MA 113 - Calculus I
SECOND MIDTERM

Spring 2002
03/05/2002

Name: ______________________ Sec.: ____

SEC.	INSTRUCTORS	T.A.'S	LECTURES	RECITATIONS
001	A. Corso	B. Bennewitz	MWF 8:00-8:50, CB 204	TR 8:00-9:15, CB 341
002	A. Corso	B. Bennewitz	MWF 8:00-8:50, CB 204	TR 9:30-10:45, CB 345
004	M. Silhavy	H. Song	MWF 10:00-10:50, CB 214	TR 8:00-9:15, CB 349
005	M. Silhavy	C. Budovsky	MWF 10:00-10:50, CB 214	TR 2:00-3:15, CB 343
006	M. Silhavy	H. Song	MWF 10:00-10:50, CB 214	TR 3:30-4:45, CB 345
007	A. Martin	M. Neu	MWF 12:00-12:50, CB 208	TR 9:30-10:45, CB 347
008	A. Martin	Y. Jia	MWF 12:00-12:50, CB 208	TR 11:00-12:15, CB 347
009	A. Martin	Y. Jia	MWF 12:00-12:50, CB 208	TR 12:30-1:45, CB 349
010	M. Silhavy	C. Budosvky	MWF 2:00-2:50, CB 204	TR 12:30-1:45, CB 345
011	M. Silhavy	M. Slone	MWF 2:00-2:50, CB 204	TR 2:00-3:15, CB 345
012	M. Silhavy	M. Slone	MWF 2:00-2:50, CB 204	TR 3:30-4:45, CB 349

Answer all of the following questions. Use the backs of the question papers for scratch paper. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. When answering these questions, please be sure to:

- check answers when possible,
- clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may receive NO credit*).

QUESTION	SCORE	TOTAL
1.		9
2.		15
3.		20
4.		10
5.		8
6.		20
7.		10
8.		8
TOTAL		100

1. The population of a bacterial colony after t hours is given by

$$n(t) = 48t - t^3 + 100.$$

(*a*) (3 pts) Determine the growth rate as a function of time.

$$n'(t) = \underline{48 - 3t^2}$$

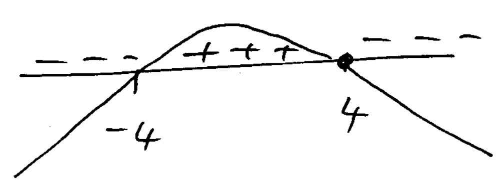
(*b*) (3 pts) Find the growth rate after 2 hours.

$$n'(2) = 48 - 3 \cdot 2^2 = \underline{36}$$

(*c*) (3 pts) Find the time t at which the population starts diminishing.

$$n'(t) \leq 0 \iff 48 - 3t^2 \leq 0$$

$$\iff 16 - t^2 \leq 0$$



$\therefore$ the population starts diminishing at $\underline{t = 4}$

pts: /9

2. Compute the following limits. Each limit is worth 5 points.
Note: Remember to simplify your answers!

(a) $\lim_{x\to\pi/6} \dfrac{3\sin(-x)}{\cos^2(2x)} = \underline{-6}$

$$= \frac{3\sin(-\pi/6)}{\cos^2(\pi/3)} = \frac{-3/2}{(1/2)^2} = -6$$

(b) $\lim_{x\to 0} \dfrac{\cos^2(3x)-1}{x^2} = \underline{-9}$

$$\frac{\cos^2(3x)-1}{x^2} = \frac{-\sin^2(3x)}{x^2} = -9\,\frac{\sin^2(3x)}{(3x)^2}$$

$$\therefore \lim_{x\to 0} = -9$$

(c) $\lim_{x\to 2} \dfrac{\sin(x-2)}{x^2-x-2} = \underline{1/3}$

$$\frac{\sin(x-2)}{(x-2)(x-1)} \quad \therefore \lim_{x\to 2} \frac{\sin(x-2)}{(x-2)} \cdot \lim_{x\to 2} \frac{1}{x+1}$$

$$= 1 \cdot \frac{1}{2+1} = \frac{1}{3}$$

pts: /15

3. Compute the derivatives of the following functions. Each derivative is worth 5 points. Do not simplify your answers.

(*a*) If $y = \pi^2 + x^2 \sin(8x)$ then $y' =$ $2x \sin(8x) + 8x^2 \cos(8x$

$$y' = 2x \sin(8x) + x^2 \cdot \cos(8x) \cdot 8$$

(*b*) If $y = \cos\sqrt{x}$ then $y' =$ $-\frac{\sin(\sqrt{x})}{2\sqrt{x}}$

$$y = \cos(x^{1/2}) \qquad y' = -\sin(x^{1/2}) \cdot \tfrac{1}{2} x^{-1/2}$$

(*c*) If $y = \tan^2 x - \tan(x^2)$ then $y' =$ $2\tan(x)\sec^2(x) - 2x\sec^2(x$

$$y' = 2\tan x \cdot \sec^2 x - \sec^2(x^2) \cdot 2x$$

(*d*) If $y = \frac{\cos x}{x-1}$ then $y' =$ $\frac{-\sin(x)(x-1) - \cos(x)}{(1-x)^2}$

$$y' = \frac{-\sin(x)(x-1) - \cos(x) \cdot 1}{(x-1)^2}$$

pts: /20

4. The volume of a ball is increasing at a rate of 10 cm^3/min.
How fast is the surface area increasing when the radius is 30 cm?

$V = \frac{4}{3}\pi R^2$

$S = 4\pi R^2$

1st

$\frac{dV}{dt} = 4\pi R^2 \frac{dR}{dt}$; $\frac{dS}{dt} = 8\pi R \frac{dR}{dt}$

$\therefore \frac{dR}{dt} = \frac{10}{4\pi\, 30^2} = \frac{1}{360\pi}$ $\quad \therefore \frac{dS}{dt} = 8\cancel{\pi} \cdot 3\cancel{0} \frac{1}{\cancel{360}_{12}\cancel{\pi}} = \boxed{\frac{2}{3}}$

2nd method: $R = \sqrt{S/4\pi}$ $\quad \therefore V = \frac{4}{3}\pi \frac{S^{3/2}}{(4\pi)^{3/2}} = \frac{1}{6\sqrt{\pi}} S^{3/2}$

$\therefore \frac{dV}{dt} = \frac{1}{6\sqrt{\pi}} \frac{3}{2} S^{1/2} \frac{dS}{dt}$ $\quad \therefore \frac{dV}{dt} = \frac{1}{4\sqrt{\pi}} \sqrt{S} \frac{dS}{dt}$

$\therefore \frac{dS}{dt} = \frac{4\sqrt{\pi}}{\sqrt{S}} \frac{dV}{dt} \rightsquigarrow \frac{dS}{dt} = 2/3$

pts: /10

5. Each problem is worth 4 points

(*a*) Find the second derivative of $f(x) = \sqrt{1-x}$.

$f(x) = (1-x)^{1/2}$

$f'(x) = \frac{1}{2}(1-x)^{-1/2} \cdot (-1)$

$f''(x) = \frac{1}{2}(-\frac{1}{2})(1-x)^{-3/2}(-1)(-1)$

$\therefore \boxed{f'' = -\frac{1}{4} \frac{1}{(1-x)}}$

(*b*) If g is a twice differentiable function, find the second derivative of $f(x) = g(x^2+1)$ in terms of g, g', g''.

$f' = g'(x^2+1) \cdot 2x$

$\boxed{f'' = g''(x^2+1) \cdot 4x^2 + g'(x^2+1) \cdot 2}$

pts: /8

[redacted]

6. Calculate the derivatives of the following functions. Each ~~derivative is worth~~ 5 points. Do not simplify your answers.

(*a*) If $F(x) = (x^3 - 5)^3$ then $F'(x) =$ $3(x^3-5)^2 \cdot 3x^2$

(*b*) If $F(x) = \sqrt{x - 4x^5}$ then $F'(x) =$ $\frac{1}{2}(x-4x^5)^{-1/2} \cdot (1-20x^4)$

(*c*) If $F(x) = \sin(\cos(\sin x))$ then $F'(x) =$ $\cos(\cos(\sin(x)))(-\sin(\sin(x))) \cdot \cos(x$

(*d*) If $F(x) = \sin\left(\frac{1-x}{1+x}\right)$ then $F'(x) =$ ________

$$F'(x) = \cos\left(\frac{1-x}{1+x}\right) \cdot \frac{-1(1+x)-(1-x)}{(1+x)^2}$$

$$= \cos\left(\frac{1-x}{1+x}\right) \cdot \frac{-2}{(1+x)^2}$$

pts: /20

7. Each problem is worth 5 points.

(a) Find the equation of the tangent line to the curve $y^3 - 2xy + x^3 = 0$ at the point $P(1,1)$.

$$3y^2 \cdot y' - 2y - 2xy' + 3x^2 = 0$$

$$y'(3y^2 - 2x) = 2y - 3x^2$$

$$y' = \frac{2y - 3x^2}{3y^2 - 2x} \qquad y'\Big|_P = \frac{2-3}{3-2} = -1$$

$$\therefore \quad y - 1 = (-1)(x-1) \quad \text{or} \quad \boxed{y = -x + 2}$$

(b) Express the derivative of y with respect to x in terms of x and y if $y^2 = \dfrac{x-1}{y-1}$.

$$y^2 \cdot (y-1) = x - 1 \iff y^3 - y^2 = x - 1$$

$$\therefore \quad 3y^2 y' - 2y\,y' = 1 \quad \text{or} \quad y' = \frac{1}{3y^2 - 2y}$$

alternative: $$2yy' = \frac{(y-1) - (x-1)y'}{(y-1)^2}$$

$$2y(y-1)^2 y' = (y-1) - (x-1)y'$$

$$y'\left[2y(y-1)^2 + (x-1)\right] = y - 1$$

$$y' = \frac{y-1}{2y(y-1)^2 + (x-1)}$$

they are the same!

pts: /10

8. Each part is worth 4 points.

(*a*) Find the linearization $L(x)$ of $f(x) = \sqrt[3]{x}$ at $a = 27$.

$$f(x) = x^{1/3} \qquad f'(x) = \frac{1}{3} x^{-2/3} = \frac{1}{3x^{2/3}}$$

$$L(x) = \sqrt[3]{27} + \frac{1}{3} \frac{1}{\left(\sqrt[3]{27}\right)^2} (x-27)$$

$$= 3 + \frac{1}{3(3^2)} (x-27)$$

$$= 3 + \frac{1}{27} (x-27)$$

(*b*) Estimate the value of $\sqrt[3]{28}$.
Note: A calculator solution is not an acceptable answer.

$$\sqrt[3]{28} \cong L(28) = 3 + \frac{1}{27} (28-27)$$

$$= 3 + \frac{1}{27} \cdot 1 = \frac{82}{27}$$

$$= 3.037$$

$$\sqrt[3]{28} = 3.0366$$

↑ calculator

pts: /8