

SEC.	INSTRUCTORS	T.A.'S	LECTURES	RECITATIONS
001	A. Corso	B. Bennewitz	MWF 8:00-8:50, CB 204	TR 8:00-9:15, CB 341
002	A. Corso	B. Bennewitz	MWF 8:00-8:50, CB 204	TR 9:30-10:45, CB 345
004	M. Silhavy	H. Song	MWF 10:00-10:50, CB 214	TR 8:00-9:15, CB 349
005	M. Silhavy	C. Budovsky	MWF 10:00-10:50, CB 214	TR 2:00-3:15, CB 343
006	M. Silhavy	H. Song	MWF 10:00-10:50, CB 214	TR 3:30-4:45, CB 345
007	A. Martin	M. Neu	MWF 12:00-12:50, CB 208	TR 9:30-10:45, CB 347
008	A. Martin	Y. Jia	MWF 12:00-12:50, CB 208	TR 11:00-12:15, CB 347
009	A. Martin	Y. Jia	MWF 12:00-12:50, CB 208	TR 12:30-1:45, CB 349
010	M. Silhavy	C. Budovsky	MWF 2:00-2:50, CB 204	TR 12:30-1:45, CB 345
011	M. Silhavy	M. Slone	MWF 2:00-2:50, CB 204	TR 2:00-3:15, CB 345
012	M. Silhavy	M. Slone	MWF 2:00-2:50, CB 204	TR 3:30-4:45, CB 349

Answer all of the following questions. Use the backs of the question papers for scratch paper. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. When answering these questions, please be sure to:

- check answers when possible,
- clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may receive NO credit*).

QUESTION	SCORE	TOTAL
1.		16
2.		16
3.		6
4.		10
5.		8
6.		16
7.		12
8.		8
9.		8
TOTAL		100

1. Compute the following limits. Each limit is worth 4 points.

$$(a) \lim_{x \rightarrow 2} \frac{x^3 - 1}{x - 1} = \frac{\cancel{\lim}_{x \rightarrow 2} (x^2 + x + 1)}{\cancel{x - 2}} = \underline{7}$$

recall $x^3 - 1 = (x - 1)(x^2 + x + 1)$

$$(b) \lim_{x \rightarrow 0} \frac{\cos^2 x - 1}{2x^2} = \frac{\cancel{\lim}_{x \rightarrow 0} -\frac{1}{2} \frac{\sin^2 x}{x^2}}{\cancel{x^2}} = -\frac{1}{2} \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right)^2 = \underline{-\frac{1}{2}}$$

$$\sin^2 x + \cos^2 x = 1 \implies \cos^2 x - 1 = -\sin^2 x \quad \downarrow 1$$

$$(c) \lim_{x \rightarrow \infty} \cot\left(\frac{2}{x} + \frac{\pi}{4}\right) = \cot\left(\frac{\pi}{4}\right) = \underline{1}$$

$$\cot x = \frac{\cos x}{\sin x} \quad \text{as } x \rightarrow \infty \quad \frac{2}{x} + \frac{\pi}{4} \rightarrow \frac{\pi}{4}$$

$\rightarrow \therefore \cot\left(\frac{\pi}{4}\right) = 1$

$$(d) \lim_{x \rightarrow \infty} \frac{\sqrt{9x^4 + 8}}{3x^2 + \sqrt{x}} = \underline{1}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{\sqrt{9x^4 + 8}}{x^2}}{\frac{3x^2 + \sqrt{x}}{x^2}} = \lim_{x \rightarrow \infty} \frac{\sqrt{9 + \frac{8}{x^4}}}{3 + \sqrt{\frac{1}{x^3}}} =$$

$$= \frac{\sqrt{9}}{3} = 1$$

pts: /16

2. Find the derivative of the following functions. Each derivative is worth 4 points. Do **not** simplify your answers.

(a) If $f(x) = (2x^8 + 7)(3x^2 + 5x)$ then $f'(x) =$ _____

$$f'(x) = 16x^7(3x^2 + 5x) + (2x^8 + 7) \cdot (6x + 5)$$

(b) If $f(x) = \sin(\sqrt[3]{x^2})$ then $f'(x) =$ _____

$f(x) = \sin(x^{2/3})$ use chain rule

$$f'(x) = \cos(x^{2/3}) \cdot \frac{2}{3} \cdot x^{-1/3}$$

(c) If $f(x) = \cos^3(x^3) + (5x^2 - 3)^3$ then $f'(x) =$ _____

$$f'(x) = 3\cos^2(x^3) \cdot (-\sin(x^3)) \cdot 3x^2 + 3(5x^2 - 3)^2 \cdot 10x$$

(d) If $f(x) = \frac{\sin(x^3 - 1)}{x^3 + 1}$ then $f'(x) =$ _____

use quotient rule and chain rule

$$f'(x) = \frac{[\cos(x^3 - 1) \cdot 3x^2](x^3 + 1) - \sin(x^3 - 1) \cdot 3x^2}{(x^3 + 1)^2}$$

pts: /16

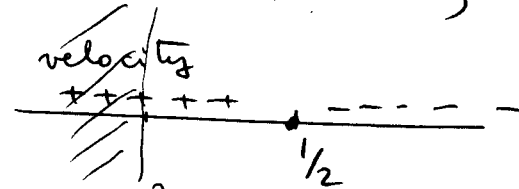
3. A particle is moving on a line such that its position after t hours is $s(t) = -t^2 + t + 2$ measured in miles.

(a) Find the velocity of the particle.

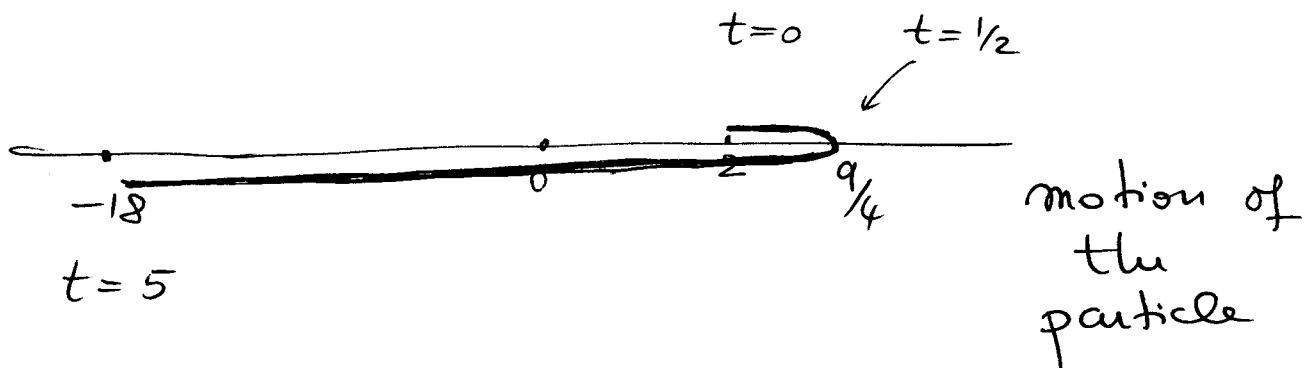
$$v(t) = -2t + 1$$

(b) When does the particle change its direction?

it changes direction when $v(t) = 0$,
i.e. at $t = \frac{1}{2}$



(c) What is the largest distance of the particle to its origin within the first 5 hours.



the position at $t=0$ is 2 ;

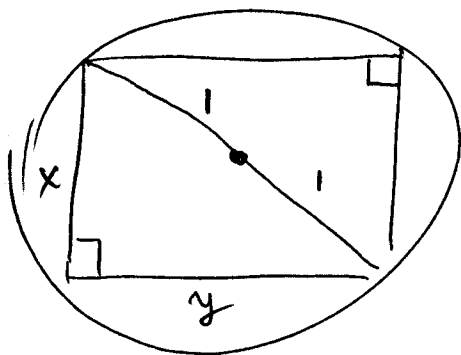
the position at $t=\frac{1}{2}$ is $\frac{9}{4}$.

The position at $t=5$ is -18 .

pts: /6

Thus the largest distance
to its origin is (in absolute value)
20

5. Find the largest area of a rectangle that can be inscribed in a circle of radius 1.



Notice that the main diagonals of any such rectangle are diameters.

Now, if x and y are the dimensions of the rectangle we have that $x^2 + y^2 = 2^2 = 4$

thus $y = \sqrt{4 - x^2}$. We need to maximize the function

$$\text{Area} = x \cdot y = x \sqrt{4 - x^2} \quad 0 \leq x \leq 2$$

$$A' = 1 \cdot \sqrt{4 - x^2} + x \cdot \frac{(-2x)}{2\sqrt{4 - x^2}} = \frac{4 - x^2 - x^2}{\sqrt{4 - x^2}} = \frac{4 - 2x^2}{\sqrt{4 - x^2}}$$

$$A' = 0 \iff 4 - 2x^2 = 0 \quad x^2 = 2 \quad x = \sqrt{2}$$

x	$A(x)$
0	0
2	0
$\sqrt{2}$	2

} value at the end points

← max

pts: 18

Notice $x = \sqrt{2} = y \therefore$ it is a square

6. Find the following indefinite integrals. Each problem is worth 4 points.

$$(a) \int (\sqrt{x} + \sin(5x)) dx = \boxed{\frac{2}{3} x \sqrt{x} - \frac{1}{5} \cos(5x) + \text{const}}$$

$$\int \sqrt{x} dx + \int \sin(5x) dx = \frac{2}{3} x^{3/2} + \frac{1}{5} (-\cos(5x)) + \text{const}$$

$\left\{ \begin{array}{l} x^{3/2} = x\sqrt{x} \end{array} \right.$

$$(b) \int \sqrt[3]{x} \cdot (x^7 - 1) dx = \frac{3}{25} x^{8\sqrt[3]{x}} - \frac{3}{4} x^{\sqrt[3]{x}} + \text{const}$$

$$\int (x^{7+1/3} - x^{1/3}) dx = \int (x^{22/3} - x^{1/3}) dx = \frac{3}{25} x^{25/3} - \frac{3}{4} x^{4/3} + \text{const}$$

$\left\{ \begin{array}{l} x^{25/3} = x^8 x^{1/3} \\ x^{4/3} = x \cdot x^{1/3} \end{array} \right.$

$$(c) \int \frac{\sin x}{\cos^5 x} dx = \frac{1}{4 \cos^4 x} + \text{const}$$

use $u = \cos x$

$du = -\sin x dx$

$$= \int -\frac{du}{u^5} = -\int u^{-5} du = -\frac{1}{-4} u^{-4} + \text{const}$$

$$= \frac{1}{4u^4} + \text{const}$$

$$(d) \int (x^6 + x^3)^7 \cdot (2x^5 + x^2) dx = \frac{1}{24} (x^6 + x^3)^8 + \text{const}$$

set $u = x^6 + x^3$

$du = (6x^5 + 3x^2) dx$

$\therefore \frac{du}{3} = (2x^5 + x^2) dx$

$$\therefore = \int u^7 \frac{du}{3} =$$

$$= \frac{1}{3} \cdot \frac{1}{8} u^8 + \text{const}$$

pts: /16

NOTE: there was a typo in problem (d) !!

7. Calculate the following definite integrals. Each problem is worth 4 points.

$$(a) \int_0^9 (x^2 - \sqrt{x}) dx = \underline{225}$$

$$= \left. \frac{1}{3} x^3 - \frac{2}{3} x^{3/2} \right|_0^9 = \frac{1}{3} 9^3 - \frac{2}{3} 9^{3/2} - 0 = 243 - 18 = 225$$

THERE WAS
A TYPO

$$(b) \int_0^1 x^3 \sqrt{1+2x^4} dx = \underline{\frac{3\sqrt{3}-1}{12}}$$

$$u = 1+2x^4$$

$$du = +8x^3 dx$$

$$+\frac{1}{8} du = x^3 dx$$

$$= +\frac{1}{8} \int_1^3 \sqrt{u} du = \frac{1}{8} \int_1^3 u^{1/2} du = \frac{1}{8} \left[\frac{2}{3} u^{3/2} \right]_1^3$$

$$= \frac{1}{8} \cdot \frac{2}{3} [3\sqrt{3} - 1] = \frac{3\sqrt{3}-1}{12}$$

$$(c) \int_0^1 (2x+1)^2 dx = \underline{13/3}$$

$$u = 2x+1$$

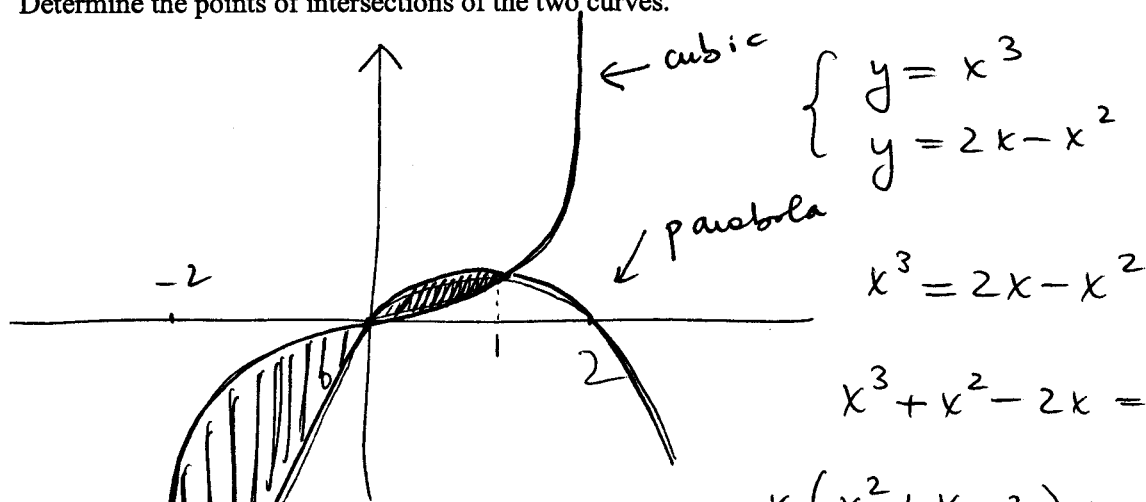
$$du = 2 dx$$

$$= \int_1^3 \frac{1}{2} u^2 du = \left. \frac{1}{6} u^3 \right|_1^3 =$$

$$= \frac{1}{6} [27 - 1] = \frac{26}{6} = \frac{13}{3}$$

pts: /12

8. (a) Sketch the region that is bounded by the graphs of the functions $f(x) = 2x - x^2$ and $g(x) = x^3$. Determine the points of intersections of the two curves.



- (b) Compute the area of the region.

$$\begin{cases} y = x^3 \\ y = 2x - x^2 \end{cases}$$

$$x^3 = 2x - x^2$$

$$x^3 + x^2 - 2x = 0$$

$$x(x^2 + x - 2) = 0$$

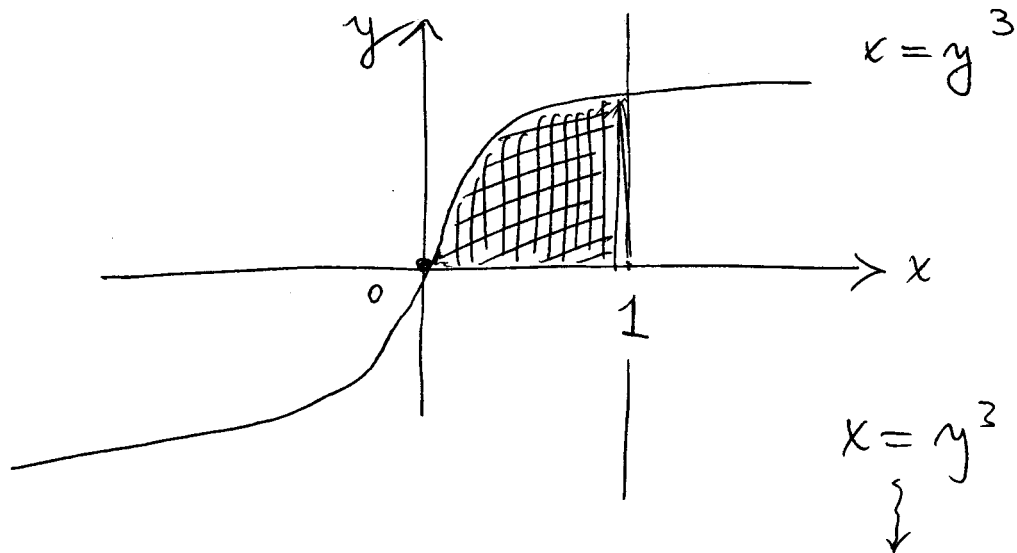
$$x(x+2)(x-1) = 0$$

$$x = 0, 1, -2$$

$$\begin{aligned} \text{Area} &= \int_{-2}^0 (x^3 - (2x - x^2)) dx + \int_0^1 ((2x - x^2) - x^3) dx \\ &= \int_{-2}^0 (x^3 - 2x + x^2) dx + \int_0^1 (-x^3 - x^2 + 2x) dx = \\ &= \left[\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2 \right]_{-2}^0 + \left[-\frac{1}{4}x^4 - \frac{1}{3}x^3 + x^2 \right]_0^1 = \\ &= 0 - \left(4 + \frac{1}{3}(-8) - 4 \right) + \left(-\frac{1}{4} - \frac{1}{3} + 1 \right) \\ &= \frac{8}{3} + \frac{-3-4+12}{12} = \frac{32+5}{12} = \frac{37}{12} \end{aligned}$$

pts: /8

9. Find the volume of the solid that is obtained by rotating about the x -axis the region bounded by the curve $x - y^3 = 0$ and the line $x = 1$.



$$\text{Volume} = \int_0^1 \pi [f(x)]^2 dx$$

$$= \int_0^1 \pi \left[\sqrt[3]{x} \right]^2 dx = \int_0^1 \pi x^{2/3} dx$$

$$= \pi \cdot \frac{2}{5} x^{5/2} \Big|_0^1 = \pi \cdot \frac{2}{5} \cdot 1^{5/2} - 0$$

$$= \frac{2}{5} \pi$$

pts: /8