MTH 234 - 02/16/99 FIRST MIDTERM	Spring 99 A. Corso and A. Minut	Name: ________________________ ________________________

PLEASE, BE NEAT AND SHOW ALL YOUR WORK; CIRCLE YOUR ANSWER.

Problem Number	Possible Points	Points Earned
1	20	
2	12	
3	12	
4	12	
5	8	
6	14	
7	12	
8	10	
TOTAL	100	

1. Let $\mathbf{u} = -4\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$. Compute

(a) $3\mathbf{u} - 2\mathbf{v} =$ __________________________;

(b) $\mathbf{u} \cdot \mathbf{v} =$ _____________;

(c) $\cos\theta =$ _____________, where θ is the angle between the two vectors;

(d) $\mathrm{proj}_{\mathbf{v}}(\mathbf{u}) =$ __________________________;

(e) Find a third vector $\mathbf{n} =$ __________________________ that is normal to the plane determined by $\mathbf{u}$ and $\mathbf{v}$ such that $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{n}$ form a right-hand system.

pts: /20

2. In each part below, find all possible values of the scalar c, or explain why no such value exists.

   (a) Is there a value of c such that the vector $\mathbf{a}_c = -2\mathbf{i} + 2\mathbf{j} + c\mathbf{k}$ is perpendicular to the vector $\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$?

   (b) Is there a value of c such that the vector $\mathbf{a}_c = -2\mathbf{i} + 2\mathbf{j} + c\mathbf{k}$ is parallel to the vector $\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$?

pts: /12

3. Find parametric equations for the line ℓ_1 which both lies on the plane with equation $2x+3y+z=6$ and is perpendicular to the line ℓ_2, where ℓ_2 has the parametric equations

$$x = 1+t, \qquad y = 1+2t, \qquad z = 1+3t.$$

pts: /12

4. (a) Give the distance from the point $P(1,1,3)$ to the plane whose equation is

$$3x+2y+6z=6.$$

(b) Give the distance from the point $Q(1,1,5)$ to the line whose parametric equations are

$$x=1+t, \qquad y=3-t, \qquad z=2t.$$

pts: /12

5. Consider the hyperbolic paraboloid

$$z = \frac{x^2}{4} - y^2$$

Give a geometric description of the cross sections of this quadric surface that are parallel to the

(a) xy-plane;

(b) yz-plane.

pts: /8

6. Let

$$\mathbf{a}(t) = -2\mathbf{i} - 4\mathbf{j}$$

be the acceleration of a particle. Compute the velocity and the position vector

$$\mathbf{v}(t) = \underline{\hspace{6cm}},$$

$$\mathbf{r}(t) = \underline{\hspace{6cm}},$$

using the fact that

$$\mathbf{v}(1) = 4\mathbf{i} \qquad \mathbf{r}(1) = 3\mathbf{i} + 3\mathbf{j}.$$

pts: /14

7. (a) Translate the equation given into spherical coordinates

$$\rho = -6\cos\varphi$$

into an equation in rectangular coordinates. Also, identify the set of points defined by the equation.

(b) Describe the sets of points in the space whose spherical coordinates satisfy the following conditions

$$\varphi = \frac{\pi}{4} \qquad\qquad 0 \le \rho \le \sqrt{2}.$$

pts: /12

8. Find the arc length parameter

$$s = \int_0^t |\mathbf{v}(\tau)|\, d\tau$$

along the curve given by

$$\mathbf{r}(t) = (\cos t + t \sin t)\,\mathbf{i} + (\sin t - t \cos t)\,\mathbf{j}.$$

Reparametrize the curve using the arc length parameter.

pts: /10