Written Assignments: 8% of course grade
- Online Homework (unlimited attempts allowed on problems): 15% of course grade
- Three Reflective Essays: 12% of course grade
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Caveat: A system like this might be the end result of a 1-3 cycle of implementing and getting comfortable with unfamiliar assessment techniques.

# An Important Aside About Course Design

Research by psychologists and educators has demonstrated that summative assessments are most effective when they are *frequent and low-stakes* rather than infrequent and high-stakes.

**Warning!**: An excellent selection of problems for quizzes and exams can be undermined by ineffective course design.

For example, in the system given on the previous slide, there are three exams at 15% of the course grade each, but also weekly quizzes which provide another 15% worth of the course grade. In effect, this takes a course with four high-stakes exams and distributes one of those exams into more frequent, lower-stakes summative assessments.

Questions? Small-group and whole-group discussion.

# Creating and Selecting Effective Problems/Tasks

A well-known framework for analyzing problems given to students is Bloom's taxonomy. Bloom's work originally outlined multiple levels of skills in the cognitive domain of learning, increasing from simple to complex. These are described by six skill levels:

- knowledge
- comprehension
- application
- analysis
- synthesis
- evaluation

# Revised Bloom's Taxonomy





Bloom's taxonomy has been extended by researchers in educational psychology to more robust frameworks. Anderson et al. introduced a two-dimensional extension of Bloom's taxonomy.

The first dimension consists of a cognitive process dimension similar to Bloom's taxonomy (serving the behavioral domain), while the second consists of a knowledge dimension (serving the intellectual domain).

When evaluating a problem or task using this taxonomy, the cognitive process is represented by the verb used when specifying the task (what the student is doing) and the knowledge process dimension corresponds to the noun (what kind of knowledge the student is working with).

# Knowledge Dimension (Intellectual)



- Factual Knowledge: The basic elements that students must know to be acquainted with a discipline or solve problems in it.
- Conceptual Knowledge: The interrelationships among the basic elements within a larger structure that enable them to function together.
- Procedural Knowledge: How to do something; methods of inquiry, and criteria for using skills, algorithms, techniques, and methods.
- Metacognitive Knowledge: Knowledge of cognition in general as well as awareness and knowledge of ones own cognition.

# Cognitive Process Dimension (Behavioral)

- Remember: Retrieving relevant knowledge from long-term memory.
- Understand: Determining the meaning of instructional messages, including oral, written, and graphic communication.
- Apply: Carrying out or using a procedure in a given situation.
- Analyze: Breaking material into its constituent parts and detecting how the parts relate to one another and to an overall structure or purpose.
- Evaluate: Making judgments based on criteria and standards.
- Create: Putting elements together to form a novel, coherent whole or make an original product.

# Small-Group Activity



As a group, create several problems from Calculus I that would fall in different categories in the following table:

| | Remember | Understand | Apply | Analyze | Evaluate | Create |
|---|---|---|---|---|---|---|
| Factual | | | | | | |
| Conceptual | | | | | | |
| Procedural | | | | | | |
| Metacognitive | | | | | | |

Time: 10-15 minutes

Pause for whole-group questions/discussion.

Possible break time.

# What's Next?

There are often SLOs for which exams, quizzes, problem sets, etc are not effective summative evaluations, or for which summative assessment is inappropriate.

Sometimes these SLOs are simply tough to evaluate through student submitted work, in which case we rely primarily on in-class formative assessment.

With others, alternative assessments such as writing assignments, group projects, and oral presentations are appropriate for in-depth formative assessments of these SLOs.

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We will discuss the following two techniques for assessment in the emotional and behavioral domains, with a focus on their use for formative assessment.

- writing assignments (reflective essays, projects, critical analyses, etc)
- using famous unsolved problems as homework

# Writing Assignment #1: Autobiography

Autobiographical writing serves as excellent formative assessment in the affective domain. All of my undergraduate courses with less than 60 students begin with the following assignment.

- Imagine that you have written a book-length autobiography about your mathematical experiences.
- Write a passage, thought of as a quote from your autobiography, that reveals something significant about you mathematically.
- Be as creative as you like.

I assign a grade based on completion, completely ignoring the quality of the writing, editing, or ideas. If students respond to the prompt in a relevant manner, they get full credit.

# Writing Assignment #2: Reflective Essays

Short reflective essays about challenges in the course promote development in both the affective and enactive domains.

- Write several paragraphs on the following topic: what was the most challenging aspect for you regarding [TOPIC]? What made this difficult for you? Did you overcome the challenge, or are you still struggling with it?

This can be graded based on completion, i.e. if students write several paragraphs that address these questions then they receive full credit for the problem.

# Writing Assignment #3: Critical Reviews of Reading

To promote critical analysis skills and develop students' reading abilities, have students write a review of selected readings from your course text.

- Write a three page critical review of [ASSIGNED READING].
- Imagine that you are writing your review for a journal for undergraduates in mathematics and the sciences.
- You must address the mathematical depth and mathematical style of [ASSIGNED READING] in addition to other topics.

In my courses, short essays are graded using a rubric with five criteria: Writing Style, Arrangement and Development, Editing and Conventions, Mathematical Depth, Mathematical Style. (We can discuss this more in a few minutes, if you want.)

Small-group activity: Discuss where you might be able to use writing assignments in courses you have taught or anticipate teaching.

Time: 5-10 minutes

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Pause for questions/discussion about writing assignments.

In addition to IP Guide section on writing assignments, see the special volume in PRIMUS on using writing in mathematics courses from 2014.

# Unsolved Problems

One interesting technique for getting students to break out of their typical habits of doing mathematics is having them work on an unsolved problem in mathematics, either in groups in class or as a homework assignment.

# An Unsolved Problem I Use As Homework

For a positive integer $n$, let $\sigma(n)$ denote the sum of the positive integers that divide $n$. For example, $\sigma(4) = 1 + 2 + 4 = 7$, and $\sigma(6) = 1 + 2 + 3 + 6 = 12$. Let $H_n$ denote the $n$-th *harmonic number*, i.e.

$$H_n = \sum_{i=1}^{n} \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} .$$

Let ln denote the natural log function. Does the following inequality hold for all $n \geq 1$?

$$\sigma(n) \leq H_n + \ln(H_n) e^{H_n}$$

NOTE: By work of Lagarias, this problem is equivalent to the *Riemann Hypothesis*.

# What I Tell Students To Do With This Problem



This is an open, i.e. unsolved, problem given by Jeffrey Lagarias in 2002 in an article in the *American Mathematical Monthly*. Make as much progress as you can on it. Your goal is to do something more than check examples; the examples should lead you to make some interesting observations about the problem, to understand it a bit better. Why do you think it might be true? Why might it be false? Are there any properties of $e$, $H_n$, ln, or $\sigma(n)$ that support your comments? Are there special values of $n$ for which this is obviously true? (Seriously, write down everything you're thinking and every idea you try, even if it doesn't go anywhere.)

# How To Grade This?

I typically tell students that I will grade this based on a combination of mathematical correctness (i.e. they should not have incorrect mathematics in their write-up) and the depth of their investigation. I tell students

> *You should do something more than merely check a few examples, you should look for special structures, or write some computer code to investigate a LOT of examples and make some thoughtful observations, or describe some interesting approach to the problem and try to carry it out, etc.*

In general, I am generous with the grading on this assignment for students who take it seriously and honestly do their best with it, i.e. I focus on growth rather than proficiency.

# How Do Students Respond?

A few weeks after giving this assignment, I have students write a reflective essay about what they found most and least challenging in the homework so far, and what their most and least favorite homework problems have been.

What follows are direct quotes from these reflective essays written by undergraduates about their experience working on unsolved problems in several junior-senior-level courses I have taught.

- I did have a favorite assignment, and that was the unsolved problem. This confuses me a bit because the problem was the essence of theoretical which, as I said before, can give me some trouble. But maybe since there was not really a correct answer I felt like I could attack it from whatever angle I wanted to without consequence. Now that I think about it, this should probably be how I approach all the theoretical problems. Instead of trying to find the correct answer right off the bat, I should write down what I know to be true about the problem and get a better understanding of it first. Anyway, I just really enjoyed how this problem challenged me to come up with my own way of approaching the problem and how I did not feel any pressure to find the correct answer.

- I was quite frustrated as I did not even have a clue what to do. I found myself merely writing "I don't know what to do" in a bunch of different ways after partially starting random thoughts. I felt somewhat insignificant while rifling through my disjointed thoughts.

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- Working on the unsolved problem involving unit fractions was automatically intimidating to me. Just knowing that this problem has not yet been solved convinced me that I would make little to no progress on the problem. Constantly my mind was reminding me that everything I had found, someone else had found before. There were moments when I was hopeful that my algebra and determination would lead me to some insight. Overall, I was unsatisfied with my work because I left feeling defeated.

- I liked that in proposing different ideas, you [the instructor] could provide feedback as to whether I was on the right track or not, but being that the problem is unsolvable, our ideas combined only made us one step closer to finding a correct solution. To me, this felt like real collaboration, and I enjoyed seeing how that may play out in real life one day.

- It was fun to explore the properties of some unproven theorem, even if I had no real idea as to what I was doing. It was interesting to gain some sort of intuitive understanding of the theorem without knowing for certain the real properties of it. I feel that, and maybe unjustifiably, that this is what real mathematicians do on a daily basis; attempt to validate a theorem with examples, then develop an intuition about how it works, and then use this intuition to approach a proof. It was quite enjoyable to remove the weight of mathematical rigor and go back to the basic mathematical idea of recognizing patterns.

Pause for questions/discussion.

If you want to read about other unsolved problems that can be used for this, I wrote a blog article on this topic: http://blogs.ams.org/matheducation/2015/05/01/famous-unsolved-math-problems-as-homework/