Ma123 Exam 3
April 16, 2003

Instructions:
All submitted work is considered part of your answer. Be sure to cross out, erase, or otherwise indicate work that you do not want graded.

Show all work and explain your answers. Unsupported answers will receive no credit.

There are 9 problems on 6 pages (including this cover page). Each problem counts 10 points. Check that you have a complete exam.

NO QUESTIONS WILL BE ANSWERED DURING THE EXAM

Fill in the information below and put your name or initials on each of the other pages.

Name: ______________________________________

Instructor: ___________________________________

Section Number/Class Meeting Time: ______________

<table>
<thead>
<tr>
<th>Problem</th>
<th>Score</th>
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\begin{bmatrix}
\text{Homework} \\
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= \begin{bmatrix}
16 \\
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\end{bmatrix}
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3. Find the values of A and B in the following:

a. Find the value of A so that $f(x) = Ax^3 - 12x^2$ has a critical point at $x = 1$.

b. Find the value of B so that $f(x) = Bx^3 - 12x^2 + 7x$ has a point of inflection at $x = -1$.

Problem from Exam II - March 12, 2003

4. A computer store can buy packages of CD blanks for $1.00 per package. If they charge $3.00 per pack then they can sell 600 packages per month. For each $.10 that they reduce the selling price they can sell an additional 60 packages per month. At what price should they sell the packages in order to realize the maximum profit?

$x = \# \text{ of times I will reduce the price of 10 cents}$

$0.1\cdot x = \text{reduction in price in dollars}$

Price charged per package $= 3 - 0.1x$

Net profit per package $= 3 - 0.1x - \frac{1}{10}$

$\# \text{ of packages sold} = 600 + 60x$

$$\text{Profit} = (2 - 0.1x)(600 + 60x) = -6x^2 + 60x + 1200$$

$0 \leq x \leq 30$ (at most I can reduce the price 30 times)

Critical #: $P'(x) = -12x + 60 = 0 \rightarrow [x = 5]$

<table>
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<tr>
<th>$x$</th>
<th>0</th>
<th>30</th>
<th>5</th>
<th>$P(x)$</th>
<th>1200</th>
<th>-2400</th>
<th>1350</th>
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<tr>
<td>$P'(x)$</td>
<td>1200</td>
<td>-2400</td>
<td>1350</td>
<td>Best selling price</td>
<td>2.5 $</td>
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1. Evaluate each of the following:

a. \[ \int (x^2 - 3x + 7) \, dx = \frac{1}{3} x^3 - \frac{3}{2} x^2 + 7x + C \]

b. \[ \int \frac{2x + 3}{x^2} \, dx = \int \left( \frac{2x}{x^2} + \frac{3}{x^2} \right) \, dx = \int \left( 2 \cdot \frac{1}{x} + 3 \cdot x^{-2} \right) \, dx \]
   \[ = 2 \ln|x| + 3 \frac{1}{-2+1} x^{-2+1} + C = 2 \ln|x| - \frac{3}{x} + C \]

c. \[ \int \frac{x}{x^2 + 1} \, dx = \int \frac{1}{2} \cdot \frac{1}{u} \, du = \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln(x^2+1) + C \]

we \[ u = x^2 + 1 \quad du = 2x \, dx \]

and hence \[ \frac{1}{2} \, du = x \, dx \]

d. \[ \int x e^{x^2} \, dx = \frac{1}{2} \int e^u \, du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C \]

we \[ u = x^2 \quad du = 2x \, dx \]

and hence \[ \frac{1}{2} \, du = x \, dx \]

2. For each of the following find the requested derivative:

a. If \( f(x) = e^{x^2+2x+1} \) then \( f'(x) = e^{x^2+2x+1} \cdot 2x+2 = 2(x+1)e^{x^2+2x+1} \)

we use the chain rule: \( f'(x) = e^{g(x)} \quad \Rightarrow \quad f'(x) = e^{g(x)} \cdot g'(x) \)

b. If \( f(x) = \ln(x^2 - x) \) then \( f'(x) = \frac{2x-1}{x^2-x} \)

we use the chain rule

\[ f(x) = \ln(g(x)) \quad \Rightarrow \quad f'(x) = \frac{1}{g(x)} \cdot g'(x) \]
3. Find the area of the region bounded by the graph of \( f(x) = x^2 + 3x + 4 \), the \( x \)-axis, the vertical lines \( x = -1 \) and \( x = 2 \).

4. Suppose \( F(x) \) and \( G(x) \) are two functions such that \( F'(x) = G'(x) \) for all \( x \). If \( F(4) = 5 \), \( G(4) = -3 \), and \( F(1) = -1 \), what is \( G(1) \)?

\[
\begin{align*}
\Rightarrow & \quad G(x) - F(x) = \text{constant} \quad \text{or} \\
& \quad G(x) = F(x) + \text{constant} \\
\text{Now} & \quad G(4) = F(4) + \text{constant} \quad \text{or} \\
& \quad -3 = 5 + \text{constant} \quad \therefore \quad \text{Constant} = -8 \\
\therefore & \quad G(x) = F(x) - 8 \quad \text{and hence} \\
& \quad G(1) = F(1) - 8 = -1 - 8 = -9
\end{align*}
\]
5. Recalling what you know about circles, calculate $\int_{-2}^{2} \sqrt{4-x^2} \, dx$

Cannot do it, yet!

6. At time $t = 0$ seconds an astronaut on the moon throws a baseball straight up. The vertical velocity of the ball $t$ seconds later is given by the formula $v(t) = -6.4t + 128$ feet per second.

a. What is the acceleration due to gravity on the moon?

$$a(t) = v'(t) = -6.4 \text{ feet/sec}^2$$

b. If the ball was 20 feet above the surface when thrown, how high will it be at time $t=10$?

We know that $h(0) = 20$ feet.

From $v(t) = -6.4t + 128$ we get that

$$h(t) = \int v(t) \, dt = -3.2t^2 + 128t + C$$

We find $C$ as $h(0) = 20$.

$20 = h(0) = -3.2 \cdot 0^2 + 128 \cdot 0 + C \implies C = 20$

$h(t) = -3.2t^2 + 128t + 20$ and $h(10) = 980$ feet.
7. Suppose that $f(x)$ is a function for which $f(2) = \ln(3)$ and $f'(2) = -5$. What is the equation of the tangent line to the graph of $h(x) = x e^{f(x)}$ at $x = 2$?

$$P(2, h(2)) = (2, 2 e^{f(2)}) = (2, 2 e^{\ln(3)}) = (2, 6)$$

$h'(x)$ use product and chain rule

$$\frac{1}{1} e^{f(x)} + x \cdot e^{f(x)} \cdot f'(x) \text{ so that}$$

$$h'(2) = \text{slope} = e^{f(2)} + 2 e^{f(2)} \cdot f'(2) = e^{\ln(3)} + 2 e^{\ln(3)} \cdot (-5)$$

$$= 3 + 2 \cdot 3 \cdot (-5) = -27$$

$$\therefore y - 6 = -27(x - 2) \quad \text{or} \quad y = -27x + 60$$

8. Suppose $f(x)$ is a function such that $f'(x) = (1 - x^2)x$ and $f(1) = 2$. Compute $f(3)$.

$$f'(x) = (1 - x^2)x = x - x^3 \quad \text{hence}$$

$$f(x) = \int (x - x^3) \, dx = \frac{1}{2} x^2 - \frac{1}{4} x^4 + C$$

We use the condition $f(1) = 2$ to find $C$.  

$$2 = f(1) = \frac{1}{2} 1^2 - \frac{1}{4} 1^4 + C \quad \therefore C = \frac{1}{2} - \frac{1}{4} + \frac{1}{4} \frac{8 - 2 + 1}{4} = \frac{7}{4}$$

Thus $f(x) = \frac{1}{2} x^2 - \frac{1}{4} x^4 + \frac{7}{4}$

and $f(3) = \frac{1}{2} 3^2 - \frac{1}{4} 3^4 + \frac{7}{4} = \frac{18 - 81 + 7}{4} = -\frac{14}{4} = -14$
9. Below is the graph of a function $f(x)$. The area of the region A is 5, the area of the region B is 4, and $\int_{0}^{6} f(x) \, dx = -3$

a. What is $\int_{0}^{4} f(x) \, dx$?

b. What is the area of the region C?

c. What is the value of $\int_{0}^{6} (2f(x) - x) \, dx$?

you cannot do it yet!