Answer all of the following questions. Use the backs of the question papers for scratch paper. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. When answering these questions, please be sure to:

- check answers when possible,
- clearly indicate your answer and the reasoning used to arrive at that answer
  *(unsupported answers may receive NO credit)*.

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1. Find $dy/dx$ for each of the following functions:

   (a) $y = \tan^{-1}(e^{3x})$
   
   $dy/dx =$ ______________________

   (b) $y = x^{\sin x}$
   
   $dy/dx =$ ______________________

   (c) $y = \ln(\ln(x^2))$
   
   $dy/dx =$ ______________________

pts: /10
2. Evaluate the following integrals. Each problem is worth 5 points.

(a) \[ \int \frac{e^{2x}}{1 + e^{2x}} \, dx = \] 

(b) \[ \int \frac{x^2 + 2x + 3}{(x^2 + 1)(x + 1)} \, dx = \] 

(c) \[ \int x^2 \ln x \, dx = \] 

\[ \text{pts: } /15 \]
2.(cont.d)

(d) \[ \int \frac{x^2}{\sqrt{1-x^2}} \, dx = \] 

(e) \[ \int x \sqrt{1+x} \, dx = \] 

(f) \[ \int \frac{3x^4 + 2x^2 + x - 1}{1 + x^2} \, dx = \] 

pts: /15
3. An isotope of strontium, Sr$^{90}$, has a half-life of 25 years.

(a) Find the mass $Q(t)$ of Sr$^{90}$ that remains from a sample of 18 mg after $t$ years.

(b) How long would it take for the mass to decay to 2 mg?

4. (5 pts each) Find the limits of the following sequences:

(a) $a_n = \left(1 - \frac{1}{n}\right)^n$;

(b) $a_n = (-1)^n \frac{n+1}{n}$;
5. (5 pts each) Determine if the following series are absolutely convergent, conditionally convergent, or divergent.

(a) \[ \sum_{n=2}^{\infty} (-1)^n \frac{1}{\sqrt{2} + n}; \]

(b) \[ \sum_{n=1}^{\infty} \frac{n^2}{4^n}; \]

(c) \[ \sum_{n=1}^{\infty} \frac{\sin(n)}{n^3}. \]
6. (a) Find the interval of convergence of the power series

\[
\sum_{n=0}^{\infty} \frac{3n}{2^n} x^{2n}.
\]

(b) Find the 4th degree Taylor polynomial centered at \( a = -1 \) for \( f(x) = \ln(2 + x) \).
7. (a) Find the area bounded by the curve

\[ x(t) = \cos t \quad y(t) = e^t \quad 0 \leq t \leq \pi/2, \]

and the lines \( y = 1 \) and \( x = 0 \).

(b) Find the length of the curve

\[ x(t) = e^t - t \quad y(t) = 4e^{t/2} \quad 0 \leq t \leq 1. \]
8. (a) Sketch the curve

\[ x(t) = t^2 \quad y(t) = t^3 - t \]

(b) Find the coordinates of the point where the curve crosses itself.

(c) Find the equations of the tangent lines at the point in part (b).
9. Solve the initial value problem \( \frac{dy}{dx} = xy + x \quad y(1) = 0. \)

**Bonus.** Sketch carefully the graphs of the following equations given in polar coordinates

\[ r = 1 - \cos \theta \quad r = \cos \theta \]

Label and give the coordinates of all the points of intersection.

Why not check all your work?