# PRACTICE PROBLEMS ( Chapter 14)

1. Let $C$ be the curve in space parametrized by

$$\vec{r}(t) = (2t+8)\,\hat{i} + (-2t)\,\hat{j} + t^2\,\hat{k} \qquad 0 \le t \le 1$$

(a) Evaluate the line integral

$$\int_C f(x,y,z)\,ds \quad , \qquad \text{where } f(x,y,z) = \sqrt{y+x+4z}$$

(b) Evaluate the work $\int_C \vec{F}\cdot\vec{T}\,ds$ , where

$$\vec{F}(x,y,z) = (x+y)\,\hat{i} + (yz)\,\hat{j} + y^2\,\hat{k}$$

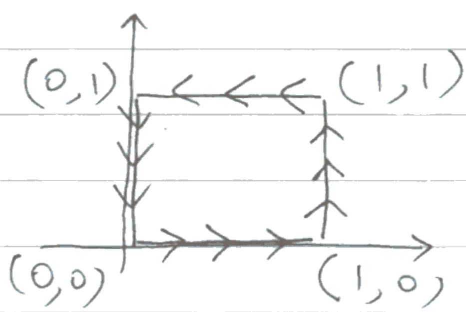
and $C$ is as above.

2. Let $\vec{F} = 2xz\,\hat{i} + z\,\hat{j} + (x^2+y)\,\hat{k}$ be a 3-dimensional vector field.

(a) Show that $\vec{F}$ is conservative.

(b) Find the work done by $\vec{F}$ over any path from $(0,1,1)$ to $(1,2,3)$.

3. Let $\vec{F} = xy^2\,\hat{i} + 3x\,\hat{j}$ be a 2-dimensional vector field. Use Green's theorem to calculate the counterclockwise circulation of $\vec{F}$ around the unit square, as shown.



4. Find the work done by the force

$$\vec{F}(x,y,z) = xy\,\hat{i} + yz\,\hat{j} + xz\,\hat{k}$$

over the path consisting of the line segment from the point $(1,1,0)$ to the point $(1,0,1)$ followed by the line segment from $(1,0,1)$ to $(1,1,1)$.

5. Evaluate $\int_C \sqrt{x^2+y^2}\,ds$ along the curve $\vec{r}(t) = 4\cos t\,\hat{i} + 4\sin t\,\hat{j} + 3t\,\hat{k}$ where $-2\pi \le t \le 2\pi$.

6. Consider the vector field

$$\vec{F}(x,y,z) = 2x\,\hat{i} + (2y+z)\,\hat{j} + (y+1)\,\hat{k}$$

(a) Show that $\vec{F}$ is a conservative vector field.

(b) Find a potential function $f$ for the field $\vec{F}$.

7. Evaluate $\displaystyle\int_{(1,1,1)}^{(0,3,2)} ye^x\,dx + e^x\,dy + 2z\,dz.$

8. Use Green's theorem to find the work done by

$$\vec{F}(x,y) = (3x-y)\,\hat{i} + (x-2y)\,\hat{j}$$

in moving a particle once counterclockwise around the space $C$, where $C$ is the boundary of the square $0 \le x \le 1$ and $0 \le y \le 1$.

9. Consider the line integral

$$(*) \qquad \int_C (xy^2 + e^{3x})\,dx + (x^2y + 2x + \sin y)\,dy .$$

(a) Compute $(*)$ along the segment from $(0,0)$ to $(2,4)$.

(b) If $C$ is the counterclockwise boundary of the triangular region with vertices $(0,0)$, $(2,0)$ and $(2,4)$, use Green's Theorem to evaluate $(*)$.

10. Consider the vector field

$$\vec{F}(x,y,z) = (\cos y - x)\,\hat{i} + (ze^{yz} - x\sin y)\,\hat{j} + ye^{yz}\,\hat{k}.$$

(a) Assuming $\vec{F} = \vec{\nabla} f$, find the potential $f$.

(b) Use $f$ calculated in part (a) to evaluate

$$\int_{(0,0,1)}^{(1,\pi,0)} \vec{F} \cdot d\vec{r}$$

11. Evaluate the line integral using Green's Theorem

$$\oint (xy^2 + 2y + 5)\,dx + (y^3 \ln y + x^2 y)\,dy$$

in counterclockwise direction around the circle $x^2 + y^2 = 9$.

12. Find the work done by the force

$$\vec{F}(x,y,z) = (y + 2xz)\,\hat{i} + x\,\hat{j} + x^2\,\hat{k}$$

over the straight line path from $(0,0,0)$ to $(1,1,1)$.

13. Consider the vector field

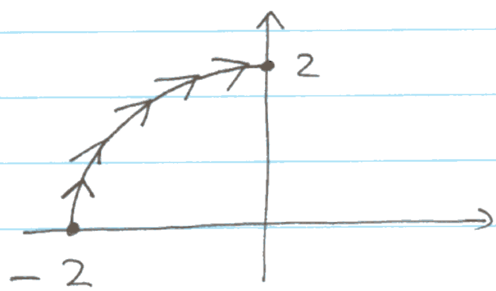
$$\vec{F}(x,y,z) = \left(2xy + e^{z-x}\right)\hat{i} + (x^2 - \sin y)\,\hat{j} - e^{z-x}\,\hat{k}.$$

(a) Find a potential function for $\vec{F}(x,y,z)$.

(b) Find the exact value of $\int_C \vec{F} \cdot d\vec{r}$

along the straight path from $(0,0,0)$ to $(1,\pi,1)$.

14. Let $C$ be the arc of the circle of radius 2 as shown in the figure




Find $$\int_C xy\,dx + dy .$$