MA 113 - Calculus I — Fall 2003
SECOND MIDTERM — 10/21/2003

Name: ______________________ Sec.: ____

SEC.	INSTRUCTORS	T.A.'S	LECTURES	RECITATIONS
001	A. Corso	D. Watson	MWF 8:00-8:50, CB 110	TR 8:00-9:15, CB 205
002	A. Corso	D. Watson	MWF 8:00-8:50, CB 110	TR 9:30-10:45, CP 103
003	A. Corso	K. Messina	MWF 8:00-8:50, CB 110	TR 2:00-3:15, CP 287
004	U. Nagel	E. Stokes	MWF 10:00-10:50, CP 220	TR 8:00-9:15, BE 206
005	U. Nagel	E. Stokes	MWF 10:00-10:50, CP 220	TR 12:30-1:45, SRB 303
006	U. Nagel	K. Messina	MWF 10:00-10:50, CP 220	TR 3:30-4:45, CP 222

Answer all of the following questions. Use the backs of the question papers for scratch paper. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. When answering these questions, please be sure to:

- check answers when possible,
- clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may receive NO credit*).

QUESTION	SCORE	TOTAL
1.		9
2.		15
3.		20
4.		8
5.		10
6.		20
7.		10
8.		8
Bonus.		5
TOTAL		100

1. A particle is moving on a line such that its position after t hours is

$$s(t) = 30t - t^3 + 50.$$

(*a*) (3 pts) Determine the velocity of the particle.

$v(t) = s'(t) = \underline{30 - 3t^2}$

(*b*) (3 pts) When does the particle change its direction?

$v(t) = 3(10 - t^2) = 30(\sqrt{10} - t)(\sqrt{10} + t) \geq 0$ if and only if $\sqrt{10} \geq t \geq -\sqrt{10}$.

The particle changes its direction after $\underline{\sqrt{10}\ h}$.

(*c*) (3 pts) Find the acceleration of the particle after 5 min.

The acceleration is $a(t) = v'(t) = -6t$

$5 \text{ min} = \frac{1}{12} h$, thus $a(\frac{1}{12}) = -\frac{1}{12} \cdot 6 = -\frac{1}{2}$.

The acceleration after 5 min is $\underline{-\frac{1}{2}\ \frac{1}{h^2}}$.

pts: /9

2. Compute the following limits. Each limit is worth 5 points.
Note: Remember to simplify your answers!

(*a*) $\displaystyle\lim_{x\to-\pi/6} \frac{\cos^2(-2x)}{3\sin(x)} = \underline{\quad -\frac{1}{6} \quad}$

$$= \frac{\cos^2(\frac{\pi}{3})}{3\sin(-\frac{\pi}{6})} = \frac{(\frac{1}{2})^2}{3\cdot(-\frac{1}{2})} = -\frac{1}{6}$$

(*b*) $\displaystyle\lim_{x\to 0} \left(\frac{\cos(3x)-1}{3x}\right)^2 = \underline{\quad 0 \quad}$

$$= \left(\lim_{t\to 0} \frac{\cos(t)-1}{t}\right)^2 = (\cos'(0))^2 = (-\sin(0))^2 = 0$$

(*c*) $\displaystyle\lim_{x\to 1} \frac{\sin(x-1)}{x^2-1} = \underline{\quad \frac{1}{2} \quad}$

$$= \left(\lim_{x\to 1} \frac{\sin(x-1)}{x-1}\right)\left(\lim_{x\to 1} \frac{1}{x+1}\right) = 1\cdot\frac{1}{2} = \frac{1}{2}$$

pts: /15

3. Compute the derivatives of the following functions. Each derivative is worth 5 points.
Do **not** simplify your answers.

(a) If $f(x) = 5 + x^3 \cos(4x)$ then $f'(x) =$ $3x^2 \cdot \cos(4x) + x^3 \cdot (-\sin(4x)) \cdot 4$

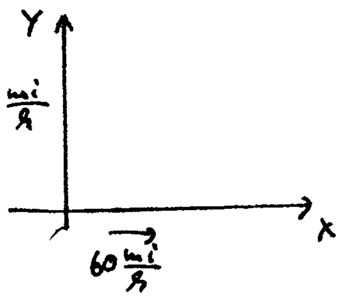
(b) If $f(x) = \sin(\sqrt[3]{x})$ then $f'(x) =$ $\cos(\sqrt[3]{x}) \cdot \frac{1}{3} \frac{1}{\sqrt[3]{x^2}}$

(c) If $f(x) = \sin^4(x) - \sin(x^4)$ then $f'(x) =$ $4\sin^3(x) \cdot \cos(x) - \cos(x^4) \cdot 4x^3$

(d) If $f(x) = \dfrac{\sin(x-1)}{x+2}$ then $f'(x) =$ $\dfrac{\cos(x-1) \cdot (x+2) - \sin(x-1)}{(x+2)^2}$

pts: /20

4. A Ford and a Chrysler car are leaving an intersection at the same time. The Ford is travelling east at $60\frac{\text{mi}}{\text{h}}$ and the Chrysler is travelling north at $80\frac{\text{mi}}{\text{h}}$. At what rate is the distance between the cars increasing 30 min later?



Let $x(t)$ be the distance of the Ford to the intersection at t and let $y(t)$ be the distance of the Chrysler to the intersection at t.

Then $x(t) = 60t$ and $y(t) = 80t$.

The distance between the cars is $z(t) = \sqrt{x^2(t) + y^2(t)}$

$= \sqrt{60^2 t^2 + 80^2 t^2} = \sqrt{3600 + 6400} \cdot t = 100t.$

Hence $z'(t) = 100$. The distance is increasing at $\underline{100 \frac{\text{mi}}{\text{h}}}$.

pts: /8

5. Find the second derivative of the following functions. Each problem is worth 5 points

(a) $f(x) = \sqrt[3]{2-3x}$

$$f'(x) = \frac{1}{3}(2-3x)^{-\frac{2}{3}} \cdot (-3) = -(2-3x)^{-\frac{2}{3}}$$

$$f''(x) = \frac{2}{3}(2-3x)^{-\frac{5}{3}} \cdot (-3) = \underline{-\frac{2}{\left(\sqrt[3]{2-3x}\right)^5}}$$

(b) $f(x) = \sin(1-x^2)$.

$$f'(x) = \cos(1-x^2) \cdot (-2x)$$

$$f''(x) = -\sin(1-x^2) \cdot (-2x)^2 + \cos(1-x^2)(-2)$$

$$= \underline{-2\left[2x^2 \cdot \sin(1-x^2) + \cos(1-x^2)\right]}$$

pts: /10

6. Calculate the derivatives of the following functions. Each derivative is worth 5 points. Do **not** simplify your answers.

(*a*) If $f(x) = (x^7 - 2)^{100}$ then $f'(x) = \underline{100 \cdot (x^7-2)^{99} \cdot 7x^6}$

(*b*) If $f(x) = \sqrt[5]{2x + 3x^4}$ then $f'(x) = \underline{\frac{1}{5}(2x+3x^4)^{-\frac{4}{5}} \cdot (2+12x^3)}$

(*c*) If $f(x) = \cos\left(\sin(x^6)\right)$ then $f'(x) = \underline{-\sin(\sin(x^6)) \cdot \cos(x^6) \cdot 6x^5}$

(*d*) If $f(x) = \cos\left(\frac{1+x}{1-x}\right)$ then $f'(x) = \underline{-\sin\left(\frac{1+x}{1-x}\right) \cdot \frac{1-x+(1+x)}{(1-x)^2}}$

pts: /20

7. Each problem is worth 5 points.

(*a*) Find the equation of the tangent line to the curve $y^2 = x^3 + 3x^2$ at the point $P(1,-2)$.

Write $y = f(x)$ near $P(1,-2)$. Then $2\cdot f(x)\cdot f'(x) = 3x^2 + 6x$, thus $f'(x) = \dfrac{3x^2+6x}{2\cdot f(x)}$,

$f'(1) = \dfrac{3\cdot 1^2 + 6\cdot 1}{2\cdot(-2)} = -\dfrac{9}{4}$. Hence the equation of the tangent at $P(1,-2)$ is

$y+2 = -\frac{9}{4}(x-1)$ or $\underline{y = -\frac{9}{4}x + \frac{1}{4}}$

(*b*) Show that the curves $x^2 - y^2 = 5$ and $4x^2 + 9y^2 = 72$ are orthogonal.
(**Recall** that two curves are called orthogonal if at each point of intersection their tangent lines are perpendicular.)

For the first curve we get $2x - 2yy' = 0$, thus $y' = \frac{x}{y}$.
For the second curve we obtain $8x + 18yy' = 0$, thus $y' = -\frac{4}{9}\frac{x}{y}$.

Now, we compute the points of intersection. Write the equation of the first curve as $x^2 = y^2 + 5$ and substitute in the equation of the second curve:

$$72 = 4(y^2+5) + 9y^2 = 20 + 13y^2, \text{ thus } 52 = 13y^2, \text{ hence } \underline{y^2 = 4}.$$

Then the first equation gives for points of intersection $\underline{x^2} = y^2 + 5 = \underline{9}$.

Therefore, at each point of intersection the product of the slopes of the tangents is

$$\frac{x}{y}\cdot\left(-\frac{4}{9}\frac{x}{y}\right) = -\frac{4}{9}\frac{x^2}{y^2} = -\frac{4}{9}\cdot\frac{9}{4} = -1.$$

Hence the curves are orthogonal.

pts: /10

8. Each part is worth 4 points. Let $f(x) = \sqrt[3]{x}$.

(a) Find the linearization $L(x)$ of $f(x)$ at 1.

$$f'(x) = \frac{1}{3} x^{-\frac{2}{3}}, \quad f'(1) = \frac{1}{3}, \quad f(1) = 1$$

$$L(x) = f(1) + f'(1)\cdot(x-1) = 1 + \frac{1}{3}(x-1) = \underline{\frac{1}{3}x + \frac{2}{3}}$$

(b) Compute the quadratic approximation $Q(x)$ of $f(x)$ at 1.

$$f''(x) = -\frac{2}{3}\cdot\frac{1}{3} x^{-\frac{5}{3}}, \quad f''(1) = -\frac{2}{9}$$

$$Q(x) = L(x) + \frac{1}{2} f''(1)\cdot(x-1)^2 = \frac{1}{3}x + \frac{2}{3} - \frac{1}{9}(x^2 - 2x + 1)$$

$$= \underline{-\frac{1}{9}x^2 + \frac{5}{9}x + \frac{5}{9}}$$

pts: /8

Bonus. Let f be an even function that is twice differentiable. **Prove** that $f'(0) = 0$.

Since f is even, we have $f(x) = f(-x)$. Thus, the chain rule provides $f'(x) = -f'(-x)$. It follows

$$f'(0) = -f'(0),$$

thus $2\cdot f'(0) = 0$, hence $f'(0) = 0$, as claimed.

pts: /5