MA 213 – Sec. 01/02 – Spring 2003 PRACTICE EXAM – A. Corso	The exam is on Wednesday February 12 CP 103 — 6:00-8:00pm	Please, be neat and show all your work!

1. Find the equation of the tangent line to the curve

$$x = t^2 \qquad y = t^3$$

at the point $t = 2$, without eliminating the parameter.

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2. Find the work done by the force $\mathbf{F} = 3\mathbf{i} + 10\mathbf{j}$ newtons in moving an object 10 meters north (i.e., in the $\mathbf{j}$ direction).

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3. $\lim\limits_{t \longrightarrow \infty} \left[\dfrac{t \sin t}{t^2}\,\mathbf{i} - \dfrac{7t^3}{t^3 - 3t}\,\mathbf{j} \right]$

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4. Consider the vector $\mathbf{a} = 3\mathbf{i} - 5\mathbf{j} + \mathbf{k}$. Find a vector $\mathbf{b}$ with the same direction as $\mathbf{a}$, opposite orientation and length 6.

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5. Consider the points $A(1,2,0)$, $B(3,0,-4)$, the midpoint M of the segment AB and the origin O. Find

   (a) the component form of $\vec{AB}$;

   (b) the component form of $\vec{OA}, \vec{OB}, \vec{OM}$;

   (c) the coordinates of M.

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6. Let $\mathbf{u} = 8\mathbf{i} + 4\mathbf{j} - 12\mathbf{k}$ and $\mathbf{v} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$. Compute

   (a) $3\mathbf{u} - 2\mathbf{v}$;

   (b) $\mathbf{u} \cdot \mathbf{v}$;

   (c) $\cos\theta$, where θ is the angle between the two vectors;

   (d) $\mathrm{proj}_{\mathbf{v}}(\mathbf{u})$.

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7. Let $\mathbf{a} = -5\mathbf{i} + \mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + 10\mathbf{j} + \sqrt{17}\mathbf{k}$. Find $\mathbf{a} \cdot \mathbf{b}$, $\mathbf{a} \times \mathbf{b}$, $|\mathbf{a}|$, $\mathrm{proj}_{\mathbf{a}}(\mathbf{b})$ and the angle between $\mathbf{a}$ and $\mathbf{b}$. Find a vector perpendicular to $\mathbf{a}$. Find a vector parallel to $\mathbf{a}$ that has length 10.

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8. Give the area of the parallelogram formed by the two vectors

$$\mathbf{a} = 2\mathbf{i} + \mathbf{j} \qquad \mathbf{b} = \mathbf{i} - 2\mathbf{k}.$$

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9. Let $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j}$, $\mathbf{b} = \mathbf{i} + 3/2\mathbf{j} - \mathbf{k}$ and $\mathbf{c} = -\mathbf{i} + \mathbf{j} + \mathbf{k}$.

   (a) find a unit vector $\mathbf{n}$ that is normal to a plane determined by $\mathbf{a}$ and $\mathbf{b}$ (any one of many such planes) such that $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{n}$ (in that order) form a right-hand system;

   (b) find $\mathrm{proj}_{\mathbf{a}}(\mathbf{c})$;

   (c) if $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are placed end-to-end they form three sides of a parallelepiped. Find the volume of this parallelepiped.

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10. In each part below, find all possible values of the scalar c, or explain why no such value exists.

(a) Is there a value of c such that the vector $\mathbf{i}-\mathbf{j}+c\mathbf{k}$ is perpendicular to the vector $2\mathbf{i}-\mathbf{k}$?

(b) Is there a value of c such that the vector $\mathbf{i}-\mathbf{j}+c\mathbf{k}$ is parallel to the vector $2\mathbf{i}-\mathbf{k}$?

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11. Find the area of the triangle determined by the points $(-2,1,0),(2,0,4)$ and $(-1,-1,3)$.

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12. Write the equations of the plane determined by the points $O(0,0,0), A(1,1,0)$ and $B(-1,1,1)$.

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13. Find the distance from the origin to the plane $x+y+z-2=0$.

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14. Find the equation of the sphere centered at $C(1,1,2)$ and tangent to the plane $3x-4y+5z=4$.

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15. Give the distance from the point $P(2,3,4)$ to the plane whose equation is $x+y+z=3$.

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16. Write the parametric equations of the line parallel to the z-axis passing through the point $(1,1,1)$.

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17. Give the parametric equations for the line through the points $P(2,3,4)$ and $Q(3,2,1)$.

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18. Give the distance from the point $P(1,1,1)$ to the line which goes through both the origin and the point $Q(6,1,4)$.

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19. Find parametric equations for the line ℓ_2 which both lies on the plane with equation $2x+3y+z=6$ and is perpendicular to the line ℓ_1, where ℓ_1 has the parametric equations

$$x=1+t, \quad y=1+2t, \quad z=1+3t.$$

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20. Let $P(-1,0,5)$, $Q(3,-1,-2)$ and $\mathbf{v}=2\mathbf{i}-3\mathbf{j}+2\mathbf{k}$.

(a) Write parametric equations for the line parallel to $\mathbf{v}$ passing through P.

(b) Determine the (exact) distance from Q to the line described in part (a).

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21. Find the parametric equations of the line through the point $P(3,-3,1)$ and parallel to the line of intersection of the planes $2x+y+z=4$ and $3x-y+z=3$.

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22. Consider the plane $x-2y+z=2$.

(a) Find the point P of intersection of this plane and the line

$$x=5+t, \quad y=5+2t, \quad z=-5-t,$$

where $-\infty<t<\infty$.

(b) Find the line that is normal to the given plane and passes through the point P found in (a).

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23. Find the equation of the line perpendicular to $5x-3y+z=5$ through the point $Q(1,2,3)$. Compute the point of intersection between this line and the plane.

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24. The position vector $\mathbf{r}(t)$ of a particle is given by

$$\mathbf{r}(t) = \sin t\,\mathbf{i} + t\,\mathbf{j} + \cos t\,\mathbf{k}.$$

Find

(a) the velocity $\mathbf{v}(t)$;

(b) the acceleration $\mathbf{a}(t)$;

(c) the distance travelled by the particle between $t=0$ and $t=2\pi$.

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25. Determine the position vector $\mathbf{r}(t)$ satisfying the initial value problem

$$\frac{d^2\mathbf{r}}{dt^2} = 4e^{2t}\,\mathbf{i} - t\,\mathbf{j},$$

subject to the initial conditions

$$\mathbf{v}(0) = 2\mathbf{i} + \mathbf{j} - \mathbf{k} \qquad \mathbf{r}(0) = 3\mathbf{k}.$$

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26. Find the length along the curve

$$\mathbf{r}(t) = (3/10t^2-1)\mathbf{i} + 2/5t^2\mathbf{j} + (2t+1)^{3/2}\mathbf{k} \qquad t \geq -1/2$$

from the point $(-1,0,1)$ to the point $(19/5, 32/5, 27)$.

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27. Consider the position vector

$$\mathbf{r}(t) = (-2t^2+1)\,\mathbf{i} + \ln(2+t^2)\,\mathbf{j} - (e^t - t)\,\mathbf{k}$$

of a particle moving in space. Find the particle's speed at time $t=3$.

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28. Set up but do not evaluate the integral to find the arc length of the curve

$$\mathbf{r}(t) = 3\sin(t^2)\,\mathbf{i} + 2\cos(3t)\,\mathbf{j},$$

for $t \in [0, \pi/2]$.

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29. Find parametric equations for the line that is tangent to the curve

$$\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + \sin(2t)\,\mathbf{k}$$

at $t_0 = \pi/2$. Find the unit tangent vector $\mathbf{T}(t)$. Set up an integral to compute the arc length of this curve for $\pi/2 \leq t \leq \pi$.

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30. Let

$$\mathbf{v}(t) = t^2\,\mathbf{i} + \mathbf{j} + t\,\mathbf{k}$$

be the velocity of a particle. Compute the position vector $\mathbf{r}(t)$, using the fact that $\mathbf{r}(1) = \mathbf{i}$.

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31. Find the curvature, the unit tangent vector, the principal normal, and binormal to the curve

$$\mathbf{r}(t) = e^{-2t}\,\mathbf{i} + e^{2t}\,\mathbf{j} + 2\sqrt{2}t\,\mathbf{k}$$

at the point $t = 0$.

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32. Find the tangential and normal vector components a_T and a_N of the acceleration vector at any time if

$$\mathbf{r}(t) = (t - 1/3t^3)\,\mathbf{i} - (t + 1/3t^3)\,\mathbf{j} + t\,\mathbf{k}.$$

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33. Name the surface represented by each of the following equations

(a) $4x^2 + 9y^2 + 4z^2 = 36;$
(b) $y = -x^2 - z^2;$
(c) $4x^2 + 9z^2 = y^2;$
(d) $y^2 - 4x^2 - z^2 = 1;$
(e) $y = -x^2 + z^2;$
(f) $x^2 + z^2 = 1.$

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34. Consider the hyperboloid of two sheets

$$\frac{x^2}{4} - y^2 - z^2 = 1.$$

Give a geometric description of the cross sections of this quadric surface that are parallel to the

(a) xy-plane;
(b) yz-plane. Indicate which x-values correspond to an empty cross section.

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35. Make a rough sketch of the surface which satisfies the equation

$$x^2 - y^2 + z^2 = 1.$$
