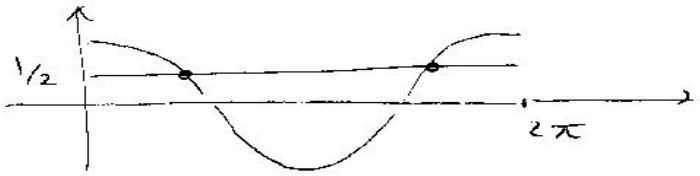
6.2. Trigonometric Equations

* Find the solutions of $\cos\theta = 1/2$ if

(a) $\theta \in [0, 2\pi)$

(b) θ is any real #.

Ans:



In $[0, 2\pi)$ there are 2 solutions; namely at $\theta = \pi/3$ and $\theta = 2\pi - \pi/3 = \frac{5}{3}\pi$

In general the solutions are

$$\theta = \pi/3 + 2\pi k \quad , \quad \theta = 5/3\,\pi + 2\pi k$$

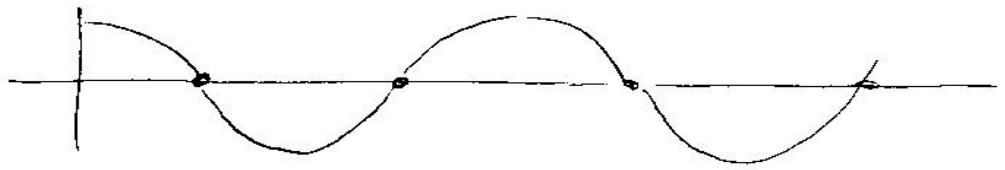
where $k \in \mathbb{Z}$.

* Solve $\cos(2x) = 0$

(a) for any real #;

(b) for $x \in [0, 2\pi)$.

Ans: in general $\cos\theta = 0$ has solutions



at $\theta = \pi/2 \; , \; \frac{3}{2}\pi \; , \; \frac{5}{2}\pi \; , \; \ldots$

i.e. when $\theta = \pi/2 + \pi k \quad k \in \mathbb{Z}$

Thus $\cos(2x) = 0$ has solutions of the form: $2x = \frac{\pi}{2} + \pi k \quad k \in \mathbb{Z}$

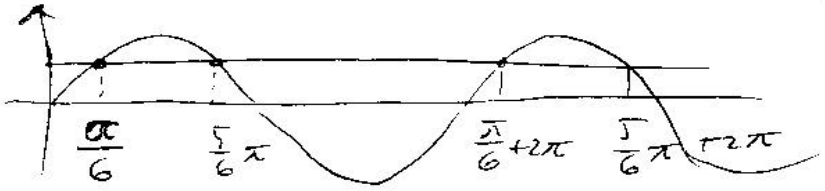
$\Longleftrightarrow \quad x = \frac{\pi}{4} + \frac{\pi}{2} k \quad k \in \mathbb{Z}$

Hence for $x \in [0, 2\pi)$ we have

$x = \frac{\pi}{4}, \frac{3}{4}\pi, 5/4\,\pi, 7/4\,\pi$

* Solve $\sin(2x - \pi/3) = 1/2$

Ans: Observe that $\sin\theta = 1/2$ has solutions



etc...

or $\theta = \pi/6 + 2\pi k$ and $5/6 + 2\pi k$

Thus $2x - \frac{\pi}{3} = \pi/6 + 2\pi k$

$2x = \frac{\pi}{3} + \frac{\pi}{6} + 2\pi k \longrightarrow x = \frac{\pi}{4} + \pi k$

or $2x - \pi/3 = 5/6\,\pi + 2\pi k$

$2x = \frac{\pi}{3} + \frac{5}{6}\pi + 2\pi k \longrightarrow x = \frac{7\pi}{12} + \pi k$

* Solve $\tan\alpha + \tan^2\alpha = 0$

Ans: $\tan\alpha + \tan^2\alpha = 0$

$\Longleftrightarrow \tan\alpha(1+\tan\alpha) = 0$

$\Longleftrightarrow \tan\alpha = 0$ or $\tan\alpha = -1$

$\downarrow$ $\qquad$ $\downarrow$

$\alpha = \pi k$ $\qquad$ $\alpha = \frac{3}{4}\pi + \pi k$

$k \in \mathbb{Z}$

* Find the solutions of the equations

$2\cos^2 t + 3\cos t + 1 = 0$

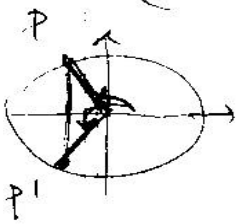
that are in the interval $[0, 2\pi)$.

Ans: $(2\cos t + 1)(\cos t + 1) = 0$

$\Longleftrightarrow \cos t = -1/2$ $\qquad$ $\cos t = -1$

$\downarrow$ $\qquad$ $\downarrow$

$t = 2/3\,\pi,\ 4/3\,\pi$ $\qquad$ $t = \pi$



* Find the solutions of the equation

$\sin^2\theta + \sin\theta - 6 = 0$

that are in the interval $[0, 2\pi)$.

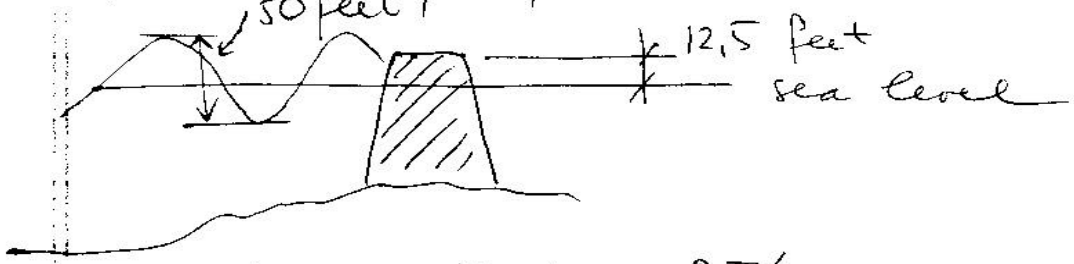
Ans: $(\sin\theta + 3)(\sin\theta - 2) = 0$

$\therefore \sin\theta = -3$ or $\sin\theta = 2$ impossible

* A tidal wave of height 50 feet and period 30 minutes is approaching a sea wall that is 12.5 feet above the sea level. From a particular point on shore, the distance y from sea level to the top of the wave is given by

$$y = 25\cos\left(\frac{\pi}{15}t\right)$$

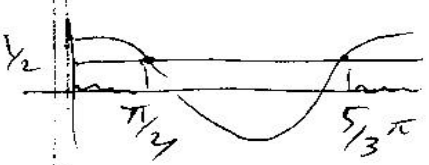
with t minutes. Find approximatively how many minutes of each 30-minute period is the top of the wave above the level of the top of the sea wall?



Observe that $2\pi/\pi/15 = 30 =$ period

Want to know for how long in a period

$$25\cos\left(\tfrac{\pi}{15}t\right) \geq 12.5 \iff \cos\left(\tfrac{\pi}{15}t\right) \geq \tfrac{1}{2}$$



$$0 \leq \frac{\pi}{15}t \leq \pi/3 \iff 0 \leq t \leq 5$$

$$5/3\pi \leq \frac{\pi}{15}t \leq 2\pi \iff 25 \leq t \leq 30$$

$\therefore$ $5 + 5 = 10$ minutes