Practice Test — 2nd Exam on Thursday Oct. 30, 1997 — Today's Date: Oct. 24,1997

Chapter 2 (everything) — Time: 9:00-10:15 am during regular class hour

Introductory Analysis I (Mathematics 223 – Fall 1997)

Exercise #1: If $f(x) = \dfrac{x+1}{x^2}$, find $f''(x)$.

Problem #2: Find the equation of the line tangent to the graph of $y = \sqrt[3]{x^5} - \dfrac{1}{\sqrt{x}}$ at $x = 1$.

Exercise #3: If $(x^2+1)^3y^3 = 8x$, evaluate $\dfrac{dy}{dx}$ at $x = 1$.

Exercise #4: Compute $\lim\limits_{\Delta x \longrightarrow 0} \dfrac{f(1+\Delta x) - f(1)}{\Delta x}$ when $f(x) = (x^2+1)^4$.

Exercise #5: Estimate the percentage change in the function $f(x) = 2x^2 - 6x + 7$ as x increases from 4 to 4.3. Use approximation by differentials. (Do <u>not</u> find <u>exact</u> value!)

Problem #6: Let $g(x)$ be a function so that $g'(-2) = 5$. If $f(x) = g(x^4 - 3)$, find $f'(-1)$. (Hint: apply chain rule.)

Exercise #7: Find the slope of the tangent to the curve $x^3y^3 + 3xy = x + 3y$ at the point whose x coordinate is 1.

Problem #8:

Given the above graph of $f(x)$, answer true or false. T/F

1. $f'(a) < 0$. ☐
2. If $c < \alpha < d$ then $f'(\alpha) < 0$. ☐
3. $f'(x) > 0$ on (a,b) and $f'(x) < 0$ on $(b,c) \cup (c,d)$. ☐
4. $\lim\limits_{\Delta x \longrightarrow 0} \dfrac{f(d+\Delta x) - f(d)}{\Delta x} > 0$. ☐
5. $\lim\limits_{\Delta x \longrightarrow 0} \dfrac{f(b+\Delta x) - f(b)}{\Delta x} = 0$. ☐

Problem #9: The radius of a spherical balloon is growing at the rate of $4/\pi$ m/min when the radius is 3 m. How fast is the volume of the balloon growing when the radius is 3m?

Problem #10: Let $s(t) = t^3 - 6t^2 + 9t - 2$ be the displacement of an object moving along a line for $0 \leq t \leq 4$.

1. Find the velocity of the object.
2. Find the total distance travelled by the object.
3. Find the acceleration of the object.
4. Determine when the object is decelerating.

Problem #11: Find the simplified form of $\dfrac{f(x+\Delta x) - f(x)}{\Delta x}$, if $f(x) = x^2 - 2x + 4$.

Problem #12: The cost is $C(q) = 1/6q^3 + 492q + 500\$$ when q units are produced. Use marginal analysis to estimate the cost of producing the 4th unit.

Exercise #13: If $f(x) = x + \dfrac{1}{x}$, find $f''(x)$.

Exercise #14: Find the derivative of each of the given functions:

1. $f(x) = x^6 - \dfrac{2}{3\sqrt{x}} + \dfrac{x+\sqrt{2}}{3}$;
2. $f(x) = \dfrac{2x^2+1}{3x^2-1}$;
3. $f(x) = x^{20}(1-2x)^{10}$.

Exercise #15: Find the equation of the line tangent to the graph of $y = \sqrt{2x+2}$ at $x = 1$.

Exercise #16: Given $2x^2y + xy^3 = 2$.

1. Use implicit differentiation to find y';
2. Find the equation of the tangent line at $P(1,1)$.

Exercise #17: It is estimated that t years after 1997 the population of a certain town will be $P(t) = t^2 + 200t + 8,000$.

1. Use calculus to estimate how much will the population increase during the first half of the year 2002?
2. What is the percentage of the population increase in (a)?

Exercise #18: A ladder of length 10 meters is sliding along two perpendicular surfaces. The bottom of the ladder slides at a constant speed of 4 meters/sec. How fast is the top sliding when $x = 2y$?