

UK

Departmental Colloquium

Some Variational Problems from Image Processing and a Remark about Doubling Measures and Rectifiable Curves

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Friday, March 26, 2010
4:00 PM CB 102

Refreshments at 3:30 p.m. in OT 745

Abstract:

I will talk about two distinct topics.

The first topic represents joint work with Triet Le of Yale and Luminita Vese of UCLA.

Let $f(x) \in L^2(\mathbb{R}^2)$ be real. The Rudin-Osher-Fatema scheme is to minimize

$$\|u\|_{BV} + \lambda \|f - u\|_2^2. \quad (1)$$

In this minimization problem one thinks of f as an image, and u as a “cartoon”, $f - u$ as “noise” or “texture”, and $\lambda > 0$ is a “tuning parameter”. We consider several variations on (1), including

$$\inf\{\|u\|_{BV} + \lambda \|K * (f - u)\|_p^q\}, \quad (2)$$

where K is a real analytic kernel, like a Gaussian. We prove several elementary results about (2), including the theorem that if f and K are radial functions, then u is a radial step function, but we have many unresolved questions.

For the second topic I will, time permitting, construct an example, done with R. Killip and R. Schul, of a doubling measure in \mathbb{R}^d that assigns positive mass to a rectifiable curve.