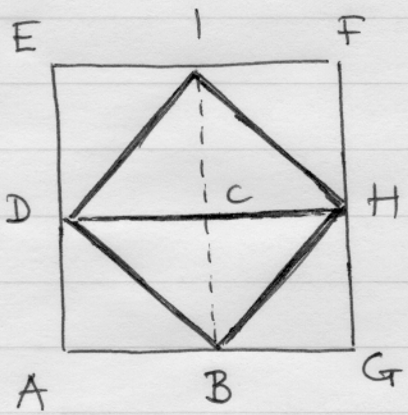
43)

It is Aristotle who hints about the nature of the first proof of the incommensurability of the diagonal of a square and its side.

He writes that "if it is assumed to be commensurable, then odd numbers will be equal to even."[4]

One possible form of the original argument looks as follows:



Assume that the side BD and diagonal DH are commensurable (≡ each is represented by the number of times it is measured by their common measure).

It may be <u>assumed</u> that <u>at least</u> <u>one</u> <u>of these numbers is odd</u> (if not there would be a larger common measure!)

The squares DBHI and AGFE represent square numbers ($\overline{DB}^2$ and $\overline{DH}^2$ respectively).

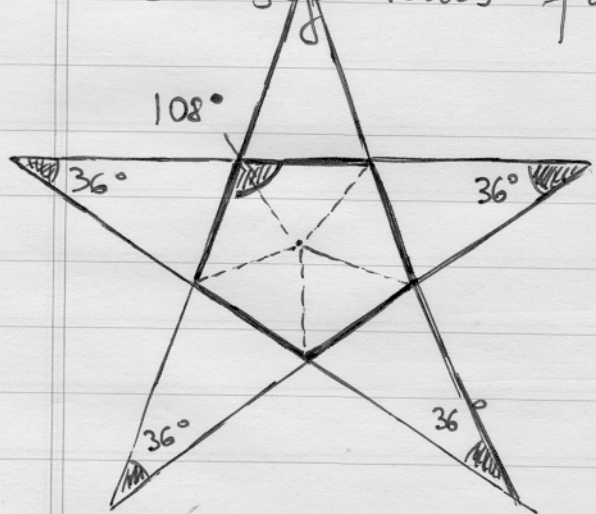
The latter square is clearly the double of the former, so it represents an even square number. Therefore its side $\overline{AG} = \overline{DH}$ represents an even number.

Thus the square AGFE is a multiple of 4. Since DBHI is half of AGFE, it must be a multiple of 2.

Thus DBHI represents an even square. Hence, its side must also be even.

But this contradicts that either $\overline{DH}$ or $\overline{BD}$ must be odd. $\blacksquare$

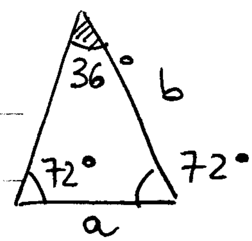
The holy symbol of the Pythagoreans was the pentagram (or 5-pointed star). They found the relationships between the segments fascinating.



Notice that the internal angles in a regular pentagon are 108° each.

Thus the internal sum of the angles is $5 \cdot 108° = 540°$
$= 3 \cdot 180°$

Also notice that the triangles at the tip of a pentagram are isosceles triangles with base angles of 72° and vertex angle of 36°.



such a triangle is called a golden triangle

and the ratio b/a is called "golden ratio"

"golden mean"

"divine proportion"

This ratio is $\frac{1+\sqrt{5}}{2} \approx$

Lots of properties of this ratio are known.

For instance the ratio of the Fibonacci numbers converges to the golden ratio

Fibonacci #s are: 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

Notice that the ratios are:

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \underset{1.615}{\frac{21}{13}}, \underset{1.619}{\frac{34}{21}}, \underset{1.6176}{\frac{55}{34}}, \underset{1.6181}{\frac{89}{55}}, \ldots$$

Later definition: a point is said to divide a segment in a golden section when the longer of the 2 segments is the mean proportion between the short and the entire segment:

$a : b = b : (a+b)$ thus

$$\frac{a}{b} = \frac{b}{(a+b)} \quad \text{or} \quad b^2 = a(a+b)$$

$$\text{or} \quad b^2 - ab - a^2 = 0 \quad \text{or} \quad \left(\frac{b}{a}\right)^2 - \frac{b}{a} - 1 = 0$$

$$\therefore \quad \frac{b}{a} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2} \rightsquigarrow \frac{1+\sqrt{5}}{2}.$$

Another group of critical men, the "sophists", approached problems of a mathematical nature as part of a philosophical investigation of the natural and moral world.
Thus developed a mathematics investigated in the spirit of understanding rather than of utility. This group could be thought of as connected to the democratic movement .... the Pythagoreans were on the other hand more related to the aristocratic faction.

The only complete mathematical fragment of this period is due to Hippocrates of Chios.

Look at the in-class presentation.