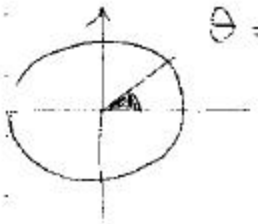
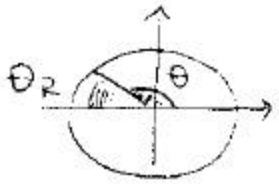
5.4 Values of the trigonometric functions

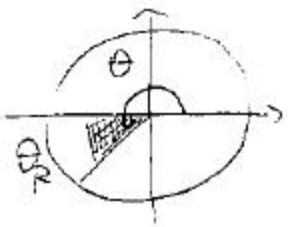
In the previous lecture we somewhat used already the following notion to be able to compute the values of the trigonometric functions for values greater than $\pi/2$.

Let θ be a non-quadrantal angle in standard position. The reference angle for θ is the acute angle that the terminal side of θ makes with the x-axis.

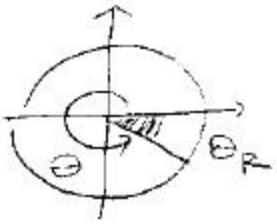




$\theta_R = \pi - \theta$



$\theta_R = \theta - \pi$



$\theta_R = 2\pi - \theta$

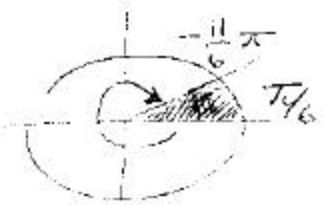
The values of the trig. functions at θ are (up to a sign) equal to the values of the trig. functions at the reference angle θ_R.

If $\theta = \frac{7}{4}\pi$ (i.e. 315°) $\quad \theta_R = 2\pi - \frac{7}{4}\pi = \pi/4$

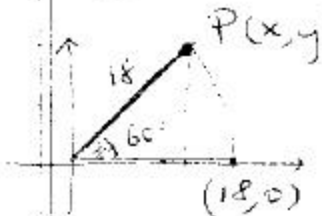
$\cos(\theta) = \cos(\theta_R) = \sqrt{2}/2 \qquad \sin\theta = -\sin\theta_R = -\sqrt{2}/2$

$\tan\theta = -\tan\theta_R = -1$

If $\theta = -\frac{11}{6}\pi$ then $\theta_R = \frac{\pi}{6}$



Robot's arm Points on the terminal sides of angles play an important part in the design of arms for robots. Suppose a robot has a straight arm 18 inches long that can rotate about the origin in a coordinate plane. If the robot's hand is located at (18, 0) and then rotates an angle of 60° what is the new location of the hand?



Observe that

$$\frac{x}{18} = \cos(60°) \qquad \frac{y}{18} = \sin(60°)$$

$$\therefore \quad x = 18\cos(60°) = \underline{9} \qquad y = 18\sin(60°) = 18\frac{\sqrt{3}}{2} = \underline{9\sqrt{3}}$$

Metereological calculation

In the mid-latitudes it is sometimes possible to estimate the distance between consecutive regions of low pressure. If φ is the latitude (in degrees) R is Earth's radius (in kilometers), and v is the horizontal wind velocity in (km/hr), then the distance d (in kilometers) from one low pressure area to the next can be estimated using the formula

$$d = 2\pi\left(\frac{vR}{0.52\cos\varphi}\right)^{1/3}$$

(a) At a latitude of 48°, Earth's radius ≅ 6,369 km
Approximate d if the wind speed is 45 km/hr

$$d \cong 2\pi\left(\frac{45 \cdot 6{,}369}{0.52\cos 48°}\right)^{1/3} = 589 \text{ km}$$

Ans: d increases

(b) if v and R are constant, how does d vary as the latitude increases?