5.6 Additional Trigonometric Graphs

For tan, cot, sec, csc there is no notion of amplitude.

Consider the equation \[ y = a \tan (bx + c) \]

- period \( = \frac{\pi}{|b|} \) since the period of \( \tan \) is \( \pi \)
- phase shift \( = -\frac{c}{b} \)

Observe that successive vertical asymptotes for the graph of one branch may be found by solving the inequality:

\[ -\frac{\pi}{2} < bx + c < \frac{\pi}{2} \]

Similarly, we can discuss the data attached to:

\[ y = a \cot (bx + c) \]

* Sketch \( \frac{1}{3} \tan (2x - \frac{\pi}{4}) \)

observe that the period is \( \frac{\pi}{2} \) and that the phase shift is \( -\frac{\pi}{4} = \frac{\pi}{8} \)
Also observe that 2 successive vertical asymptotes for \( y = \frac{1}{3} \tan(2x - \frac{\pi}{4}) \) are at the end points of the interval

\[-\frac{\pi}{2} < 2x - \frac{\pi}{4} < \frac{\pi}{2}\]

\(\iff\)

\[-\frac{\pi}{4} < x < \frac{\pi}{4} + \frac{\pi}{4}\]

\[-\frac{\pi}{8} < x < \frac{3\pi}{8}\]

period = \(\frac{2\pi}{2} = \pi\)

amplitude of this interval

\(= \frac{3\pi}{8} - (-\frac{\pi}{8}) = \frac{4\pi}{8} = \frac{\pi}{2}\)

Find an equation using the cotangent that has the same graph as \( y = \tan x \)

\(\text{Ans: } y = -\cot(x + \frac{\pi}{2})\)

Sketch the graph of \( y = e^x \sin x \)

Observe that \(-1 \leq \sin x \leq 1\), therefore

\[-e^x \leq e^x \sin x \leq e^x\]
Hence the graph of $e^x \sin x$ lies in between $e^x$ and $-e^x$ at the values of $x$ such that $\sin x = 1$ or $\sin x = -1$. It also touches these 2 curves.

Thus the graph of $y = e^x \sin x$ looks like.

* Sketch the graph of $y = |\sin x|$.

* Sketch the graph of $y = |x| \cos x$. 