Ma123 Exam I
February 11, 2004

Instructions:
All submitted work is considered part of your answer. Be sure to cross out, erase, or otherwise indicate work that you do not want graded.

Show all work and explain your answers.
Unsupported answers will receive no credit.

There are 9 problems on 8 pages (including this cover page). Each problem counts 10 points. Check that you have a complete exam.

NO QUESTIONS WILL BE ANSWERED DURING THE EXAM

Fill in the information below and put your name or initials on each of the other pages.
Name: ______________________
Instructor: ___________________
Section Number/Class Meeting Time: ______________________

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Test Total ___ of 90

Homework ___ of 10

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1. Determine each of the following limits or show that they do not exist.

a. \( \lim_{{x \to -3}} \frac{x^2 + 4x + 3}{x + 3} = \lim_{{x \to -3}} \frac{(x+3)(x+1)}{x+3} = \lim_{{x \to -3}} x + 1 = -3 + 1 = -2 \)

b. \( \lim_{{x \to -1}} \frac{x^2 + 4x + 3}{x + 3} = \frac{(-1)^2 + 4(-1) + 3}{-1 + 3} = \frac{1 - 4 + 3}{2} = 0 \)

\[ \]

\[ \]

\[ \]

c. \( \lim_{{x \to \infty}} \sqrt{\frac{4x^4 - 10x^3 - 1}{100x^4 - 7x + 5}} = \sqrt{\lim_{{x \to \infty}} \frac{4x^4 - 10x^3 - 1}{100x^4 - 7x + 5}} = \sqrt{\frac{4}{100}} \)

\[ = \frac{2x}{10} = \frac{1}{5} \]

\[ \]

\[ \]

d. \( \lim_{{x \to \infty}} \frac{5x^3 - 10x - 6}{2x^2 + 3} = \infty \) as the degree of the numerator is larger than the one of the denominator

\[ \]

\[ \]

e. \( \lim_{{x \to 0}} \frac{5x^3 - 10x - 6}{2x^2 + 3} = -\frac{6}{3} = -2 \)
2. Find both coordinates of the points of intersection of the graphs of \( y = 4x^2 + 5x - 10 \) and \( y = x^2 - x + 14 \). Make sure you show your work.

\[
4x^2 + 5x - 10 = x^2 - x + 14 \quad \implies \quad 3x^2 + 6x - 24 = 0
\]

\[
\implies x^2 + 2x - 8 = 0 \quad \implies (x + 4)(x - 2) = 0
\]

\( x = -4, x = 2 \)

When \( x = 2 \), \( y = (2)^2 - 2 + 14 = 11 \)

When \( x = -4 \), \( y = (-4)^2 - (-4) + 14 = 34 \).

3. Compute the equation for the tangent line to the graph of \( f(x) = 5x^3 - 4x^2 + x + 6 \) at \( x = -2 \).

\[
f(-2) = 5(-2)^3 - 4(-2)^2 + (-2) + 6 = -40 - 16 - 2 + 6 = \sqrt{-52}
\]

\[
f'(x) = 15x^2 - 8x + 1 \quad \implies \text{slope} = f'(-2) = 15 \cdot 4 + 16 + 1
\]

\[
\text{y} + 52 = 77(x + 2)
\]

4. The equation of the tangent line to the graph of \( f(x) \) at \( x = 2 \) is \( y = 3x - 7 \).

a. The tangent line meets the x-axis at the point \( \left( \frac{7}{3}, 0 \right) \).

b. The tangent line meets the y-axis at the point \( (0, -7) \).

c. \[ f(2) = \frac{-1}{3} \]

\[ f'(2) = 3 \]
5. Determine the unknowns in each of the following:

a. Determine the unknown \( A \) so that 
\[
\lim_{x \to \infty} \frac{3x^2 - 10x + 1}{Ax^2 + 5x + 27} = 12
\]

\[
\lim_{x \to \infty} \frac{3x^2 - 10x + 1}{Ax^2 + 5x + 27} = \frac{3}{A} = 12
\]

\( \rightarrow A = \frac{3}{12} = \frac{1}{4} \)

b. Determine the value \( B \) for which the piecewise defined function 
\( f(x) = \begin{cases} 
B - x^2 & x < 1 \\
2(x - B) & 1 \leq x
\end{cases} \) is continuous at \( x = 1 \).

\[
f(1) = 2(1 - B) = \lim_{x \to 1^-} (B - x^2) = B - 1
\]

\( \therefore 2 - 2B = B - 1 \quad 3B = 3 \quad \therefore B = 1 \)

c. Find \( C \) so that the function \( g(x) = Cx^2 + 3x + 5 \) satisfies \( g'(2) = 11 \)

\[
g'(x) = 2Cx + 3
\]

\[
g'(2) = 4C + 3 = \text{WANT} \quad 11
\]

\( \therefore 4C = 11 - 3 \)

\( C = \frac{8}{4} = 2 \)
6. Do the following calculations. For (a) and (b) you do not need to simplify.

a. If \( f(x) = 5x^4 - \frac{x^2}{6} + 2x + 7 \) then \( f'(x) = \frac{5}{6} \cdot 4x^3 - \frac{1}{6} \cdot 2x + 2 \).

b. If \( g(x) = 14x^3 + 10x^{-2} + \frac{3}{x} - 7 \) then \( g'(x) = 14 \cdot 3x^2 + 10 \cdot (-2)x^{-3} = -\frac{3}{x^2} \).

Note: \( \frac{3}{x} = 3x^{-1} \) so its derivative is \( -3 \cdot x^{-2} = -\frac{3}{x^2} \).

c. If \( h(x) = x^{\frac{5}{3}} \) then \( h'(8) = \)

\[
h'(x) = \frac{5}{3} x^{\frac{5}{3} - 1} = \frac{5}{3} x^{\frac{2}{3}}
\]

\[
h'(8) = \frac{5}{3} \cdot 8^{\frac{2}{3}} = \frac{5}{3} \left( \sqrt[3]{8} \right)^2 = \frac{5}{3} \cdot 2^2 = \frac{20}{3}
\]
7. Consider the function $f(x) = \begin{cases} 
-x - 3 & x \leq -2 \\
-1 & -2 < x \leq 1 \\
-2x + 6 & 1 < x 
\end{cases}$

a. Sketch the graph of $f(x)$ using the graph paper below.

b. Give all the values of $x$ at which $f(x)$ is **not** continuous. **Explain your answer.** Your answer counts one point; your explanation counts two.

\[ \text{it is not continuous at } x = 1 \]

since there is a jump

c. Give all the values of $x$ at which $f(x)$ is **not** differentiable. **Explain your answer.** Your answer counts one point; your explanation counts two.

\[ \text{it is not differentiable at } x = 1 \]

since there is a jump in the graph

\[ \text{it is not differentiable at } x = -2 \]

since there is an angular point
8. Consider the function $g(x) = x^2 - 3x + 1$ and the points A and B on the graph of $g(x)$ with x-coordinate 0 and 6 respectively. The sketch given below is not to scale.

\[ g(0) = 1 \quad g(6) = 36 - 18 + 1 = 19 \]

\[ \therefore A(0, 1) \quad B(6, 19) \]

\[ \text{slope} = \frac{19 - 1}{6 - 0} = \frac{18}{6} = 3 \]

b. Compute a number $c$ such that the tangent line to the graph of $g(x)$ at $x = c$ is parallel to the secant line connecting A and B.

\[ \text{slope of } g \text{ line at } c = g'(c) = 2c - 3 \]

\[ \therefore 2c = 6 \]

\[ \therefore c = 3 \]
9. The diagram below is a sketch of the graph of the function \( f(x) \). From the graph estimate each of the following.

a. \( f(2) = 3 \)

b. \( \lim_{x \to 2^+} f(x) = 1 \)

c. \( f'(0) = 0 \)

d. \( \lim_{x \to -3} f(x) = -\infty \)

e. \( \lim_{x \to -\infty} f(x) = 0 \)