1.6 Inequalities

* Solve the inequality and express the solution in terms of intervals :

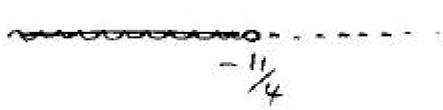
$$x-8 > 5x+3$$

!! (For inequalities we have essentially the same properties as for equations. However we have to be careful when divide by a number both sides of the inequality)

$$x-8 > 5x+3 \iff -8-3 > 5x - x$$

$$\iff -11 > 4x \iff -\frac{11}{4} > x \quad \text{or better}$$

$$x < -\frac{11}{4}$$



or $\left(-\infty, -\frac{11}{4}\right)$

Other way: $x-8 > 5x+3 \iff x-5x > 8+3$

$$\iff -4x > 11 \iff x < -\frac{11}{4}$$

i.e. we need to change the sign of the inequality!

In general the important properties are:

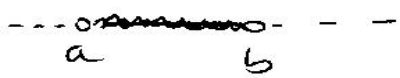
* $a < b$ and $b < c \Longrightarrow a < c$

* $a < b \longrightarrow a \pm c < b \pm c$ c any number

* $a < b$ and $c > 0 \Longrightarrow ac < bc$

$a < b$ and $c < 0 \Longrightarrow ac > bc$ (!!!)

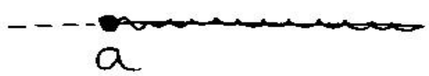
Notations:

$a < x < b$ ⇝ 

(a, b)

$a < x \leq b$ ⇝ 

$(a, b]$

$x \geq a$ ⇝ 

$[a, +\infty)$

etc...

* Other examples:

$$-2 < \frac{4x+1}{3} \leq 0 \iff -6 < 4x+1 \leq 0$$

$$\iff -6-1 < 4x \leq -1 \iff -7 < 4x \leq -1$$

$$\iff -\frac{7}{4} < x \leq -\frac{1}{4} \quad \text{or} \quad \left(-\tfrac{7}{4}, -\tfrac{1}{4}\right]$$

* $$\frac{-3}{2-x} < 0 \iff 2-x > 0 \iff x < 2$$

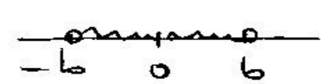
or $(-\infty, 2]$

* $$\frac{2}{(1-x)^2} > 0 \iff (1-x)^2 > 0 \iff x \neq 1$$

or $(-\infty, 1) \underset{\text{union}}{\cup} (1, +\infty)$

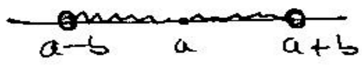
Inequalities with absolute values

$$|x| < b \iff -b < x < b$$



More generally $|x-a| < b \iff -b < x-a < b$

$$\iff \boxed{a-b < x < a+b}$$



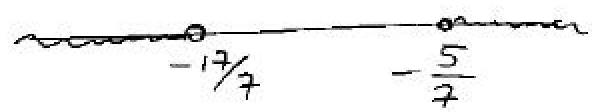
26

* Solve the inequality $2|-11-7x|-2>10$

$\iff 2|-11-7x|>12 \iff |-11-7x|>6$

We have $-11-7x>6$ or $-11-7x<-6$

$\iff -\frac{17}{7}>x$ or $-\frac{5}{7}<x$

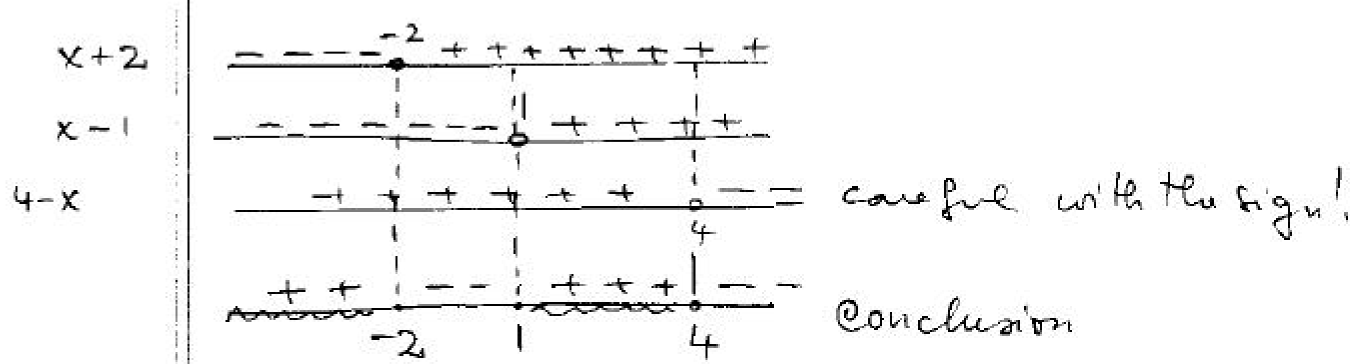


or

$\left(-\infty,-\frac{17}{7}\right) \cup \left(-5/7,+\infty\right)$

* Quadratic (and higher degree) inequalities:

$(x+2)(x-1)(4-x) \geqslant 0$



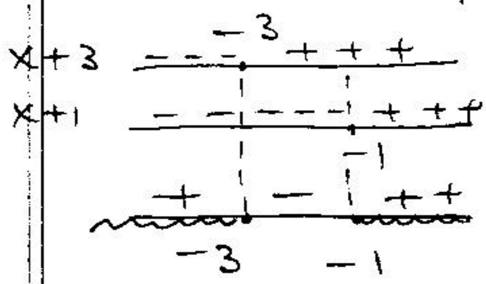
$(-\infty,-2] \cup [1,4]$

* If we wanted to solve $(x+2)(x-1)(4-x)\leq 0$ then the answer would have been

$[-2,1] \cup [4,+\infty)$

* Solve $x^2+4x+3 \geq 0$

Factor the polynomial $(x+3)(x+1) \geq 0$



conclusion $(-\infty,-3] \cup [-1,+\infty)$

* Solve $\dfrac{2}{2x+3} \leq \dfrac{2}{x-5}$

Write first as a single faction :

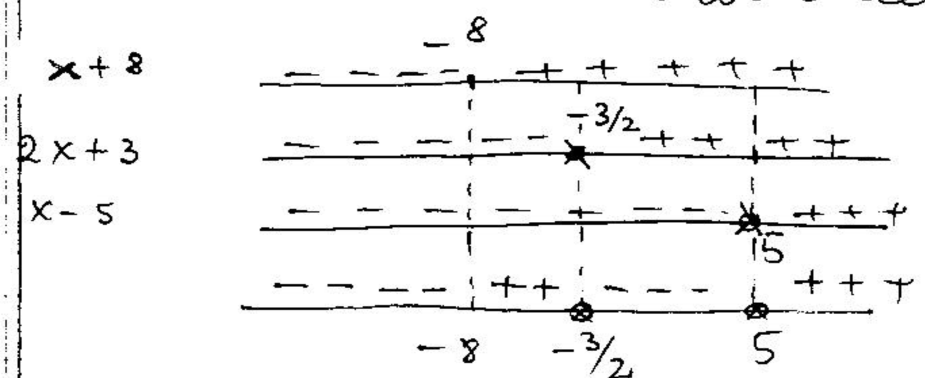
$$\frac{2}{2x+3} - \frac{2}{x-5} \leq 0 \iff \frac{2(x-5)-2(2x+3)}{(2x+3)(x-5)} \leq 0$$

$$\iff \frac{2x-10-4x-6}{(2x+3)(x-5)} \leq 0 \iff \frac{-2x-16}{(2x+3)(x-5)} \leq 0$$

We don't like the sign on top!!!

$$\iff \frac{x+8}{(2x+3)(x-5)} \geq 0$$

↑ we changed the sign!!



answer $\left[-8,-\frac{3}{2}\right) \cup (5,+\infty)$

28

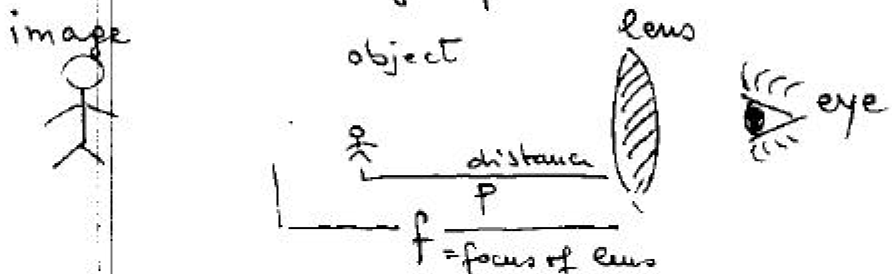
* Translate the following statement into an inequality:

"The radius of a ball bearing must be within 0.01 centimeter of 1 centimeter"

ans: $$|\underbrace{r}_{\text{radius}} - 1| \leq 0.01$$

infact $$\iff -0.01 \leq r-1 \leq 0.01$$
$$\iff 0.99 \leq r \leq 1.01$$

* Shown in the figure is a simple magnifier consisting of a convex lens. The object to be magnified is positioned so that the distance p from the lens is less than the focal length f. The linear magnification M is the ratio of the image size to the object size. It is shown in physics that



$$M = \frac{f}{f-p}$$. If f is 6 cm, how far should the object be placed from the lens so that its image appears at least 3 times as large?

Want: $3 \leq M = \dfrac{6}{6-p}$ as $6-p \geq 0 \iff$

$3(6-p) \leq 6$ or $\boxed{4 \leq p < 6}$