MA 113 - Calculus I — Fall 2003 — Name: ______________________ Sec.: ____
FIRST MIDTERM — 09/23/2003

SEC.	INSTRUCTORS	T.A.'S	LECTURES	RECITATIONS
001	A. Corso	D. Watson	MWF 8:00-8:50, CB 110	TR 8:00-9:15, CB 205
002	A. Corso	D. Watson	MWF 8:00-8:50, CB 110	TR 9:30-10:45, CP 103
002	A. Corso	K. Messina	MWF 8:00-8:50, CB 110	TR 9:30-10:45, CP 287
004	U. Nagel	E. Stokes	MWF 10:00-10:50, CP 220	TR 8:00-9:15, BE 206
005	U. Nagel	E. Stokes	MWF 10:00-10:50, CP 220	TR 12:30-1:45, SRB 303
006	U. Nagel	K. Messina	MWF 10:00-10:50, CP 220	TR 3:30-4:45, CP 222

Answer all of the following questions. Use the backs of the question papers for scratch paper. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. When answering these questions, please be sure to:

- check answers when possible,
- clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may receive NO credit*).

QUESTION	SCORE	TOTAL
1.		6
2.		6
3.		25
4.		10
5.		5
6.		8
7.		20
8.		10
9.		10
Bonus.		5
TOTAL		100

1. Given the functions: $f(x) = \sqrt{x^2 - 1}$ and $g(x) = \sqrt{1 - x}$, find a formula for the composition functions $(g \circ f)(x)$ and $(f \circ g)(x)$.

$(g \circ f)(x) =$ $\sqrt{1 - \sqrt{x^2 - 1}}$

domain

$-\sqrt{2}$ -1 1 $\sqrt{2}$

$-\sqrt{2} \le x \le -1$ and

$1 \le x \le \sqrt{2}$

$(f \circ g)(x) =$ $\sqrt{-x}$

0

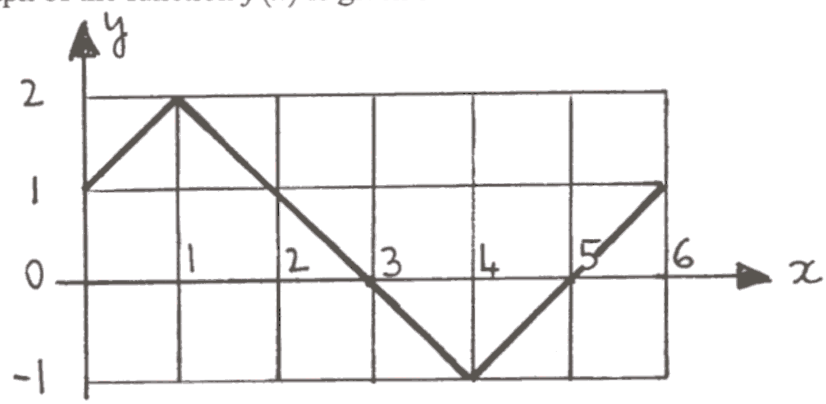
domain

$x \le 0$

the domain was not required

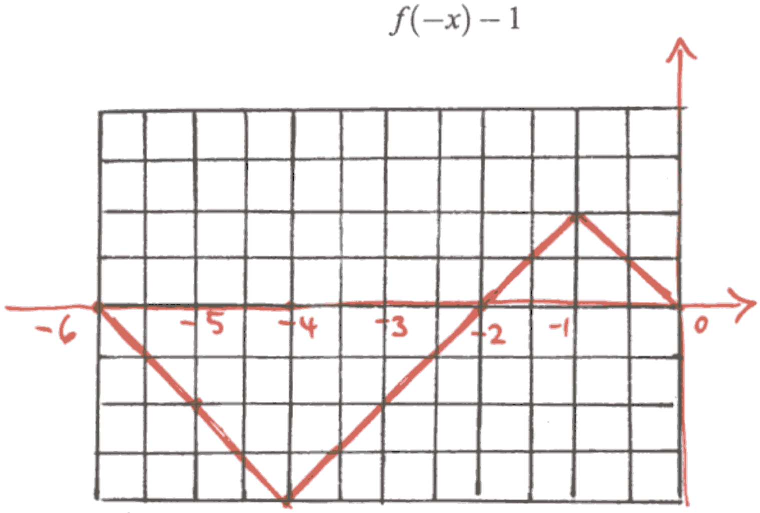
pts: /6

2. The graph of the function $f(x)$ is given below

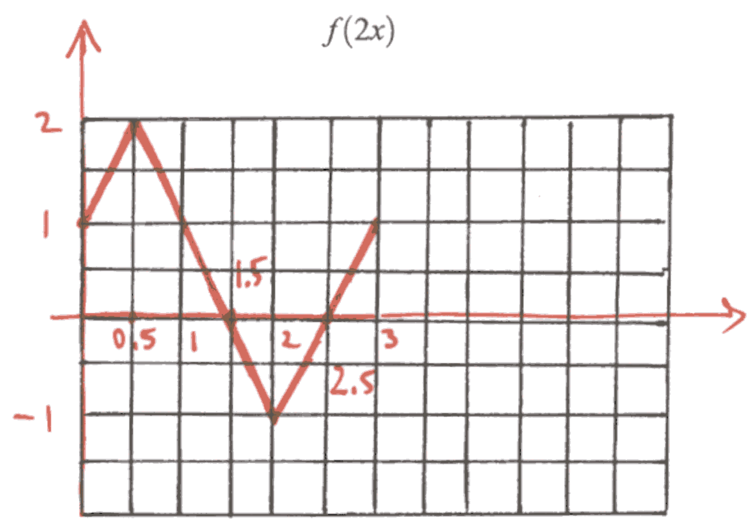


Use the above picture to graph the following functions

$f(-x) - 1$



$f(2x)$



pts: /6

3. Compute the following limits. Each limit is worth 5 points.
Note: Remember to simplify your answers!

(a) $\lim_{x \to -2} \dfrac{x^2+3x+2}{x+2} = \underline{\lim_{x \to -2} (x+1)} = \boxed{-1}$

$$\frac{x^2+3x+2}{x+2} = \frac{\cancel{(x+2)}(x+1)}{\cancel{(x+2)}} = (x+1)$$

(b) $\lim_{s \to a} \dfrac{1-f(s)}{2h(s)+g(s)-2} = \boxed{2/7}$

given that: $\lim_{s \to a} f(s) = -3$, $\lim_{s \to a} g(s) = 0$ $\lim_{s \to a} h(s) = 8$.

$$\frac{1 - \lim_{s \to a} f(s)}{2 \lim_{s \to a} h(s) + \lim_{s \to a} g(s) - 2} = \frac{1-(-3)}{2 \cdot 8 + 0 - 2} = \frac{4}{14}$$

(c) $\lim_{x \to 1} \left[\dfrac{1}{x-1} - \dfrac{2}{x^2-1} \right] = \underline{\lim_{x \to 1} \frac{1}{x+1}} = \boxed{1/2}$

(hint: give common denominators)

$$\frac{1}{x-1} - \frac{2}{(x-1)(x+1)} = \frac{x+1-2}{(x-1)(x+1)} = \frac{\cancel{x-1}}{\cancel{(x-1)}(x+1)}$$

$$= \frac{1}{x+1}$$

(d) $\lim_{x \to 0^+} \dfrac{x-1}{x^2(x+2)} = \boxed{-\infty}$ $\quad \lim_{x \to 0^-} \dfrac{x-1}{x^2(x+2)} = \boxed{-\infty}$

$$\frac{-1}{(0^+)^2 \cdot 2} = -\infty \qquad \frac{-1}{(0^-)^2 \cdot 2} = -\infty$$

(e) $\lim_{h \to 0} \dfrac{(1+h)^2-1}{h} = \underline{\quad 2 \quad}$ $= f'(1)$ when $f(x) = x^2$

so that $f'(x) = 2x$

$$\frac{(1+h)^2-1}{h} = \frac{\cancel{1}+2h+h^2-\cancel{1}}{h} = 2+h$$

$$\therefore \lim_{h \to 0} 2+h = 2$$

pts: /25

4. (a) (5 pts.) Find the value of C that makes the function $h(x)$ continuous for all real numbers

$$h(x) = \begin{cases} 2x - C & x < 2 \\ -Cx^2 + 10 & 2 \leq x \end{cases}$$

(b) (2 pts.) Using the value for C found in part (a), graph the function $h(x)$.

(c) (3 pts.) With your choice for C found in part (a), is the function $h(x)$ differentiable at $x = 2$? Explain.

(a)

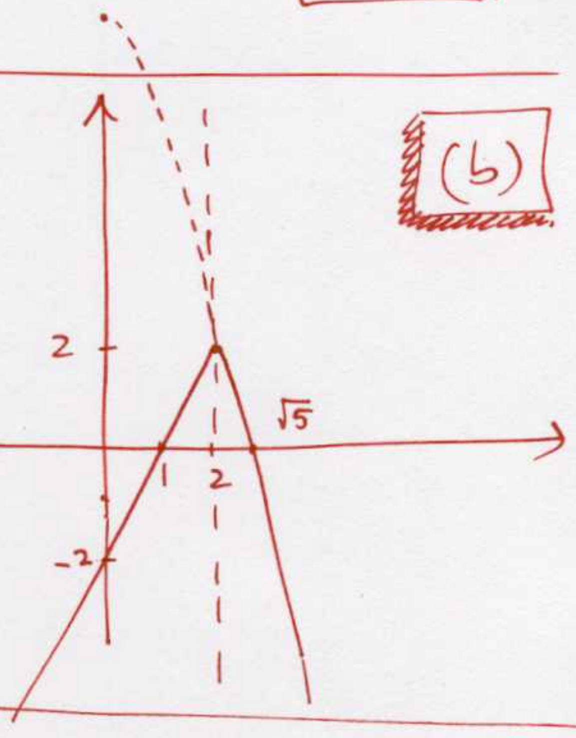
Want $\lim_{x \to 2^-} 2x - C = \lim_{x \to 2^+} -Cx^2 + 10$

$\therefore \quad 4 - C = -4C + 10$ $\quad 4C - C = 10 - 4$

$3C = 6 \quad \boxed{C = 2}$

(b)

$$h(x) = \begin{cases} 2x - 2 & x < 2 \\ -2x^2 + 10 & 2 \leq x \end{cases}$$



(c)

The function is not differentiable at $x = 2$ since the derivative does not exist.

Coming from the left it would be $\underline{2}$, whereas coming from the right it would be $\underline{-8}$.

pts: /10

5. Does the equation

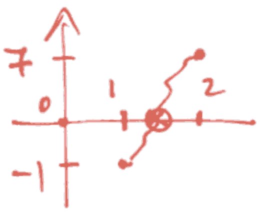
$$2x^3 - x^2 - 3x + 1 = 0$$

have a root c between 1 and 2? Explain. (**Note:** A calculator solution is not an acceptable answer.)

Let $f(x) = 2x^3 - x^2 - 3x + 1$ on $1 \le x \le 2$.

This function is continuous on that interval

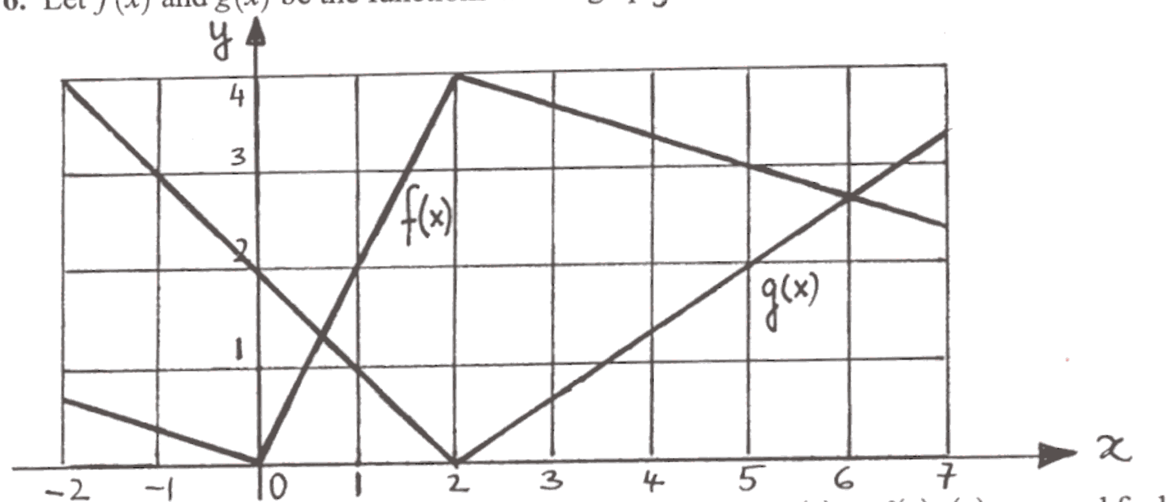
and $f(1) = 2 - 1 - 3 + 1 = \underline{\underline{-1}}$ & $f(2) = 16 - 4 - 6 + 1 = \underline{\underline{7}}$



By the intermediate value theorem there exists $1 < c < 2$ s/t $\boxed{f(c) = 2c^3 - c^2 - 3c + 1 = 0}$

pts: /5

6. Let $f(x)$ and $g(x)$ be the functions whose graphs are shown below



Define the functions $u(x) = f(x) - 2g(x)$ $v(x) = f(x)g(x)$ and find

$u'(5) = \boxed{-5/3}$

$v'(1) = 2 \cdot 1 + 2(-1) = \boxed{0}$

$u'(5) = f'(5) - 2g'(5) = -\frac{1}{3} - \frac{4}{3} = \boxed{-\frac{5}{3}}$

$f'(5) = -1/3$

$g'(5) = \frac{2}{3}$

$v'(1) = f'(1)g(1) + f(1)g'(1)$

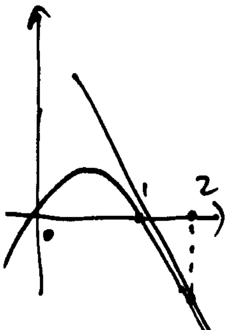
$f(1) = 2$ $f'(1) = 2$

$g(1) = 1$ $g'(1) = -1$

pts: /8

7. Each problem is worth 5 points. Do not simplify your answers in $(b)-(d)$.

(a) Find the equation of the tangent line to the graph of the function $f(x)=x-x^2$ at the point on the graph with $x=2$.



$f'(x) = 1-2x \qquad f'(2) = -3 \qquad f(2) = 2-4 = -2$

$\boxed{(y+2) = -3(x-2)}$ or $\boxed{y = -3x+4}$

(b) If $F(u)=\dfrac{1+u^2}{1-u^2}$ then $F'(u)=$ $\dfrac{4u}{(1-u^2)^2}$

$$\frac{2u(1-u^2)-(1+u^2)(-2u)}{(1-u^2)^2} = \frac{2u - \cancel{2u^3} + 2u + \cancel{2u^3}}{(1-u^2)^2}$$

(c) If $g(x)=(2x^2+1)(2-x^2-\frac{1}{4}x^4)$ then $g'(x)=$ $4x(2-x^2-\frac{1}{4}x^4)+(2x^2+1)\cdot(-2x-x^3$

(d) If $h(x)=\dfrac{x^2-\sqrt{x}+1}{\sqrt{x}}$ then $h'(x)=$ $\dfrac{3}{2}\sqrt{x}-\dfrac{1}{2x\sqrt{x}} = \dfrac{3x^2-1}{2x\sqrt{x}} = \dfrac{(2x-\frac{1}{2\sqrt{x}})\sqrt{x} - (x^2-\sqrt{x}+1)\frac{1}{2\sqrt{x}}}{x}$

$$h(x) = \frac{x^2}{\sqrt{x}} - 1 + \frac{1}{\sqrt{x}} = x^{3/2} - 1 + x^{-1/2}$$

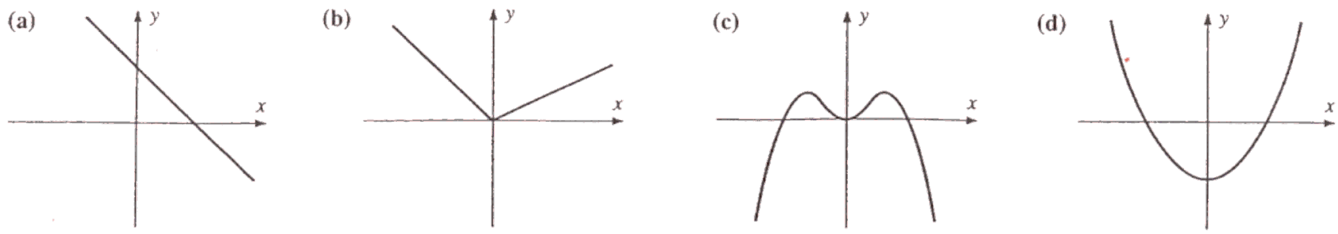
$$h'(x) = \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-3/2} = \frac{3}{2}\sqrt{x} - \frac{1}{2x\sqrt{x}} = \frac{3x^2-1}{2x\sqrt{x}}$$

or

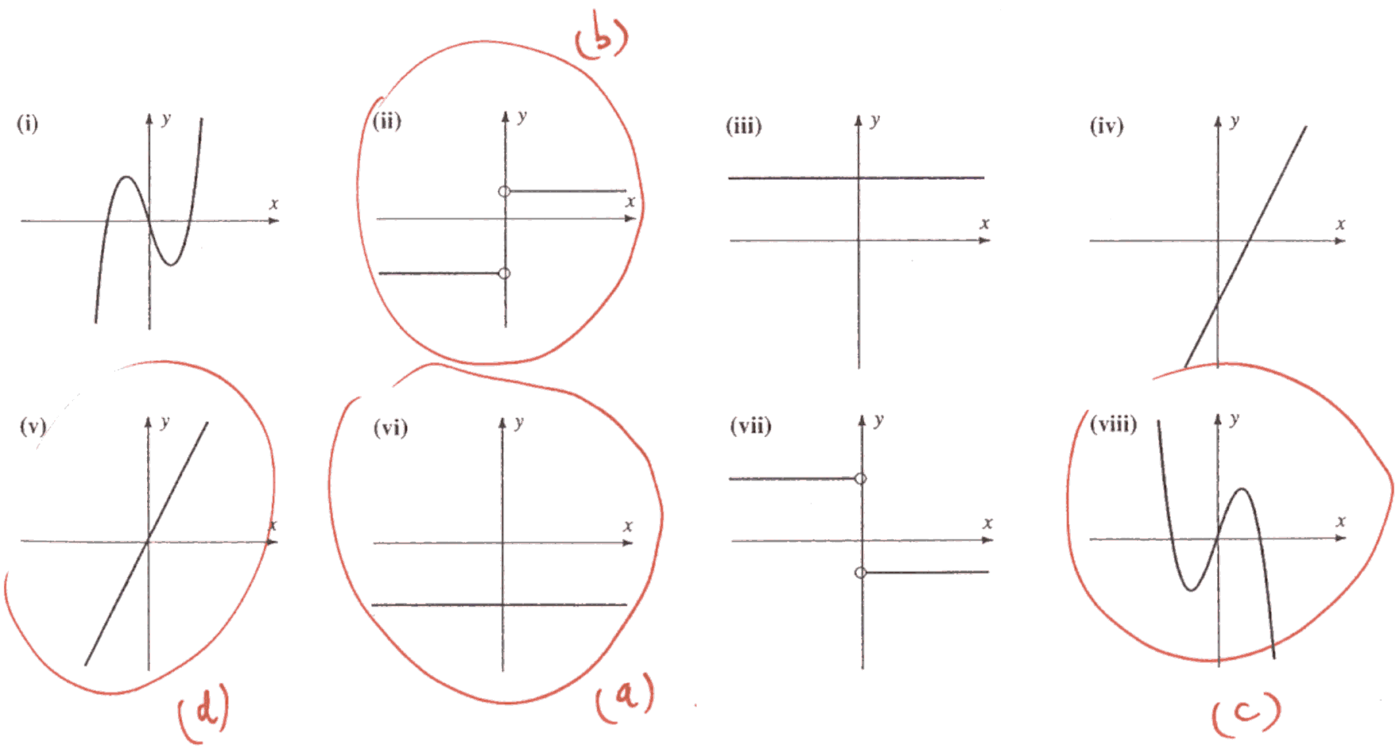
$$h'(x) = \frac{(2x-\frac{1}{2\sqrt{x}})\sqrt{x} - (x^2-\sqrt{x}+1)\frac{1}{2\sqrt{x}}}{x} = \frac{2x\sqrt{x} - \cancel{\frac{1}{2}} - \frac{x}{2}\sqrt{x} + \cancel{\frac{1}{2}} - \frac{1}{2\sqrt{x}}}{x} = \frac{4x^2-x^2-1}{2x\sqrt{x}}$$

pts: /20

8. For each function graphed in figures $(a) - (d)$



indicate which of the graphs in figures $(i) - (viii)$ is the graph of its derivative.



Answers:

$f(x)$	(a)	(b)	(c)	(d)
$f'(x)$	(vi)	(ii)	(viii)	(v)

pts: /10

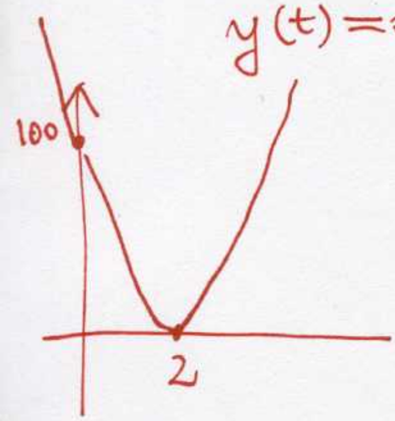
9. A spaceship approaching touchdown on the planet *Gzyx* has height y (meters) at time t (seconds) given by

$$y = 100 - 100t + 25t^2.$$

(a) When does the spaceship hit the ground?

(b) With what velocity does it hit the ground?

(a)



$y(t) = 0$ $\quad 100 - 100t + 25t^2 = 0 \iff$

$4 - 4t + t^2 = 0 = (t-2)^2$

$\boxed{t = 2}$

(b)

$y' = -100 + 50t$

$y'(2) = -100 + 50 \cdot 2 = \boxed{0}$

pts: /10

Bonus. Observe that if $f(x) = x^4 + x^2$ then $f'(x) = 4x^3 + 2x$ is an odd function. Similarly, if $g(x) = \frac{x^2}{x^2+1}$ then $g'(x) = \frac{2x}{(x^2+1)^2}$ is again an odd function. These and other examples suggest that the derivative of an even function is an odd function.

Using the definition of derivative, **prove** that if $f(x)$ is an even function then its derivative $f'(x)$ is an odd function.

as $f(\cdot)$ is even

$$f'(-x) = \lim_{h \to 0} \frac{f(-x+h) - f(-x)}{h} = \lim_{h \to 0} \frac{f(x-h) - f(x)}{h}$$

$$= \lim_{-h \to 0} -\frac{f(x-h) - f(x)}{-h} = -f'(x)$$

as $h \to 0$ then $-h \to 0$ as well

$\therefore \boxed{f'(-x) = -f'(x)}$

pts: /5