MA114 Worksheet for Section 6.1

Skills

1. Find the indefinite integrals.

   (a) $\int (2x+3)\,dx$

   (b) $\int (x^3 - \sin(x))\,dx$

   (c) $\int (1 - e^x)\,dx$

   (d) $\int \frac{1}{x}\,dx$

   (e) $\int \sec^2\left(\frac{x}{2}\right)\,dx$

   (f) $\int \frac{dx}{1+9x^2}$

   (g) $\int \frac{dx}{4+x^2}$

   (h) $\int (2^x + x^{-2})\,dx$

   (i) $\int \frac{5}{\sqrt{4-25x}}dx$

   (j) $\int x\sqrt{x}\,dx$

2. Verify each antiderivative by differentiation.

   (a) $\int \sec x\,dx = \ln|\sec x + \tan x| + C$
   [Note that this problem must be done twice. Once for $\sec x + \tan x \geq 0$ and once for $\sec x + \tan x \leq 0$.]

   (b) $\int \arcsin x\,dx = x \arcsin x + \sqrt{1-x^2} + C$

   (c) $\int \arctan x\,dx = x\arctan x - \frac{1}{2}\ln\left(1+x^2\right) + C$

   (d) $\int \ln x\,dx = x\ln x - x + C$

   (e) $\int \frac{x}{2+3x}dx = \frac{1}{9}(2 + 3x - a\ln|2+3x|) + C$

3. Find antiderivatives, $F(x)$, for the following functions, $f(x)$, satisfying the given initial conditions.

   (a) $f(x) = e^{2x}$ where $F(0) = 3$.
   (b) $f(x) = x + 3$ where $F(3) = -2$.
   (c) $f(x) = \cos(2x)$ where $F(\pi/3) = 1$.

Concepts

1. (a) Find the derivatives for the following functions:

      i. $\ln(1+x) - \ln(2+x)$
      ii. $\ln(\cos(x))$
      iii. e^{x^2}
      iv. $(1+x^2)^{15}$

(b) Using your answers in part (a), find antiderivatives for the following functions:

i. $\tan(x)$

ii. $xe^{x^2/2}$

iii. $x(1+x^2)^{14}$

iv. $\dfrac{1}{(1+x)(2+x)}$

v. $x(1+x^2)^{12}$

vi. $\dfrac{1}{(1+x)(3+x)}$

2. Confirm that $\frac{d}{dx}(\sin^2(x)) = 2\sin(x)\cos(x) = \sin(2x)$. Therefore, one antiderivative of $\sin(2x)$ is $\sin^2(x)$. Confirm that $\frac{d}{dx}((-1/2)\cos(2x)) = \sin(2x)$. Explain why the function $\sin^2(x)$ can have two different antiderivatives.

3. Verify each antiderivative by differentiation.

(a) $\displaystyle\int \frac{\sin x \cos x}{\sqrt{1+\sin x}}dx = \frac{2}{3}(\sin x + 1)^{3/2} - 2\sqrt{1+\sin x} + C.$

(b) $\displaystyle\int \sin^2 x \cos x \ln(\sin x)\, dx = \frac{1}{9}\sin^3 x[3\ln(\sin x) - 1] + C.$

(c) $\displaystyle\int e^x \ln(1+e^x)\, dx = (1+e^x)\ln(1+e^x) - (1+e^x) + C.$

(d) $\displaystyle\int \frac{\sqrt{1-x^2}}{x}dx = \sqrt{1-x^2} - \ln\left|\frac{1+\sqrt{1-x^2}}{x}\right| + C.$