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Calculus I
SKILLS TEST I
Practice Test
(Key)

Directions: You have two hours to complete this test. READ ALL DIRECTIONS CAREFULLY! Partial credit will be given, but ONLY IF YOU SHOW YOUR WORK!

Unless otherwise specified, you may assume that each problem is worth 5 points. Please indicate ALL answers clearly by drawing a box around the final answer.

1. (2 points each) **TRUE OR FALSE:** For each of the following, indicate whether the statement is true or false.

   (a) $\sqrt[3]{x^3} = |x|$ **False: when x is negative, so is its cube root.**

   (b) $\ln(A^n + B) = n \ln A + \ln B$ **False: the log of the sum is not the sum of the logs**

   (c) $\cos(2x) = 1 - 2\sin^2(x)$ **True: this follows from the identity $\sin^2(x) + \cos^2(x) = 1$ and the addition formula $cos(x+y) = \sin(x)\sin(y) - \cos(x)\cos(y)$.**

   (d) Any rational function has a domain of all real numbers. **False: the domain does not include numbers x where the function takes on the value 0.**

   (e) If $f(x)$ is a polynomial such that $f(a) = 0$, then $x - a$ is a factor of $f(x)$. **True: This is called the remainder theorem.**

2. What is the **Domain** of $f(x) = \tan(x)$.

   Since the tangent is the sine over the cosine, its domain is the set of all numbers x where $\cos(x) \neq 0$.

3. What is the **Range** of the function $f(x) = x^2 - 4$.

   By inspection, the minimum value is $f(0) = -4$, and the range is the closed ray $[0, \infty)$.

4. Which of the following functions has a **Range** which is equal to its **Domain**?

   (a) $f(x) = \dfrac{1}{x^2}$ **the domain is all $x \neq 0$, the range is all $x > 0$**

   (b) $f(x) = \sin x$ **domain all real x, range $[-1, 1]$.**

   (c) $f(x) = \dfrac{1}{x}$ **domain all $x \neq 0$, range all $x \neq 0$**

   (d) $f(x) = 5$ **dom all x, range $\{5\}$.**

   (e) None of the above

   The function $\dfrac{1}{x}$ has domain and range the same.

5. Do the lines given by the equations $7y + 3x - 17 = 0$ and $3y - 2x + 15 = 0$ ever intersect? If so, where? If not, why not?

   The first line has slope $\dfrac{-3}{7}$ and the second has slope $\dfrac{2}{3}$, so they will intersect. The point of intersection is $\{y = \dfrac{-11}{23}, x = \dfrac{156}{23}\}$, obtained by solving simultaneously.

6. Find the equation of the line whose x-intercept is 10 and whose y-intercept is 5.

   An equation for this line is $\dfrac{x}{10} + \dfrac{y}{5} = 1$, as is easily checked by substituting in the point $(10, 0)$ and $(0, 5)$.

7. Which of the following functions is **even**?

   (a) $f(x) = \sin x$ **the sin function is odd, since $\sin(-x) = -\sin(x)$.**

   (b) $f(x) = \dfrac{\sin^2(x)}{\cos(x)}$ **This function is even: It is the product of two even functions.**

(c) $f(x) = \dfrac{1}{x}$

(d) $f(x) = \ln(x^2 - 2)$

(e) None of the above

The first even function in the list is item b.

8. (2 pts each) Given that $\ln A = f$, $\ln B = g$, and $\ln C = h$, evaluate the following expressions in terms of f, g, and h.

   (a) $\ln ABC = \ln A + \ln B + \ln C = f + g + h$.

   (b) $\ln((AB)^2) = 2\ln(AB) = 2(\ln(A) + \ln(B)) = 2(f + g)$.

   (c) $\ln(\frac{1}{\sqrt{C}}) = -\ln(\sqrt{C}) = -\frac{1}{2}h$.

9. If $f(g(x)) = (x^2 + 4x + 4) + 2x + 4$ and $g(x) = x + 2$, what is $f(x)$?

   (a) $f(x) = (x^2 + 4x + 4) + 2x + 2$

   (b) $f(x) = x^2 + 4$

   (c) $f(x) = x^2 + 2x$

   (d) $f(x) = (x + 2) + 2$

   (e) None of the above

   By inspection, the most likely candidate is item c $f(x) = x^2 + 2x$. We check by composing

   $$f(g(x)) = (g(x))^2 + 2g(x) = (x + 2)^2 + 2(x + 2) = (x^2 + 4x + 4) + 2x + 4$$

   which checks out.

10. If $h(x) = \sqrt{x}$, $k(x) = x^2$, and $l(x) = x + 1$, what composition of the functions h, k and l would give us $|x + 1|$?
    We know that $\sqrt{x^2} = |x|$, so the answer is $h(k(l(x + 1))) = |x + 1|$.

11. Given the graph of **any** function $f(x)$, which of the following choices would produce a graph that is identical in shape to $f(x)$ but that is moved 2 units up and 2 units to the left?

    (a) $f(x) + 2$ **Here the graph of f is raised up 2 units.**

    (b) $f(x) - 2$ **Here it's lowered 2 units.**

    (c) $f(x+2)+2$ **Here the graph is shifted 2 units to the right and up 2 units.**

    (d) $f(x - 2) + 2$ **Here the graph is shifted 2 units to the left and raised up 2 units.**

    (e) None of the above

    item d

12. Find the following limits.

    (a) $\lim_{x\to 2} \dfrac{x^2 - 4}{x - 2} = \lim_{x\to 2} \dfrac{(x - 2)(x + 2)}{x - 2} = \lim_{x\to 2} x + 2 = 4$.

(b) $\lim_{x \to 2} \frac{x^2 - 4}{x^2 + 8} = \frac{2^2 - 4}{2^2 + 8} = \frac{0}{12} = 0$.

(c) $\lim_{x \to 0} 12\pi$

13. Find all points where the graphs of the functions $f(x) = x^2 + 2$ and $g(x) = 2x + 1$ intersect.

Solve $f(x) = g(x)$ or $x^2 - 2x + 1 = 0$. This factors into $(x - 1)^2 = 0$ and so the solution is $x = 1$. The graphs intersect in the point $(1, 3)$.

14. (2 points each) From their general shapes match each of the following functions with one of the graphs. Write down the color of the correct graph.

(a) **red** $f(x) = x^3$

(b) **green** $f(x) = \sin(x)$

(c) **yellow** $f(x) = \frac{1}{x}$

(d) **blue** $f(x) = |x|$