The Beginnings

Our knowledge of the very early developments of mathematics is largely speculative, pieced together from archeological fragments, architectural remains, and educated guesses.

Clearly, with the invention of agriculture in the years 15,000 – 10,000 BC (≡ New Stone Age or Neolithic), humans had to address 2 fundamental concepts of mathematics:

* $\boxed{\text{numbers}}$ ⇝ algebra
  - this was related to counting animals, distributing crops, etc.
  - the first occurrence of numbers was qualitative rather than quantitative. The qualitative aspect can still be detected from the fact that some indigenous populations of Australia still compute higher numbers by addition: $3 = 2+1$, $4 = 2+2$, $5 = 2+3$, etc...
  - the development of commerce helped in crystallizing the number concept.
  - numbers were arranged and bundled into larger units usually by the

use of fingers of one hand (≡ quinary system), of both hands (≡ decimal system) ... and fingers and toes (≡ vigesimal system).

- numerical records were kept by means of strokes on a stick, knots on a string, etc...

- area (of fields), length (of things) ⇝ geometry

- the standard lengths were often taken from parts of the human body: fingers, feet, etc...

This was not an harmonious coexistence. A recurrent feature has been a prevailing tension between arithmetic and geometry. Some times one branch overshadowed the other. Then a new discovery, or a new point of view would turn the tables.

Neolithic ornamentation rejoiced in the revelation of congruences, symmetries, similarities. There were magic numbers (3, 4, 7, ...) and magical figures. Modern numerology is a left over of ancient magical rites.

During the 5th, 4th, 3rd millenia BC newer and more technologically advanced societies evolved from well-established Neolithic communities along the banks of great rivers in Africa and Asia, in subtropical or nearly subtropical regions: Egypt, Mesopotamia, India and China.

Societies evolved and social classes were finally established. Kingdoms were born and the knowledge ended up in the hands of few people.

The Egyptians

We find clear signs of mathematical development in the civilization of ancient Egypt. Emphasis was always on the practical side (motion of the sun, moon, stars, motion of time, use of constellation in navigation required more knowledge of mathematics).

Our major sources of Egyptian mathematics come from

- Moscow Papyrus (contains 25 problems)
- Rhind Mathematical Papyrus ($\approx$ 1650 BC) (contains much older material and it was written by the scribe Ahmes. It was discovered in 1858 by A.H. Rhynd. Contains 85 problems)

- only a few short fragments of other original <u>Egyptian mathematical papyri</u> still remain.

There are archeological records that show that, by 2000 BC, Egyptians had a <u>primitive number system</u>. They used a decimal system with special signs for higher decimal units ... like the Romans (e.g. Romans would write 1878 as MDCCCLXXVIII).
They also had <u>geometric ideas</u> about triangles, pyramides, and the like.

The Egyptian algorithm for multiplication was based on a continual doubling process. To multiply the number b by the number a, the scribe would first write down the pair 1, b. He would then double each number in the pair repeatedly, until the next doubling would cause the first number of the pair to exceed a. Then, after having determined the powers of 2 that add to a, the scribe would add the corresponding multiples of b to get his answer.

<u>Eg.</u>: multiply 11 by 13

*	1	11
	2	22
*	4	44
*	8	88

As $13 = 8+4+1$, the scribe would add $11 + 44 + 88 = 143$ (indeed 13×11).

Interesting was also their calculus of fractions. Fractions are reduced to sums of so-called unit fractions ($\equiv \frac{1}{\text{something}}$).
For instance, the would have special symbols for the unit fractions

$$\overline{10} = \frac{1}{10} \qquad \overline{3} = \frac{1}{3}$$

But they also had a special symbol for $2/3$ say (in our language) $\overline{\overline{3}}$.
The reduction to sums of unit fractions was made possible by tables which gave decomposition of fractions of the type $2/n$. This was the only type of decomposition needed because of their dyadic multiplication.
Eg:

$$2/5 = \overline{3} + \overline{15}$$

$$2/7 = \overline{4} + \overline{28}$$

$$2/15 = \overline{10} + \overline{30}$$

however $2/19 = \overline{12} + \overline{76} + \overline{114}$ instead of $\overline{12} + \overline{57} + \overline{228}$

(actually the was no "+" sign ... simply
$\overline{3}\ \overline{15}$ ... or $\overline{4}\ \overline{28}$ ... or $\overline{10}\ \overline{30}$)

"Problem 3 of the Rhind Mathematical Papyrus:"

it asks how to divide 6 loaves of bread among 10 men. The answer given is that each man gets $\overline{2}\ \overline{10}$ loaves (i.e. $\frac{1}{2}+\frac{1}{10} = 3/5$ ... the correct answer!)

The scribe checks that $\overline{2}\ \overline{10}$ is the correct answer by multiplying it by 10.

Well, in multiplying whole numbers the scribe perform a repeated doubling step. So too in multiplying fractions ... hence he needs to be able to express the double of each unit fraction.

In the bread problem: $\overline{2}\ \overline{10}$ times 10
he writes

	1	$\overline{2}\ \overline{10}$	
*	2	$1\ \overline{5}$	$(2 \cdot \frac{1}{10} = 1/5)$
	4	$2\ \overline{3}\ \overline{15}$	(as $2/5 = \frac{1}{3}+\frac{1}{15}$)
*	8	$4\ \overline{\overline{3}}\ \overline{10}\ \overline{30}$	

As $10 = 2 + 8$, he then adds the fractions

$1\ \overline{5}$ and $4\ \overline{\overline{3}}\ \overline{10}\ \overline{30}$ $\longrightarrow$ 6 !!