Please, be neat and show all your work; circle your answer.

♣ Good Luck ♣

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Noteworthy values of trigonometric functions

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<th>Angle $\varphi$ $\left\langle \begin{array}{c} \text{degrees} \ \text{radians} \end{array} \right.$</th>
<th>$\cos \varphi$</th>
<th>$\sin \varphi$</th>
<th>$\tan \varphi$</th>
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<td>$0^\circ$</td>
<td>0</td>
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<td>0</td>
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<td>$30^\circ$</td>
<td>$\frac{\pi}{6}$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$\frac{1}{2}$</td>
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<td>$45^\circ$</td>
<td>$\frac{\pi}{4}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
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<td>$60^\circ$</td>
<td>$\frac{\pi}{3}$</td>
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<td>$\frac{\sqrt{3}}{2}$</td>
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<td>$90^\circ$</td>
<td>$\frac{\pi}{2}$</td>
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The addition and subtraction formula

- $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$
- $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$
- $\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$

Double angle identities

- $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$
- $\sin 2\alpha = 2 \sin \alpha \cos \alpha$
- $\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$

Half-angle identities

- $\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$
- $\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$
- $\tan \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$

where the choice of the sign is determined by the quadrant in which the angle $\alpha/2$ is located.
1. Let $A(3, -2)$ and $B(-3, 1)$. Write an equation for all points $P(x, y)$ such that

$$(\text{dist}(A, P))^2 = (\text{dist}(B, P))^2 + 1$$

2. Find $b$ such that the line $3x + by + 2 = 0$ has $y$-intercept $1/2$. 

pts: /6
3. Write an equation for the circle such that the points $A(6, 0)$ and $B(0, 8)$ are endpoints of a diameter.

4. Let $f(x) = x^2 - 6x + 8$. Sketch the graph of $y = f(x)$ using the method of completing the squares. Indicate the $x$- and $y$-intercepts as well as the coordinates of the vertex of the parabola.
5. Sketch the graph of

\[ f(x) = \frac{x^2 - 5x + 4}{x^2} \]

Indicate domain, horizontal and vertical asymptotes, \( x \)- and \( y \)-intercepts, sign variation, etc...

6. Find the domain of

\[ F(x) = \sqrt{\frac{1 - 2x}{3x + 2}}. \]
7. For \( f(x) = -2x^2 + 7x - 1 \), find the difference quotient
\[
\frac{f(a + h) - f(a)}{h}.
\]

8. Find the inverse of
\[ f(x) = \sqrt{3 - x}. \]
Plot both the graph of \( f(x) \) and the one of \( f^{-1}(x) \) on the same coordinate plane. State the domain and the range of both functions.
9. Find the value of $x$ for which:

$$\log_3 2x = 4; \quad 4^x = 64; \quad \log_2 (\log_4 256) = 2.$$

10. The formula

$$p(h) = 14.7 e^{-0.12h}$$

gives the atmospheric pressure in pounds per square inch at an altitude of $h$ miles above sea level. At what altitude will the atmospheric pressure be one half of the sea level pressure?
11. Solve the logarithmic equation

\[ \log_3(\log_2(x + 3)) = \log_3(\log_2(4x - 3)). \]

12. (a) Write in a single expression the following

\[ 4 \log_3(x + 1) - 2 \log_3(x - 1) - \frac{1}{2} \log_3(x + 3). \]

(b) Write the following

\[ \log \sqrt[5]{\frac{x^2(x + 1)^3}{(x + 2)^5}} \]

in terms of \( \log x \), \( \log(x + 1) \) and \( \log(x + 2) \).
13. If a central angle of $30^\circ$ is subtended by a circular arc of length 12 meters, find the radius of the circle and the area of the sector.

14. If $\tan \theta = 4$ and $\sin \theta < 0$, find the remaining 5 trigonometric functions.
15. Find the amplitude, period and phase shift of

\[ y = -2 \sin(2(x - \pi/3)). \]

Graph the function and label the \( x \)- and \( y \)-intercepts.

16. (a) Verify the identity

\[ \sec u = \frac{\cos u}{1 + \sin u} + \tan u \]

(b) Compute the exact value of

\[ \frac{\cos(\pi/3)}{1 + \sin(\pi/3)} + \tan(\pi/3). \]
17. Solve the equation

\[ \sin(2u) - \sqrt{3} \cos u = 0. \]

in the interval \([0, 2\pi]\).

18. Solve the triangle ABC if

\[ \beta = 60^\circ \quad \gamma = 45^\circ \quad a = 8. \]
19. Two joggers starting from the same point run along two directions that make an angle of 30°. One of them runs at 8mph and the other at 10mph. How far will they be after 2 hours?

20. If α and β are acute angles such that \( \csc \alpha = \frac{13}{12} \) and \( \cot \beta = \frac{4}{3} \), find

   (a) \( \sin(\alpha + \beta) \)
   
   (b) \( \tan(\alpha + \beta) \)
   
   (c) the quadrant containing \( \alpha + \beta \).