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2.5 Graphs of functions

* A function $f(x)$ is <u>even</u> if

$f(-x) = f(x)$ for all x in the domain of f.

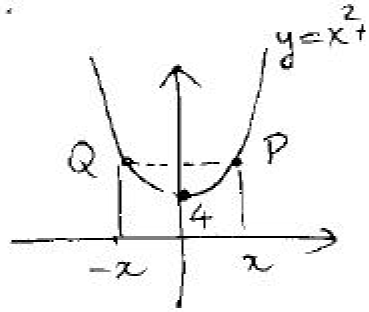
It means that the graph of f is symmetric with respect to the y-axis

* A function $f(x)$ is odd if

$f(-x) = -f(x)$ for all x in the domain of f

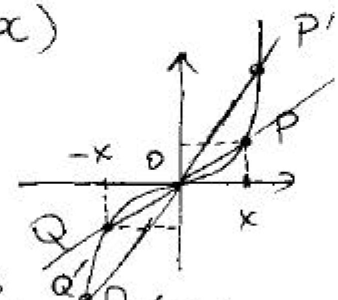
It means that the graph of f is symmetric with respect to the origin.

Eg



* $f(x) = x^2 + 4$ is <u>even</u>

$f(-x) = (-x)^2 + 4 = x^2 + 4 = f(x)$



* $f(x) = x^3$ is <u>odd</u>

$f(-x) = (-x)^3 = (-1)^3 (x)^3 = -x^3 = -f(x)$

note that $\overline{QO} = \overline{OP}$ those 2 segments are the same.

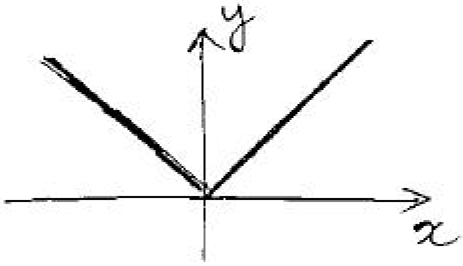
Another even function is the absolute value function:

$$f(x) = |x| \quad . \quad \text{In fact}$$

$$f(-x) = |-x| = |x| = f(x)$$

The graph is

$$y = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$



There are several techniques that helps graphing a more complex function when basic graphs are known.

(1) * <u>vertical shifting</u>

(2) * <u>horizontal shifting</u>

(3) * <u>vertical stretching or compressing</u>

(4) * <u>horizontal stretching or compressing</u>

In vertical shifting we relate the graph of

$y = f(x) + c$ to the graph of $y = f(x)$

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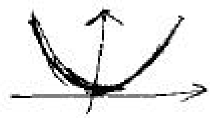
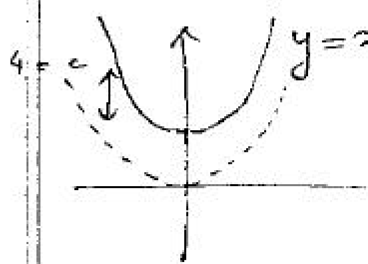
We either shift up or down the graph of f by c units



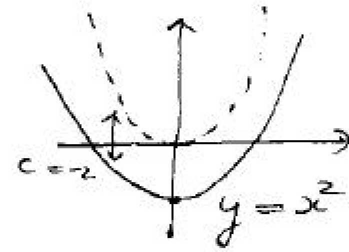
$f(x)=x^2$

eg: $f(x)+4 = x^2+4$

$f(x)-2 = x^2-2$



$y=x^2+4$



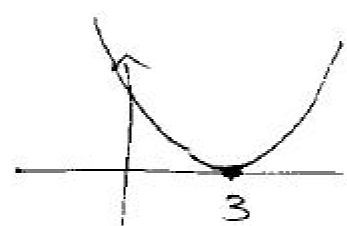
$y=x^2-2$

In horizontal shifting we relate the graph of $y=f(x+c)$ to the graph of $y=f(x)$.

We either shift left or right the graph of f by c units

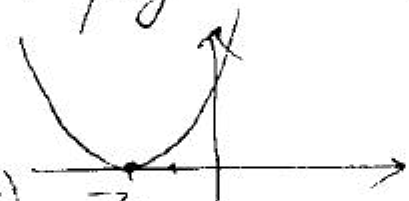
Let $f(x)=x^2$ then

$f(x-3) = (x-3)^2$



(to get the vertex we need to plug in $x=3$)

$f(x+2) = (x+2)^2$



(to get the vertex we plug in $x=-2$)

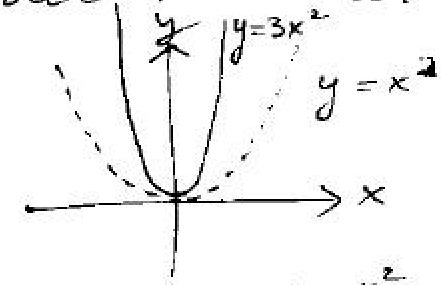
In vertical stretching or compressing we relate the graph of

$y = cf(x)$ to the graph of $y = f(x)$.

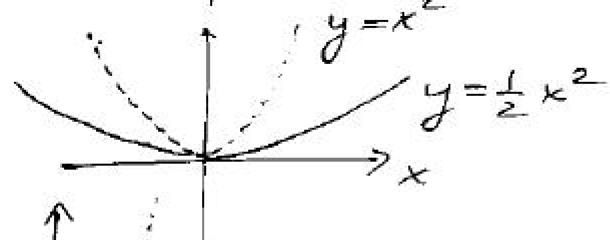
We either stretch or compress the graph of f by a factor of c (if $c > 0$).

We also flip about the x-axis if $c < 0$

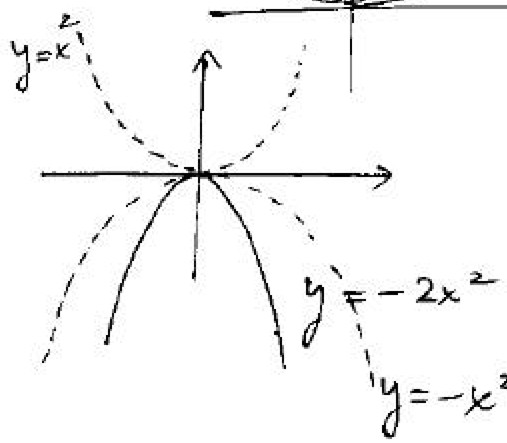
$y = 3x^2$



$y = \frac{1}{2}x^2$



$y = -2x^2$



In horizontal stretching or compressing we relate the graph of

$y = f(cx)$ to the graph of $y = f(x)$

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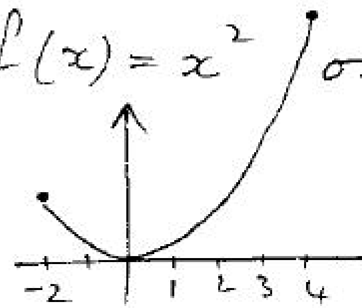
For instance if $f(x)$ is defined on the interval $a \le x \le b$ then the function $f(cx)$ is defined on the interval $a \le cx \le b$ OR $\frac{a}{c} \le x \le \frac{b}{c}$

* Eg.

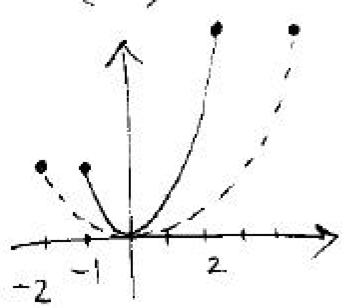
Consider $f(x) = x^2$ on $-2 \le x \le 4$



whereas the graph of $f(2x) = (2x)^2$ is defined on

$-2 \le 2x \le 4$ OR

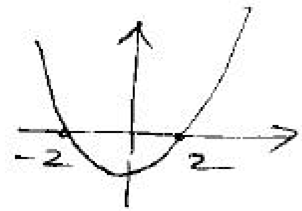
$-1 \le x \le 2$



So the graph was squeezed !

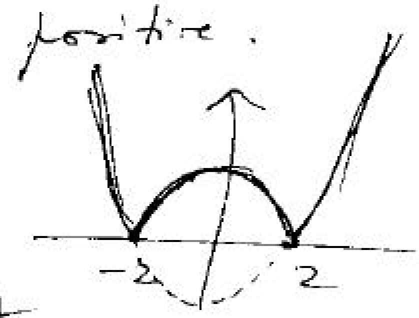
Graphs involving the absolute value:

$\therefore$ $y = |x^2 - 4|$ well $y = x^2 - 4$



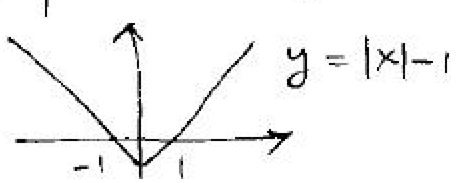
Now, when the value is positive the absolute value of the # is the same. If the value is negative, then the absolute value makes it positive.

$\therefore$ the graph of $y = |x^2-4|$ is



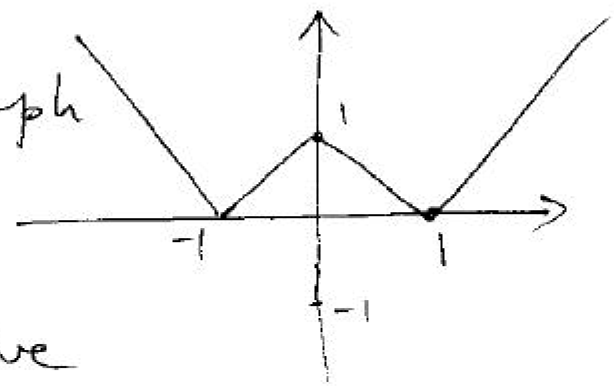
i.e. we flip the negative part about the x-axis !!!

$y = \big|\,|x|-1\,\big|$ well $y = |x|-1$ has graph



hence

$y = \big||x|-1\big|$ has graph



i.e. we flip the negative graph about the x-axis.