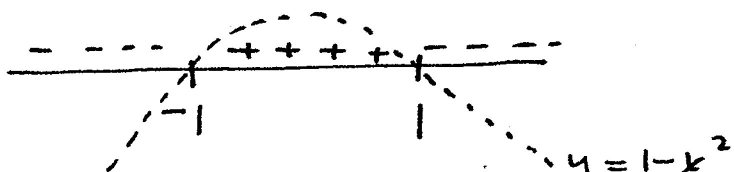
1. (*a*) (5 pts) If $f(x) = 1 - x^2$ and $g(x) = \sqrt{x}$ find a formula for $(g \circ f)(x)$.
Give the domain of $(g \circ f)(x)$.

$$\boxed{(g \circ f)(x) = g(f(x)) = \sqrt{1 - x^2}}$$

domain : all x's s/t $1 - x^2 \geq 0$



$\therefore$ domain : all x's s/t $\boxed{-1 \leq x \leq 1}$

(*b*) (5 pts) Which of the following functions are even, odd, neither? Explain your answer

(1) $f(x) = 3 + |x| - x^4$

it is even

$$f(-x) = 3 + |-x| - (-x)^4 = 3 + |x| - x^4 = f(x)$$

(2) $g(x) = 2x^3 - x^2 + 1$

it is neither

e.g. $g(-1) = 2(-1)^3 - (-1)^2 + (1) = -2$

$g(1) = 2(1)^3 - (1)^2 + (1) = 2$

but $g(2) = 2 \cdot 8 - 4 + 1 = 13$

$g(-2) = 2 \cdot (-8) - 4 + 1 = -19$

pts: /10

2. Compute the following limits. Each limit is worth 5 points.

(*a*) $\lim\limits_{x\to-1}\frac{x^2-3x-4}{x+1} = \lim\limits_{x\to-1}(x-4) = \boxed{-5}$

$$\frac{x^2-3x-4}{x+1} = \frac{(x-4)(x+1)}{x+1} = x-4$$

(*b*) $\lim\limits_{h\to0}\frac{1}{h}\left[(h-4)^2-16\right] = \lim\limits_{h\to0}(h-8) = \boxed{-8}$

$$\frac{(h-4)^2-16}{h} = \frac{h^2-8h+\cancel{16}-\cancel{16}}{h} = h-8$$

(*c*) $\lim\limits_{x\to6^+}\frac{(x-5)(3-x)}{(x-6)(x-1)} = \frac{1\cdot(-3)}{0^+\cdot5} = \boxed{-\infty}$ $\qquad\lim\limits_{x\to6^-}\frac{(x-5)(3-x)}{(x-6)(x-1)} = \frac{1\cdot(-3)}{0^-\cdot5} = \boxed{+\infty}$

(*d*) $\lim\limits_{x\to2}\frac{x^4-16}{x-2} = \lim\limits_{x\to2}(x+2)(x^2+4) = (2+2)(2^2+4) = \boxed{32}$

$$\frac{x^4-16}{x-2} = \frac{(x^2-4)(x^2+4)}{x-2} = \frac{\cancel{(x-2)}(x+2)(x^2+4)}{\cancel{(x-2)}}$$

(*e*) Find $c =$ 11 so that $\lim\limits_{x\to1}\frac{x^2+cx-x-c}{x^2+2x-3} = 3.$

$$\frac{x^2+cx-x-c}{x^2+2x-3} = \frac{x(x+c)-(x+c)}{(x+3)(x-1)}$$

$$= \frac{(x+c)\cancel{(x-1)}}{(x+3)\cancel{(x-1)}} = \frac{x+c}{x+3} \qquad \lim_{x\to1}\frac{x+c}{x+3} = \frac{1+c}{4}$$

Want

$$\frac{1+c}{4} = 3$$

$\therefore\ c = 11$

pts: /25

3. Find all the values of the constant c that make the function

$$h(x) = \begin{cases} c^2 - x^2 & \text{if } x < 1 \\ 2(x-c)^2 & \text{if } x \geq 1 \end{cases}$$

continuous everywhere. Graph these functions.

The only problem is at $x = 1$. We want to make sure that

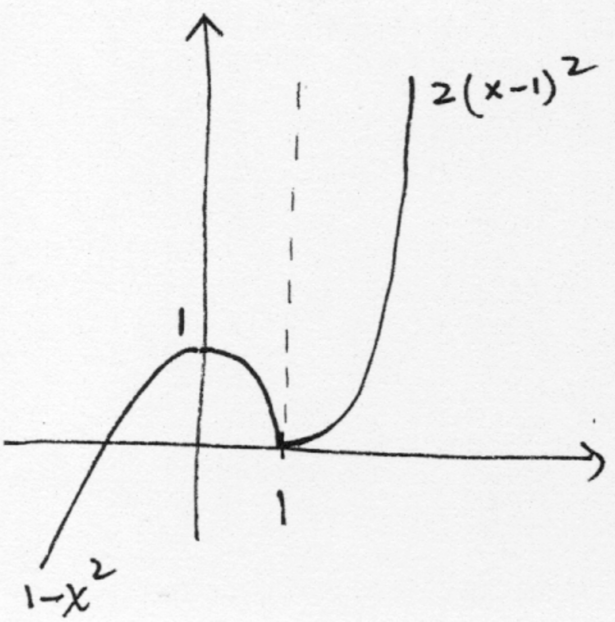
$$\lim_{x \to 1^-} h(x) \overset{\downarrow\downarrow}{=} \lim_{x \to 1^+} h(x).$$

But this means that $c^2 - 1 \overset{\downarrow\downarrow}{=} 2(1-c)^2$ or

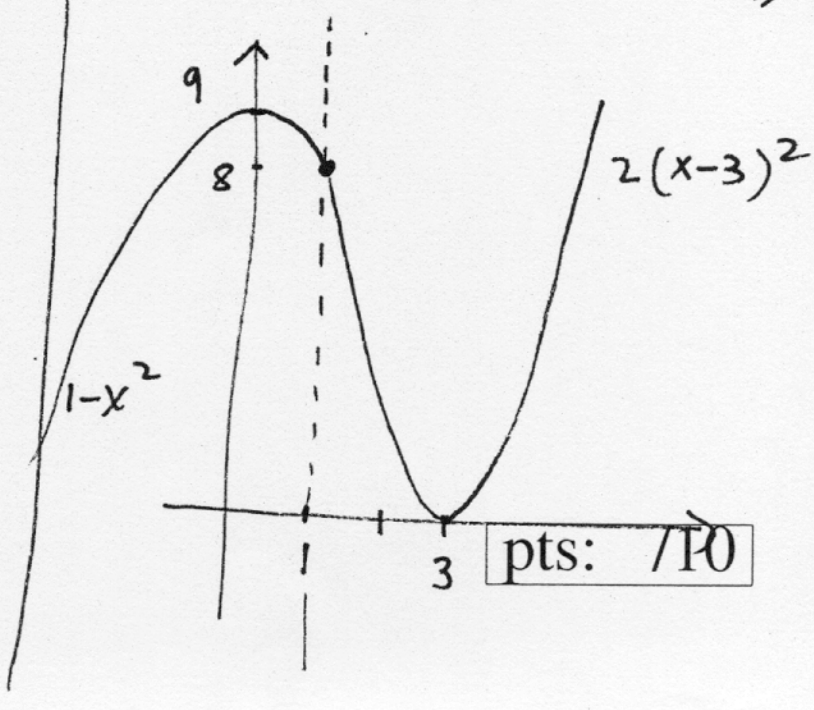
$2 - 4c + 2c^2 - c^2 + 1 = 0 \quad \therefore \quad c^2 - 4c + 3 = 0$

$(c-3)(c-1) = 0 \quad \therefore \quad \boxed{c = 1, \ c = 3}$

$$h_1(x) = \begin{cases} 1 - x^2 & x \leq 1 \\ 2(x-1)^2 & x \geq 1 \end{cases}$$



$$h_2(x) = \begin{cases} 9 - x^2 & x \leq 1 \\ 2(x-3)^2 & x \geq 1 \end{cases}$$



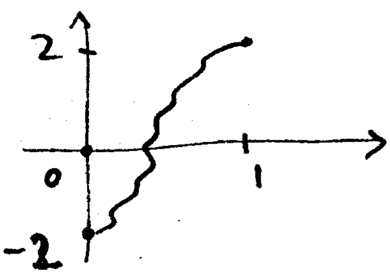
pts: 7/10

4. Does the equation $x^3+3x-2=0$ have a root between 0 and 1. Explain.
(**Note:** A calculator solution is not an acceptable answer.)

Use the Intermediate value theorem !

$f(x) = x^3+3x-2$ is continuous on $[0,1]$

and $f(0) = -2$ while $f(1) = 1^3+3\cdot 1-2 =$



$\therefore$ the graph of $f(x)$ intersects the x- axis between $x=0$ an[d] $x=1$. Thus there is a root

pts: /7

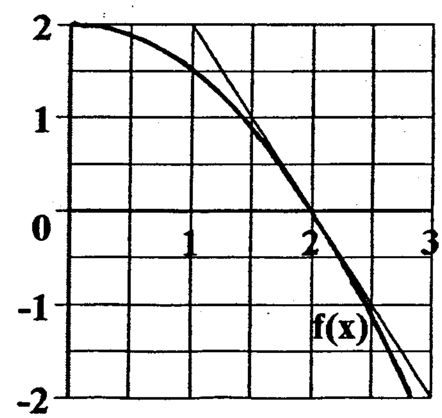
5. A segment of the tangent line to the graph of $f(x)$ at $x=2$ is shown in the diagram. Using information from the graph we can estimate that

$f(2) =$ 0 $\qquad f'(2) =$ -2

Hence the equation of the tangent line to the graph of

$g(x) = 5x + f(x)$

at $x=2$ is $y =$ $3x+4$



need $P(2, g(2)) = (2, 10)$

as $g(2) = 5\cdot 2 + f(2) = 10+0=10$

need $m = g'(2)$. But $g'(x) = 5 + f'(x)$

So $g'(2) = 5 + (-2) = 3$

$\therefore$ $y - 10 = 3(x-2)$

or $\boxed{y = 3x+4}$

pts: /8

6. Calculate the following derivatives. Each derivative is worth 5 points.

Do not simplify your answers.

(*a*) If $f(x) = 3x^2 - \frac{x}{\pi} + \pi^2$ then $f'(x) =$ $6x - \frac{1}{\pi}$.

notice : $\frac{1}{\pi}, \pi^2$ are numbers !!!

(*b*) If $f(x) = (x^3-3)(-3x-x^2)$ then $f'(x) =$ __________.

$$3x^2 \cdot (-3x - x^2) + (x^3 - 3)(-3 - 2x)$$

(*c*) If $g(t) = \frac{2t-1}{t+1}$ then $g'(t) =$ __________.

$$\frac{2 \cdot (t+1) - (2t-1) \cdot 1}{(t+1)^2}$$

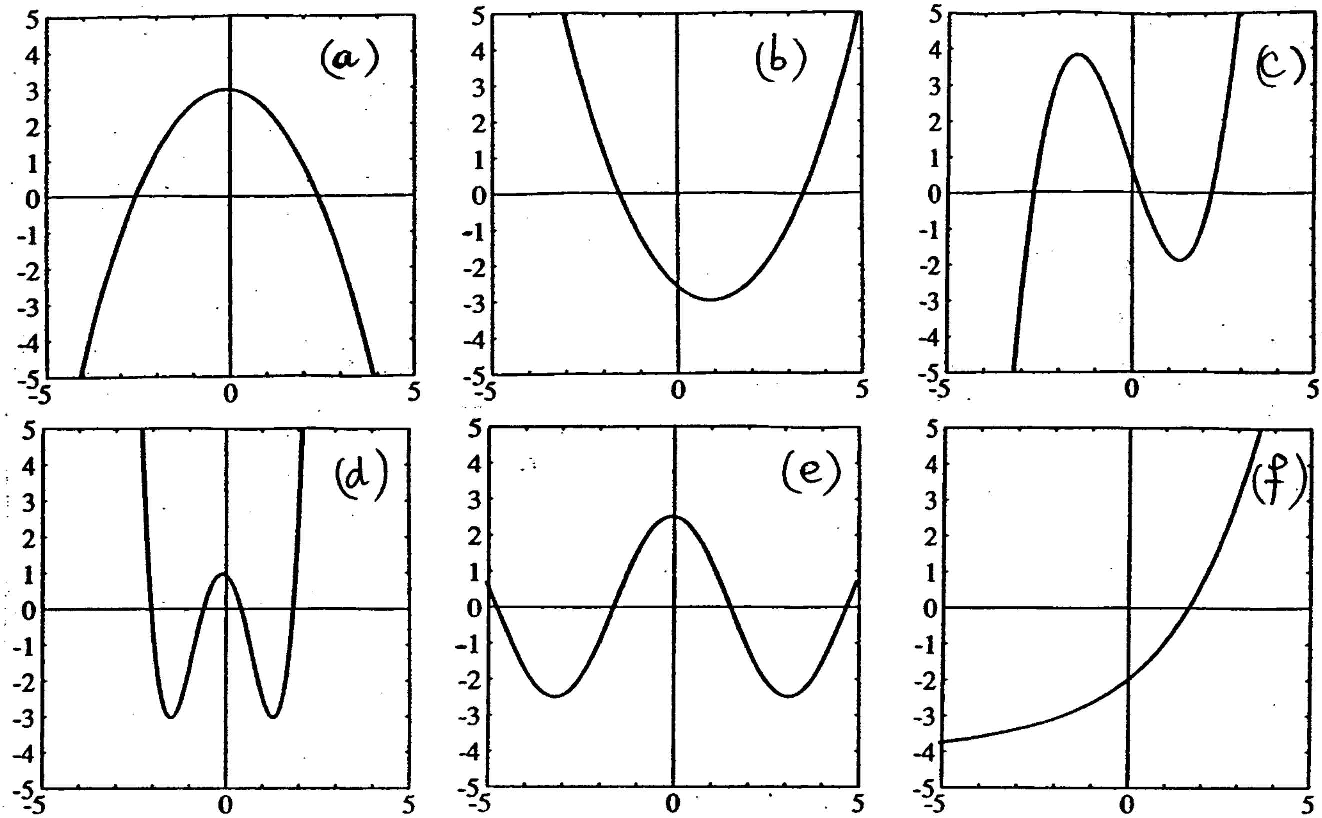
(*d*) If $p(t) = t\sqrt{t} - \frac{1}{\sqrt{t}} - 3$ then $p'(t) =$ __________.

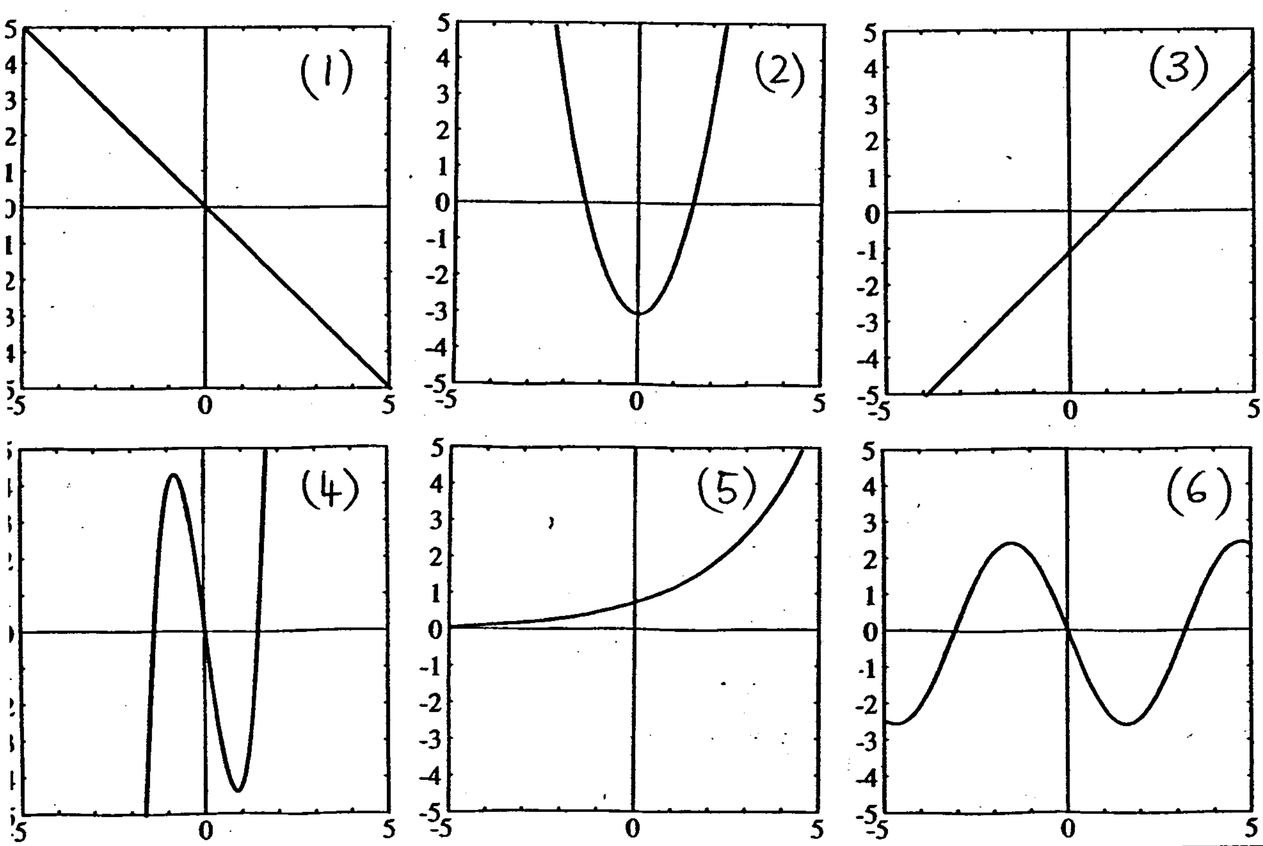
$$p(t) = t^{3/2} - t^{-1/2} - 3$$

$$\therefore \boxed{p'(t) = \frac{3}{2} t^{1/2} + \frac{1}{2} t^{-3/2}}$$

pts: /20

7. Match the graph of each function labelled (a)-(f) with the graph of its derivative (1)-(6).



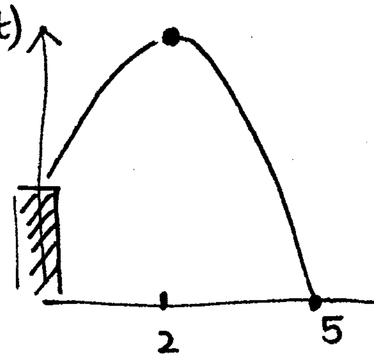


(a)	1
(b)	3
(c)	2
(d)	4
(e)	6
(f)	5

pts: /10

8. A ball is thrown upward at 64 feet per second from a height of 80 feet. In the absence of air resistance it will have height

$$h(t) = -16t^2 + 64t + 80 \text{ feet.}$$



(*a*) (3 pts) After how many seconds will the ball hit the ground?

$h(t) = 0$

$-16t^2 + 64t + 80 = 0 \rightsquigarrow t^2 - 4t - 5 = 0$

$(t-5)(t+1) = 0 \quad \therefore \quad t = 5 \quad \cancel{t = -1}$

(*b*) (2 pts) What will the velocity of the ball be 2 seconds after it is thrown?

$v(t) = h'(t) = -32t + 64$

$v(2) = -32 \cdot 2 + 64 = 0 \text{ ft/sec}$

$\therefore t = 2$ corres

to the max

of the parabol

(*c*) (2 pts) What will the velocity of the ball be when it hits the ground?

$v(5) = -32 \cdot 5 + 64 = -160 + 64 = -96 \text{ ft/s}$

from part (a), $t = 5$

(*d*) (3 pts) How high will the ball go?

from part (b) the highest point occurs when $t = 2$.

$h(2) = -16 \cdot 2^2 + 64 \cdot 2 + 80$

$= \dots = 144$ feet

pts: /10