Ma123 Exam IV
May 5, 2003

Instructions:
All submitted work is considered part of your answer. Be sure to cross out, erase, or otherwise indicate work that you do not want graded.

Show all work and explain your answers.
Unsupported answers will receive no credit.

There are 9 problems on 7 pages (including this cover page). Each problem counts 10 points.
Check that you have a complete exam.

NO QUESTIONS WILL BE ANSWERED DURING THE EXAM

Fill in the information below and put your name or initials on each of the other pages.

Name: [Answer Key]
Instructor: ____________________________
Section Number/Class Meeting Time: ____________

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| Homework Percentage | 22 | 23 | 24 |

[ ]
1. Calculate each of the following derivatives. It is not necessary to simplify your answers.

a. \[
\frac{d}{dx} \left( \frac{x^2}{1+x^4} \right) = \frac{2x(1+x^4) - x^2(4x^3)}{(1+x^4)^2} = \frac{2x - 2x^5}{(1+x^4)^2}
\]

we used the quotient rule

b. \[f'(7) \text{ where } f(x) = (x-7)e^{(2x-14)}\]

\[
f'(x) = 1 \cdot e^{2x-14} + (x-7) \cdot e^{2x-14} \cdot 2 = 2x-14, \quad \therefore f'(7) = e^{2 \cdot 7 - 14} = e^0 = 1
\]

used the product rule and the chain rule

c. \[h'(2) \text{ where } h(x) = x^2f(x) \text{ with } f(2) = -1 \text{ and } f'(2) = 3\]

\[
h'(x) = 2x \cdot f(x) + x^2 \cdot f'(x) \quad \text{use product rule}
\]

\[
h'(2) = 2 \cdot 2 \cdot f(2) + 2 \cdot f'(2) = 4 \cdot (-1) + 4 \cdot 3 = 8
\]

2. Calculate the equation of the tangent line to the graph of \(f(x) = x^2 + \frac{1}{x}\) at \(x = 2\).

point \(P(2, f(2)) = (2, 2^2 + \frac{1}{2}) = (2, \frac{9}{2})\)

slope: \(f'(2) = \ldots\)

\[f'(x) = 2x - \frac{1}{x^2} \quad \rightarrow \quad f'(2) = 2 \cdot 2 - \frac{1}{4} = 4 - \frac{1}{4} = \frac{15}{4}
\]

\[
y - \frac{9}{2} = \frac{15}{4} (x - 2) \quad \rightarrow \quad y = \frac{15}{4} x - \frac{15}{2} + \frac{9}{2} = \frac{15}{4} x - 3^2
\]
3. Calculate each of the following:

a. \[ \int \frac{e^x}{2 + e^x} \, dx \]

\[ u = 2 + e^x \Rightarrow du = e^x \, dx \]

\[ \int \frac{du}{u} = \ln |u| + C = \ln |2 + e^x| + C \]

b. \[ \int \frac{x^2 + 2}{x} \, dx \]

\[ = \int \left( \frac{x^2}{x} + \frac{2}{x} \right) \, dx = \int \left( x + \frac{2}{x} \right) \, dx = \frac{1}{2} x^2 + 2 \ln |x| + C \]

\[ = 2e(e-1) \]

c. \[ \int_{1}^{4} e^{\frac{\sqrt{x}}{x}} \, dx \]

\[ u = \sqrt{x} = x^{1/2} \Rightarrow \int e^{u} \, du = 2 e^u + C = 2 e^{\sqrt{x}} + C \]

\[ du = \frac{1}{2} x^{-1/2} \, dx = \frac{1}{2 \sqrt{x}} \, dx \Rightarrow 2 \, du = \frac{dx}{\sqrt{x}} \]

4. Find the area of the enclosed region bounded by the curves \( y = 2x + 1 \) and \( y = 5 - 2x^2 \)

Find the intersection points

\[ 5 - 2x^2 = 2x + 1 \Rightarrow 2x^2 + 2x - 4 = 0 \]

\[ x^2 + x - 2 = 0 \Rightarrow (x+2)(x-1) = 0 \]

\[ \text{Area} = \int_{-2}^{1} \left[ (5 - 2x^2) - (2x + 1) \right] \, dx = \int_{-2}^{1} (-2x^2 - 2x + 4) \, dx = \]

\[ = -\frac{2}{3} x^3 - x^2 + 4x \Big|_{-2}^{1} = \left( -\frac{2}{3} - 1 + 4 \right) - \left( \frac{16}{3} - 4 - 8 \right) = \frac{27}{3} = 9 \]
5. The CDC has found that the SARS epidemic in Toronto was spreading so that, if \( P(t) \) was the number of infected persons on day \( t \), then \( P(t) \) was increasing at a rate of \( 80 - \frac{1}{3}t \) persons per day. On day 0 there were 10 infected persons. How many persons were infected after 8 days?

\[
P'(t) = 80 - \frac{1}{3}t \quad \Rightarrow \quad P(t) = \text{antiderivative} = 80t^{\frac{3}{4}} + C
\]

\[
P(0) = 10 \quad \Rightarrow \quad 10 = C
\]

\[
P(t) = 80t^{\frac{3}{4}} + 10 \quad \Rightarrow \quad P(8) = 80 \cdot 8^{\frac{3}{4}} + 10
\]

\[
= 640 - \frac{3}{4} \cdot 6 + 10 = 638
\]

6. Calculate \( R_4 \), the right Riemann sum approximation, for \( \int_{-2}^{6} (x^2 - 4x - 5) \, dx \)

"didn't do it this semester"

b. On the diagram below carefully sketch in the rectangles corresponding to your answer to (a)
7. Suppose \( F(x) = 5 + \int_{2}^{x} t^2 f(t) \, dt \) and the line \( y = 3x - 2 \) is tangent to the graph of \( f(x) \) at \( x = 2 \).

Determine each of the following:

a. \( F(2) = \frac{5}{4} \)

\[
F(2) = 5 + \int_{2}^{2} t^2 f(t) \, dt = 5
\]

b. \( f(2) = 4 \)

\[ f(2) = \text{value of } \text{tg. line at } x = 2 \]

c. \( f'(2) = 3 \)

\[ f'(2) = \text{slope of } \text{tg. line at } x = 2 \]

d. \( F'(2) = 16 \)

\[
F'(x) = \frac{d}{dx} \left( 5 + \int_{2}^{x} t^2 f(t) \, dt \right) = x^2 f(x)
\]

\[ F'(2) = 2^2 \cdot f(2) = 4 \cdot 4 = 16 \]

e. \( F''(2) = 28 \)

\[
F''(x) = 2x \cdot f(x) + x^2 \cdot f'(x)
\]

\[
F''(2) = 4 \cdot f(2) + 4 \cdot f'(2)
\]

\[ = 4 \cdot 4 + 4 \cdot 3 = 16 + 12 = 28 \]
8. Consider the function \( f(x) = 2x^3 - 3x^2 - 36x \).

a. What are all of the critical numbers of \( f(x) \)?

\[
\frac{df}{dx} = 6x^2 - 6x - 36 = 0 \implies x^2 - x - 6 = 0
\]

\[
(x - 3)(x + 2) = 0 \implies x = 3, \quad x = -2
\]

b. Determine the interval(s) on which \( f(x) \) is increasing.

\[
\text{sign of } \frac{df}{dx}:
\]

\[
\begin{array}{cccc}
\text{inc.} & -2 & \text{dec.} & 3 & \text{inc.}
\end{array}
\]

c. Determine the interval(s) on which \( f(x) \) is concave up.

\[
\frac{d^2f}{dx^2} = 12x - 6 = 0 \implies x = \frac{1}{2}
\]

\[
\begin{array}{cccc}
\text{conc down} & \frac{1}{2} & \text{conc. up}
\end{array}
\]

d. Determine the absolute maximum value of \( f(x) \) on the interval \([-1, 5]\).

In this situation, we need to check the end points and the critical numbers inside the given interval:

\[
\begin{array}{c|c|c|c}
\text{end points} & -1 & 31 \\
\hline
5 & -5 \\
\hline
\text{c.ritical} & 3 & -81
\end{array}
\]

The absolute max value of \( f(x) \) on \([-1, 5]\) is \( 31 \).
9. Evaluate the following limits:

a. \( \lim_{{x \to -6}} \frac{x^2 - 36}{x + 6} = \lim_{{x \to -6}} \frac{(x-6)(x+6)}{(x+6)} = \lim_{{x \to -6}} x - 6 = -12 \)

b. \( \lim_{{x \to \infty}} \frac{(1 - 2x)^2 - 17x - 100}{(3x)^2} = \lim_{{x \to \infty}} \frac{4x^2 - 21x - 99}{9x^2} = \frac{4}{9} \)

Since the numerator and denominator have the same degree.

c. \( \lim_{{h \to 0}} \frac{\ln(2+h) - \ln(2)}{h} = \text{derivative of } \ln x \text{ at } x = 2 \)

\( \frac{d}{dx} \ln x = \frac{1}{x} \)

\( \therefore \quad \frac{1}{x} \bigg|_{x=2} = \frac{1}{2} \)