MATH 115 - FINAL - DECEMBER 18, 1995

Fall 95–Sections 27-30 Corso – Jones	Name:

BE NEAT - SHOW ALL YOUR WORK - CIRCLE YOUR ANSWER

The addition and subtraction formulas

- $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$;
- $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$;
- $\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$.

Product identities

- $\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta$;
- $\sin(\alpha+\beta) - \sin(\alpha-\beta) = 2\cos\alpha\sin\beta$;
- $\cos(\alpha-\beta) + \cos(\alpha+\beta) = 2\cos\alpha\cos\beta$;
- $\cos(\alpha-\beta) - \cos(\alpha+\beta) = 2\sin\alpha\sin\beta$;

Double-angle identities

- $\sin 2\alpha = 2\sin\alpha\cos\alpha$;
- $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$;
- $\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$.

Half-angle identities

- $\sin\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{2}}$;
- $\cos\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1+\cos\alpha}{2}}$;
- $\tan\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$,

where the choice of the sign is determined by the quadrant in which the angle $\alpha/2$ is located.

Midpoint formula
Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ then M=midpoint=$((x_1+x_2)/2, (y_1+y_2)/2)$.

Distance formula
Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ then $\text{dist}(P,Q) = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$.

Population growth
$P(t) = P_0 e^{rt} = P_0 2^{t/T}$, where $r > 0$ is the rate of growth and T is the doubling time.

Radioactive decay
$Q(t) = Q_0 e^{rt} = Q_0 (1/2)^{t/T}$, where $r < 0$ is the rate of decay and T is the half-life.

1. Let $A(3,-2)$ and $B(-3,1)$. Write an equation for all points $P(x,y)$ such that

$$(\mathrm{dist}(A,P))^2 = (\mathrm{dist}(B,P))^2 + 1.$$

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total: /100

2. Find b such that the line $3x + by + 2 = 0$ has y-intercept $1/2$.

$\boxed{\quad /5}$

3. Write an equation for the circle such that the points $A(6, 0)$ and $B(0, 8)$ are endpoints of a diameter.

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4. Let $f(x) = x^2 - 6x + 8$. Sketch the graph of $y = f(x)$ using the method of completing the squares (indicate x- and y-intercepts, and the vertex of the parabola).

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5. Sketch the graph of

$$f(x) = \frac{x^2 - 5x + 4}{x^2}$$

(indicate domain, horizontal and vertical asymptotes, x- and y-intercepts, sign variation, etc...).

$\boxed{\quad /5}$

6. Find the domain of

$$F(x) = \sqrt{\frac{1-2x}{3x+2}}.$$

$\boxed{\quad /5}$

7. For $f(x) = -2x^2 + 7x - 1$, find the difference quotient

$$\frac{f(a+h) - f(a)}{h}.$$

$\boxed{\quad /5}$

8. Find the inverse of

$$f(x) = \sqrt{3-x}.$$

Plot both the graph of $f(x)$ and the one of $f^{-1}(x)$ on the same coordinate plane. State the domain and range of both functions.

$\boxed{\quad /5}$

9. Using the various graphing techniques you know, graph the function

$$f(x) = 3^{x-4} - 1.$$

Find the x- and y-intercepts.

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10. Find the x for which

$$\log_{3/2} x = 4; \qquad 4^x = 64; \qquad \log_x(\log_4 256) = 2.$$

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11. The formula $p(h) = 14.7e^{-0.12\,h}$ gives the atmospheric pressure in pounds per square inch at an altitude of h miles above sea level. At what altitude will the atmospheric pressure will be one half of the sea level pressure?

$\boxed{\quad /5}$

12. Solve the logarithmic equation

$$\log_3(\log_2(x+3)) = \log_3(\log_2(4x-3)).$$

$\boxed{\quad /5}$

13. – Write in a single expression

$$4\log_3(x+1) - 2\log_3(x-1) - \frac{1}{2}\log_3(x+3).$$

– Write in terms of $\log x$, $\log(x+1)$, and $\log(x+2)$

$$\log\sqrt{\frac{x^2(x+1)^3}{(x+2)^5}}.$$

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14. – If a central angle of $30°$ is subtended by a circular arc of length 12m, find the radius of the circle.
   – Convert $\theta = 5$ radians to degrees, minutes, and seconds.

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15. It $\tan\theta = 4$ and $\sin\theta < 0$, find the remaining 5 trigonometric functions.

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16. Find the amplitude, period, and phase shift of

$$y = -2\sin(2(x - \pi/3)).$$

Graph the function and label the x- and y-intercepts.

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17. – Verify the identity

$$\sec u = \frac{\cos u}{1+\sin u} + \tan u.$$

– Compute the exact value of

$$\frac{\cos(\pi/3)}{1+\sin(\pi/3)} + \tan(\pi/3).$$

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18. Find the exact values of

$$\sin(105^\circ), \qquad \tan\left(\frac{\pi}{8}\right),$$

$$\csc\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right).$$

$\boxed{\quad /5}$

19. Solve the equation

$$\sin(2u) - \sqrt{3}\cos u = 0$$

in the interval $[0, 2\pi]$.

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20. Solve the triangle ABC if

$$\beta = 60^\circ, \qquad \gamma = 45^\circ, \qquad a = 8.$$

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21. Two joggers starting from the same point run along two directions that make an angle of $24°$. One of them runs at 8mph and the other at 10mph. How far will they be after 2 hours?

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22. **EXTRA CREDIT: Heron's area formula**

If a, b, and c are the lengths of the sides of a triangle, then the area A of the triangle is given by

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where $s = 1/2(a+b+c)$. Why? (Hint: it can be derived using the law of the cosines.)

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